End of the Year Seminars 2025

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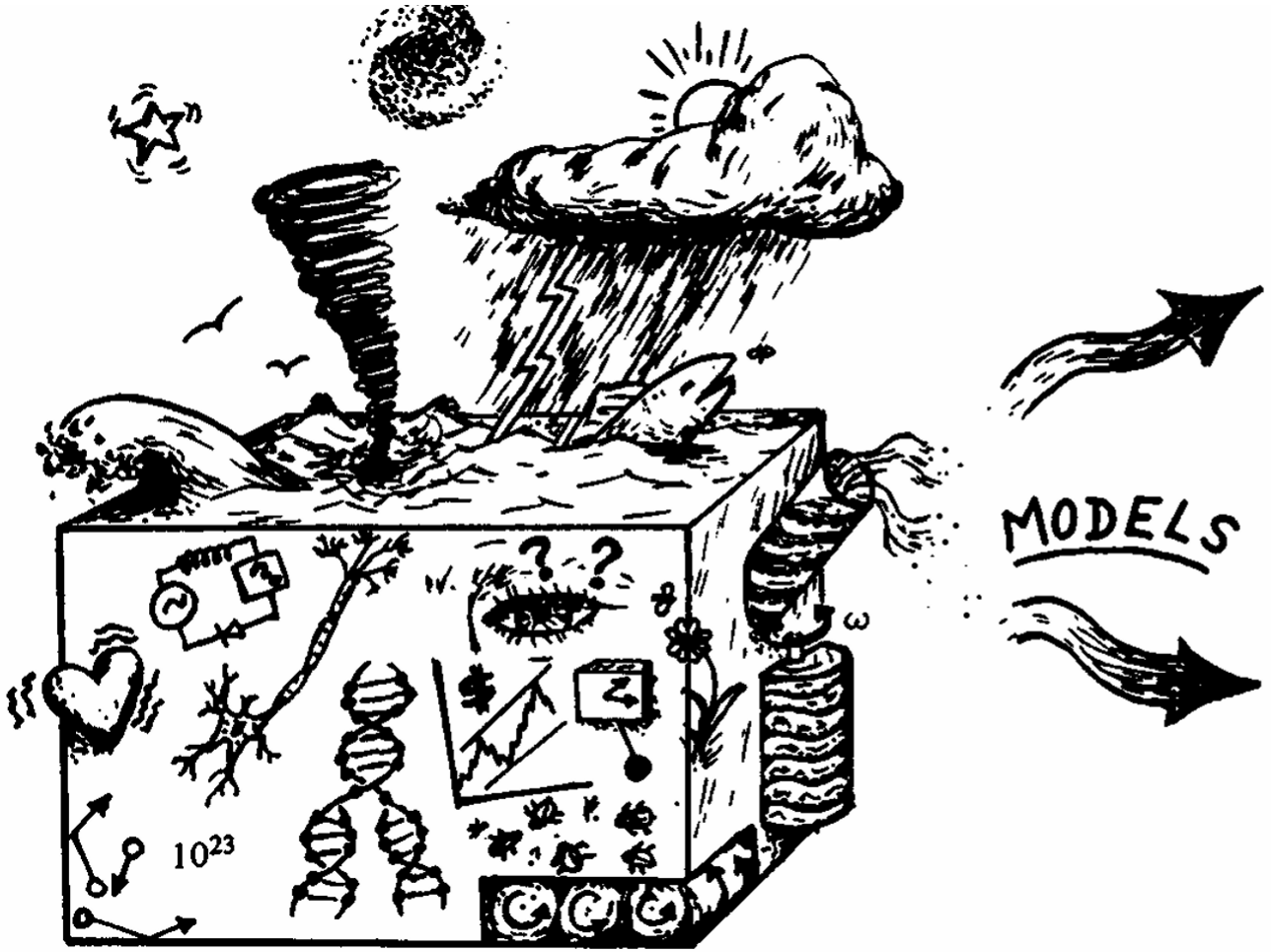
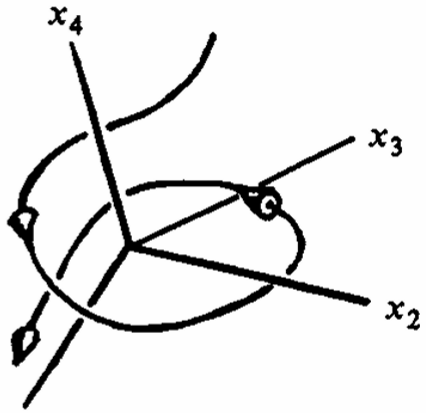
Quite **C**onfused **A**ctually



MODELS

$$\frac{dx_k}{dt} = F_K(x_1, \ldots, x_4) \quad (K = 1, \ldots, 4)$$



$$N(t+1) = CN(t)(1 - N(t))$$

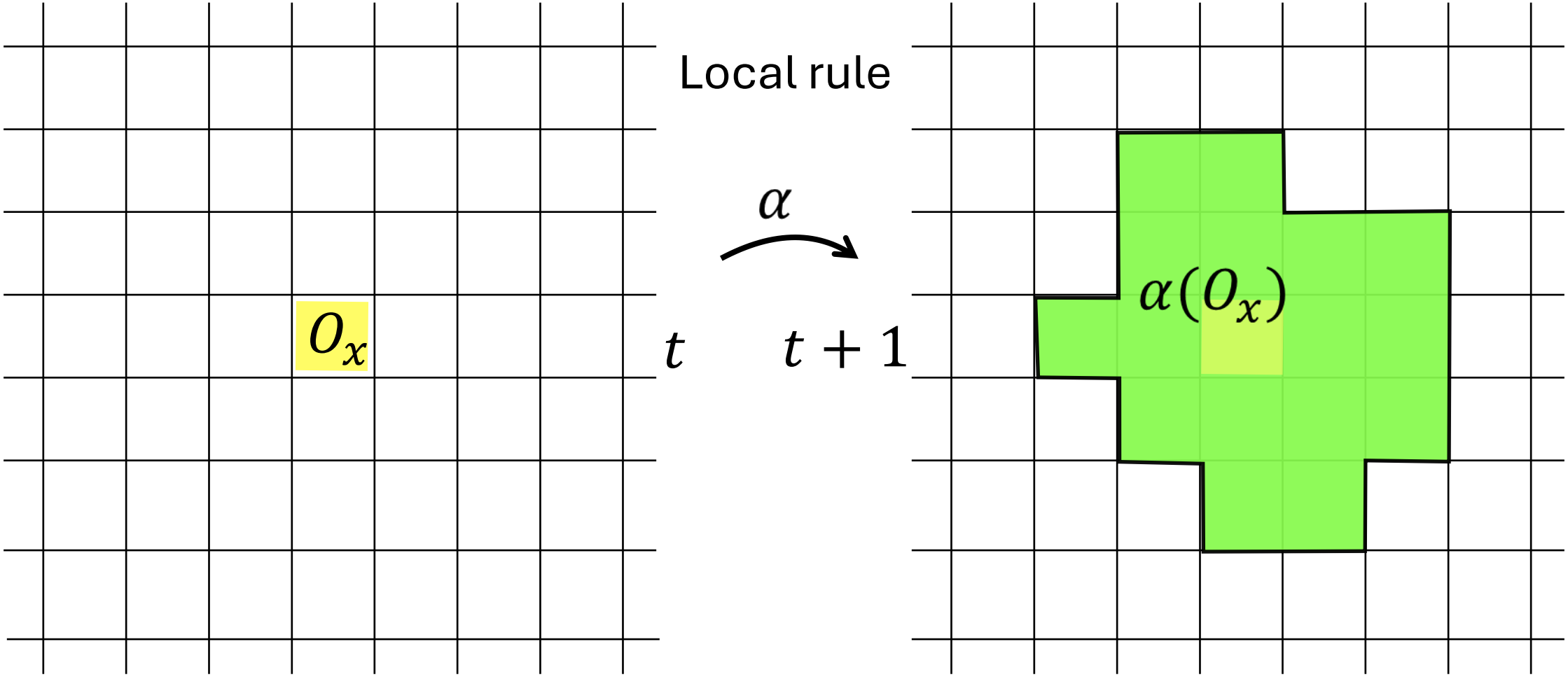
$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^3} = 0$$

$$S_i(t+1) = F(S_{i-1}(t), S_i(t), S_{i+1}(t))$$

ODE
PDE
SDE
FDE
MC
CA
PI
TN
VP
…

Perspective of nonlinear dynamics – E. Atlee Jackson 1989

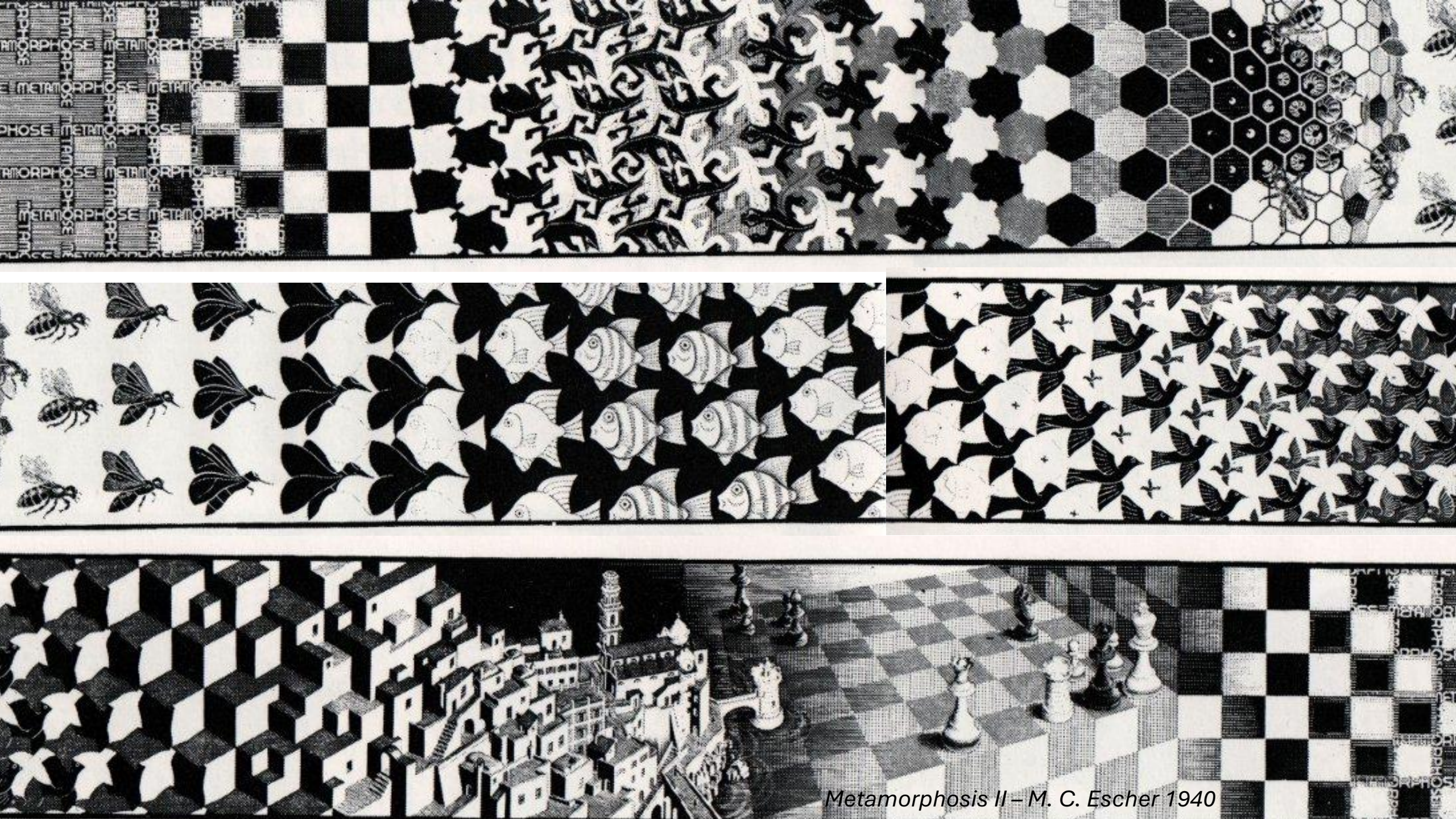
Quantum Cellular Automata



$$\alpha(O_x) = U^{\dagger}(\, O_x \otimes I)\, U$$

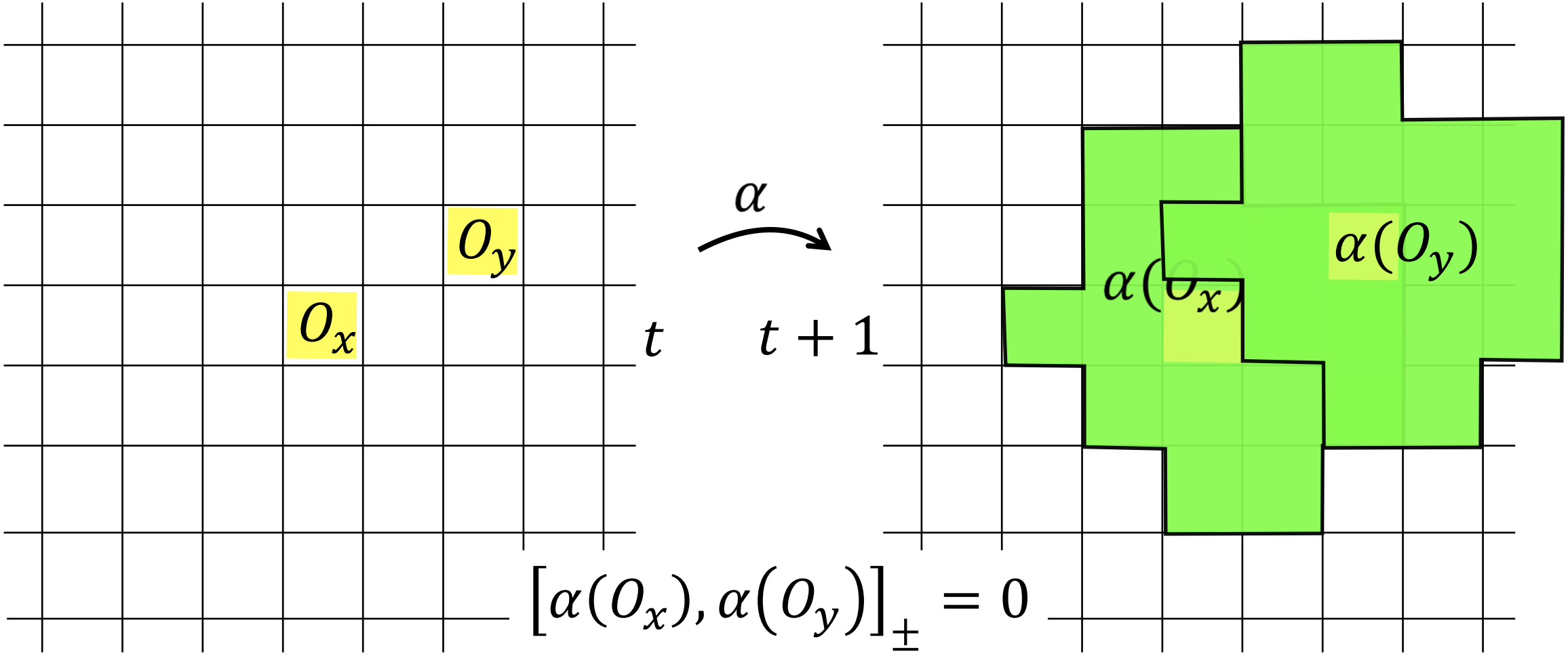
Goal of my PhD:

classify QCA beyond 1d



Metamorphosis II – M. C. Escher 1940

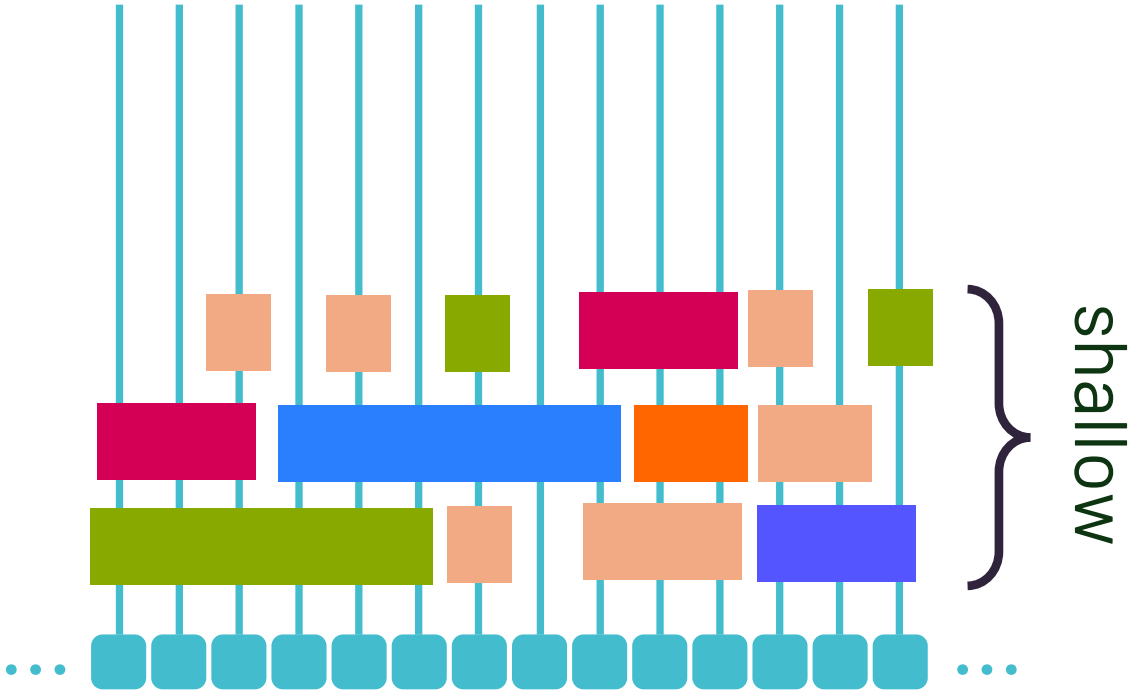
Classify: allowed Rules



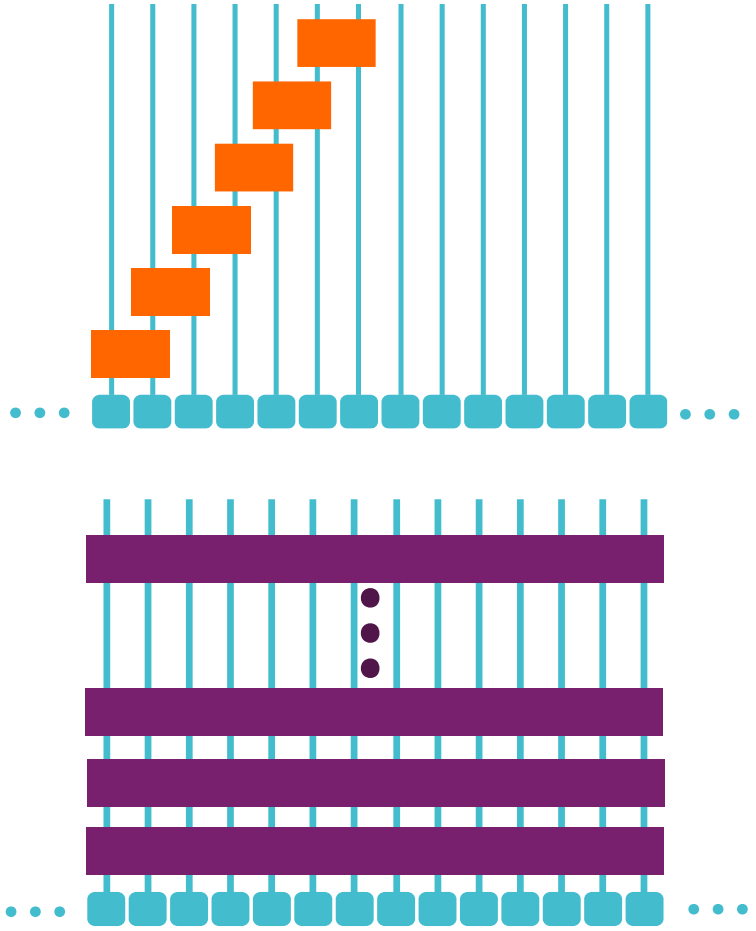
$$[\alpha(O_x), \alpha(O_y)]_{\pm} = 0$$

Classify: QCA modulo circuits

Any circuit is a QCA, trivial.



but, not all QCA are circuits!



Quest for **C**omplete **A**nswers

- **1d** Done-ish. Safe to cite, unsafe to extend.
- **2d** “Define done”, there can be dragons.
- **3d+** Indistinguishable from folklore.

- **Topological order** Still arguing, politely.
- **Interacting QFT** Bless your heart.

Quick **C**heckpoint on **A**dvances

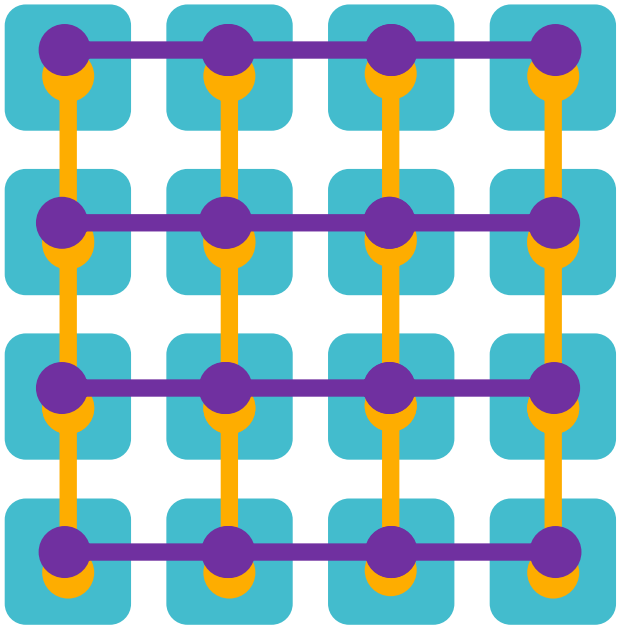
✓ Classified translation-invariant nearest-neighbor QCA on $\mathbb{Z}^d$

- of qubits
- of spinless (Majorana) fermions

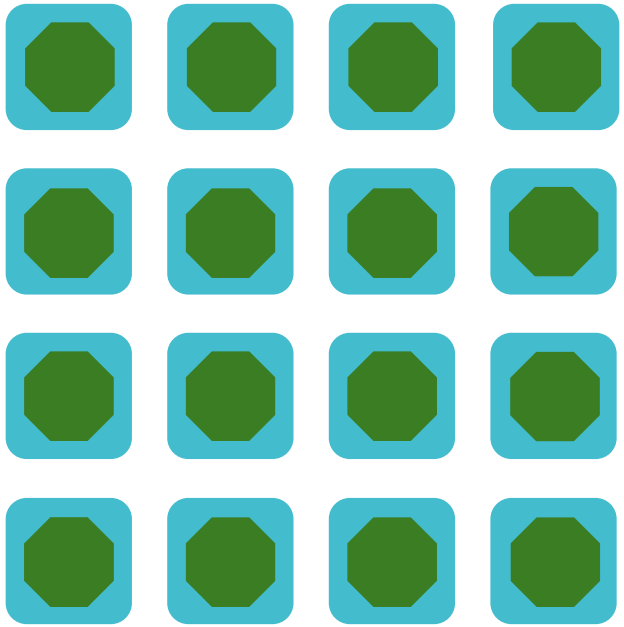
✓ Classification modulo circuits in 2d (no fermions)

Update Rules for Qubits

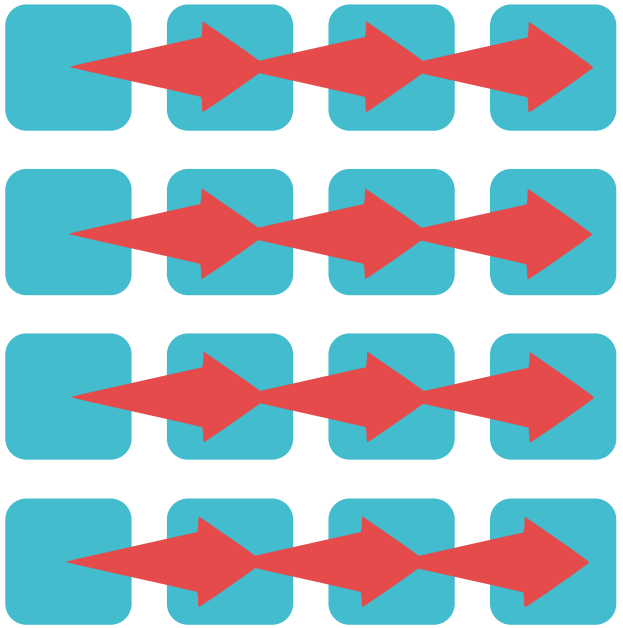
Multiply Controlled-Phase



Site-wise unitary



Shift



+ Update Rules for Majoranas

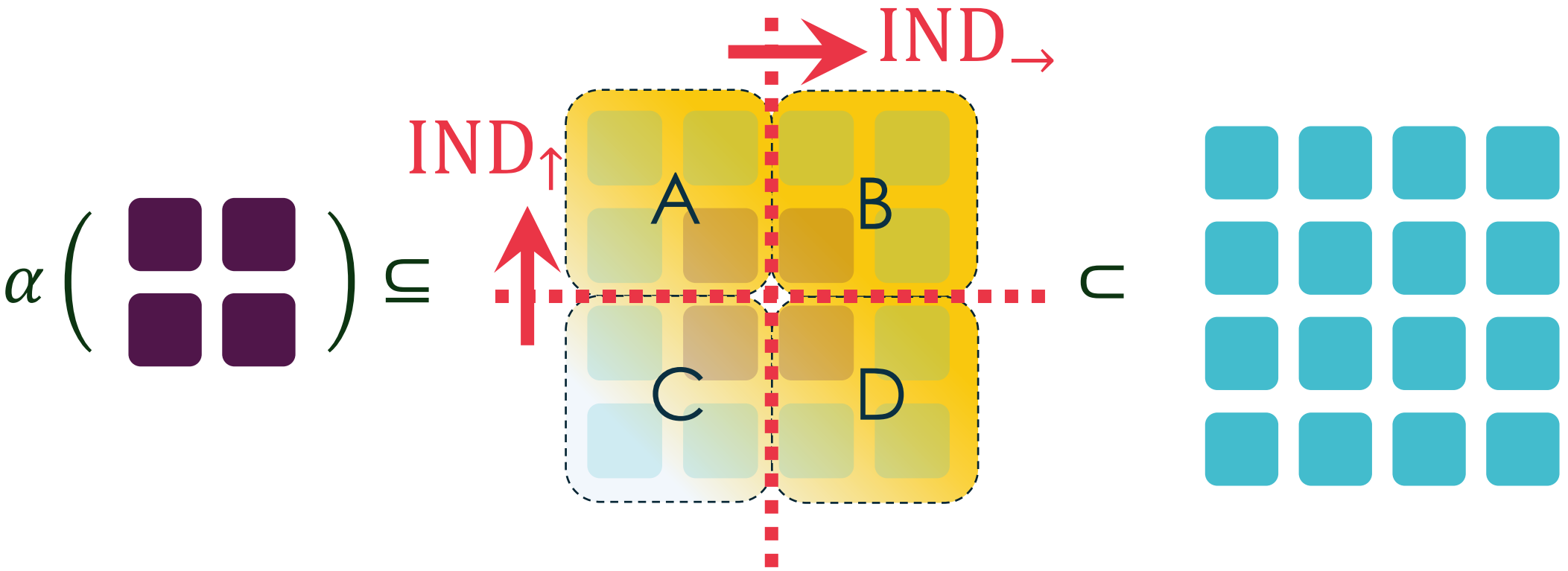
$$X = f^\dagger + f$$

$$Y = i(f^\dagger - f)$$

Majorana Shift

Forking

Index: track information flow

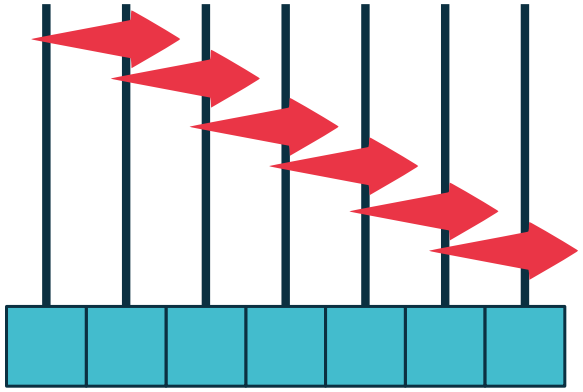


Index ↔ classification mod circuits

Unique Equivalence Classes:

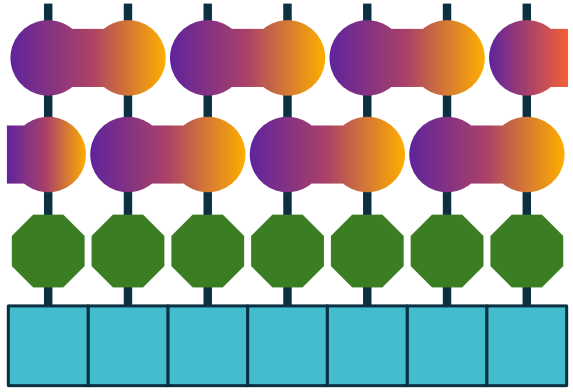
$$\overrightarrow{\mathrm{IND}}(\alpha) \neq \vec{0}$$

Shifts



$$\overrightarrow{\mathrm{IND}}(\alpha) = \vec{0}$$

Shallow Circuits



Non-compact 2d (no fermions)

Any 2d QCA is a circuit followed by an ∞ bunch of shifts

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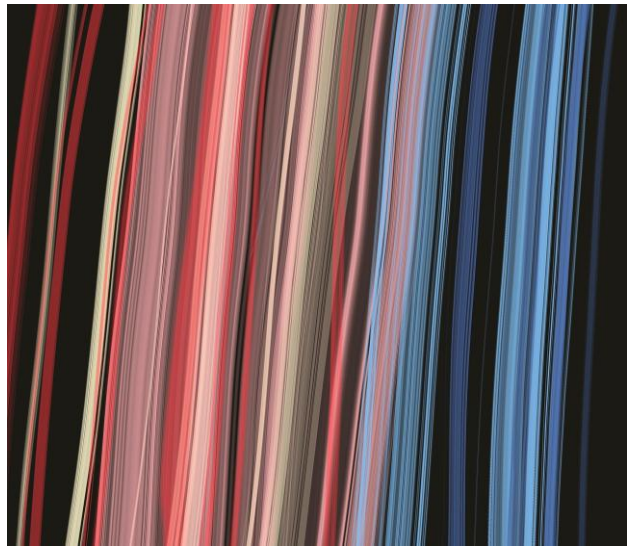
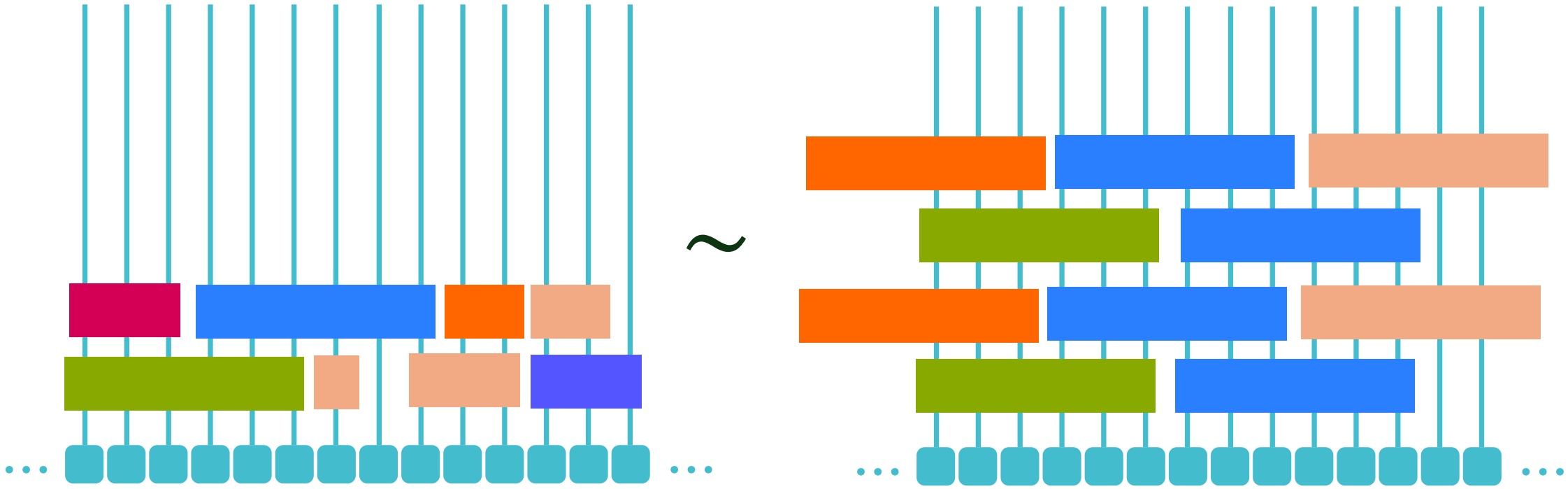


No 1d invariants (unless for transl-inv case)

Non-compact 2d

- *Any* d circuit is equivalent to a regular circuit




Moka Express