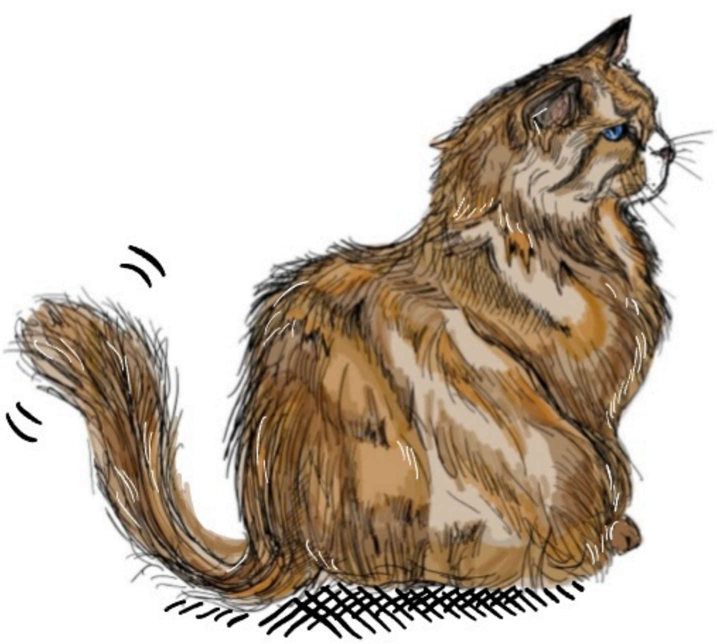
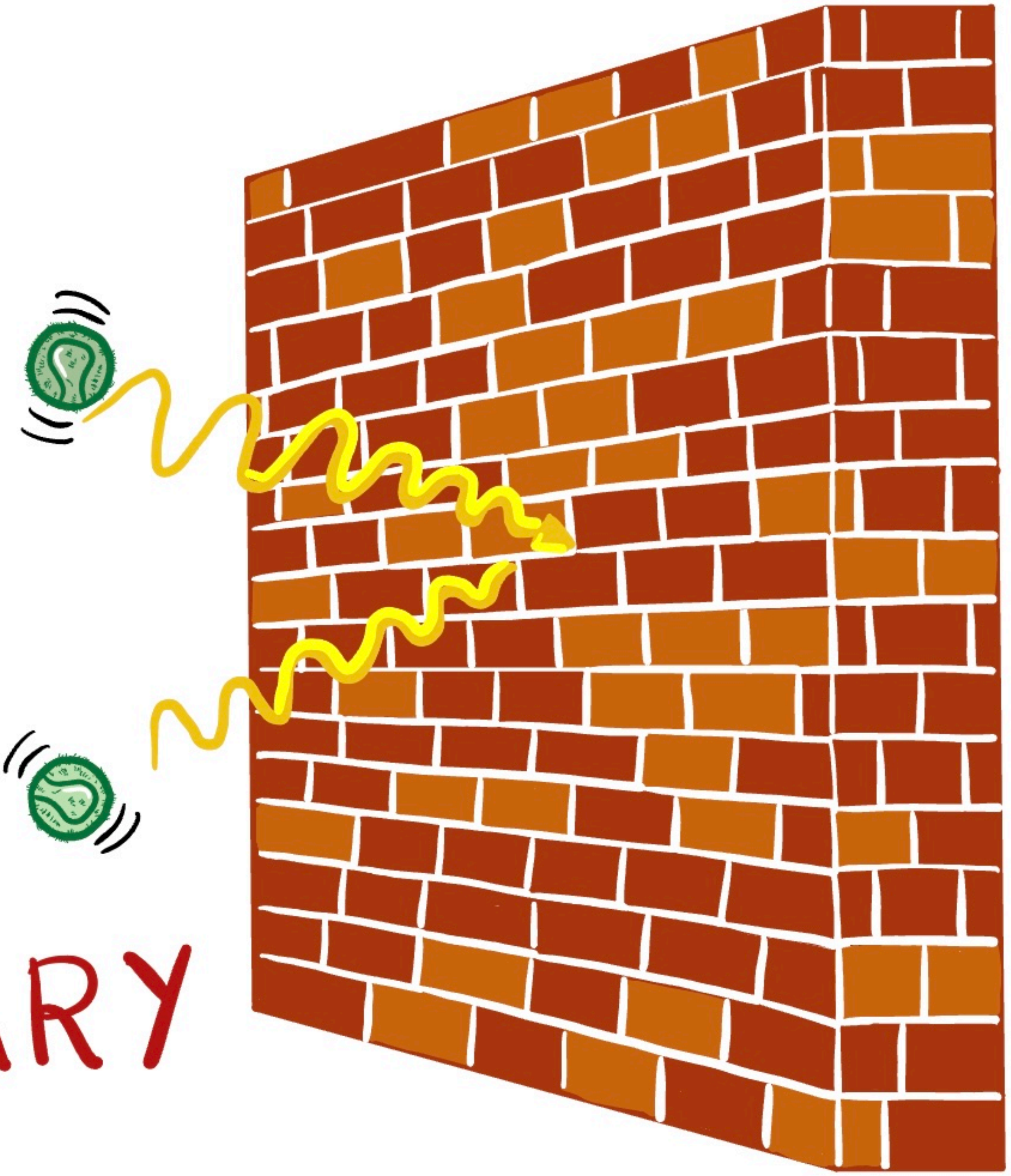
Bouncing OFF the wall: Quantum Fields at the BOUNDARY



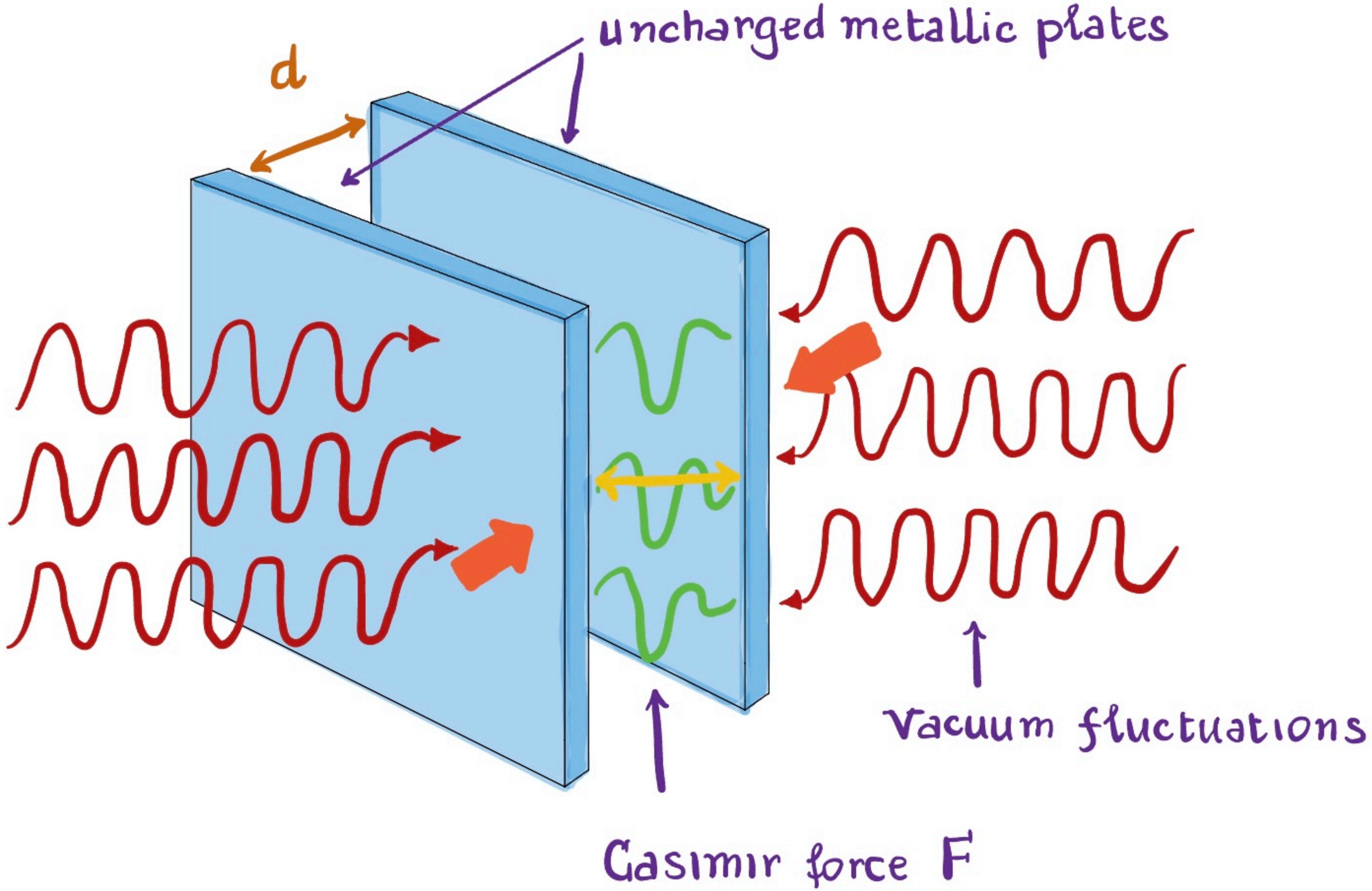


Beatrice Costeri

17.09.25

E.O.Y. seminars

Casimir Effect (I)



Casimir Effect (II)

Hendrik Casimir

Casimir force

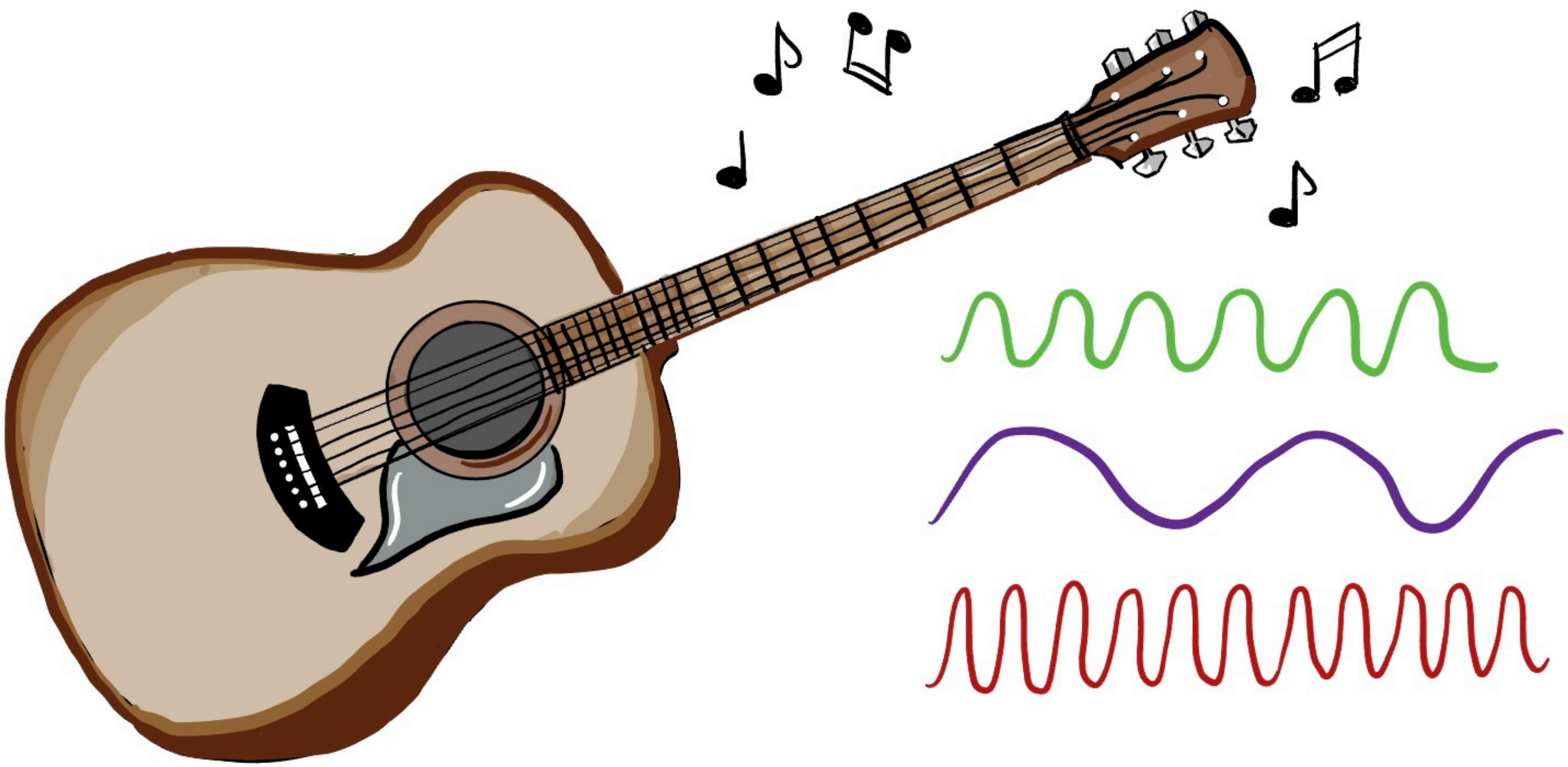
area of the mirrors

$$\boxed{F = \frac{\hbar c \pi^4}{240} \frac{A}{d^4}}$$

distance

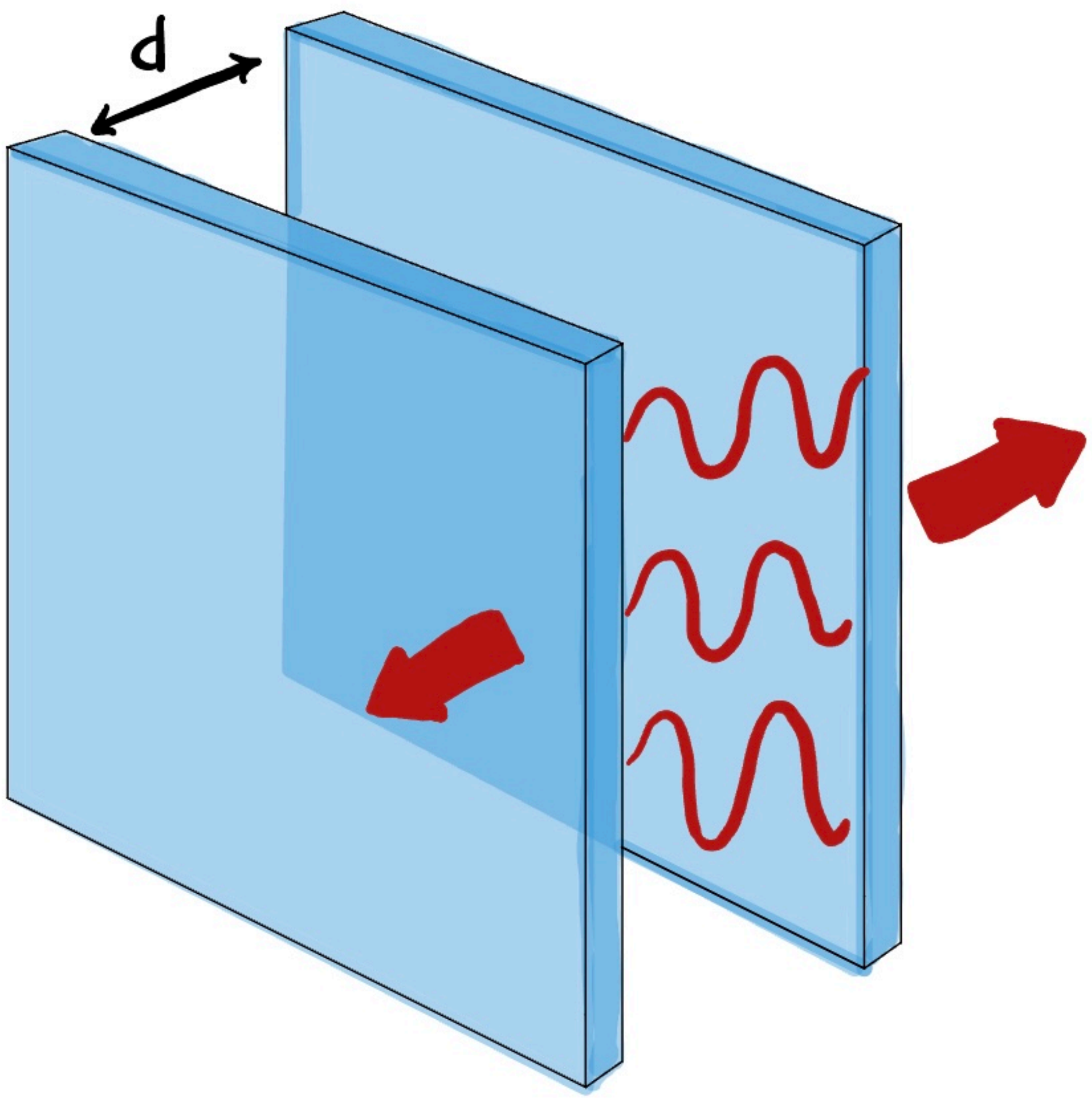
Casimir Effect (III)

Quantum fields $\longleftrightarrow$ ∞ Harmonic oscillators



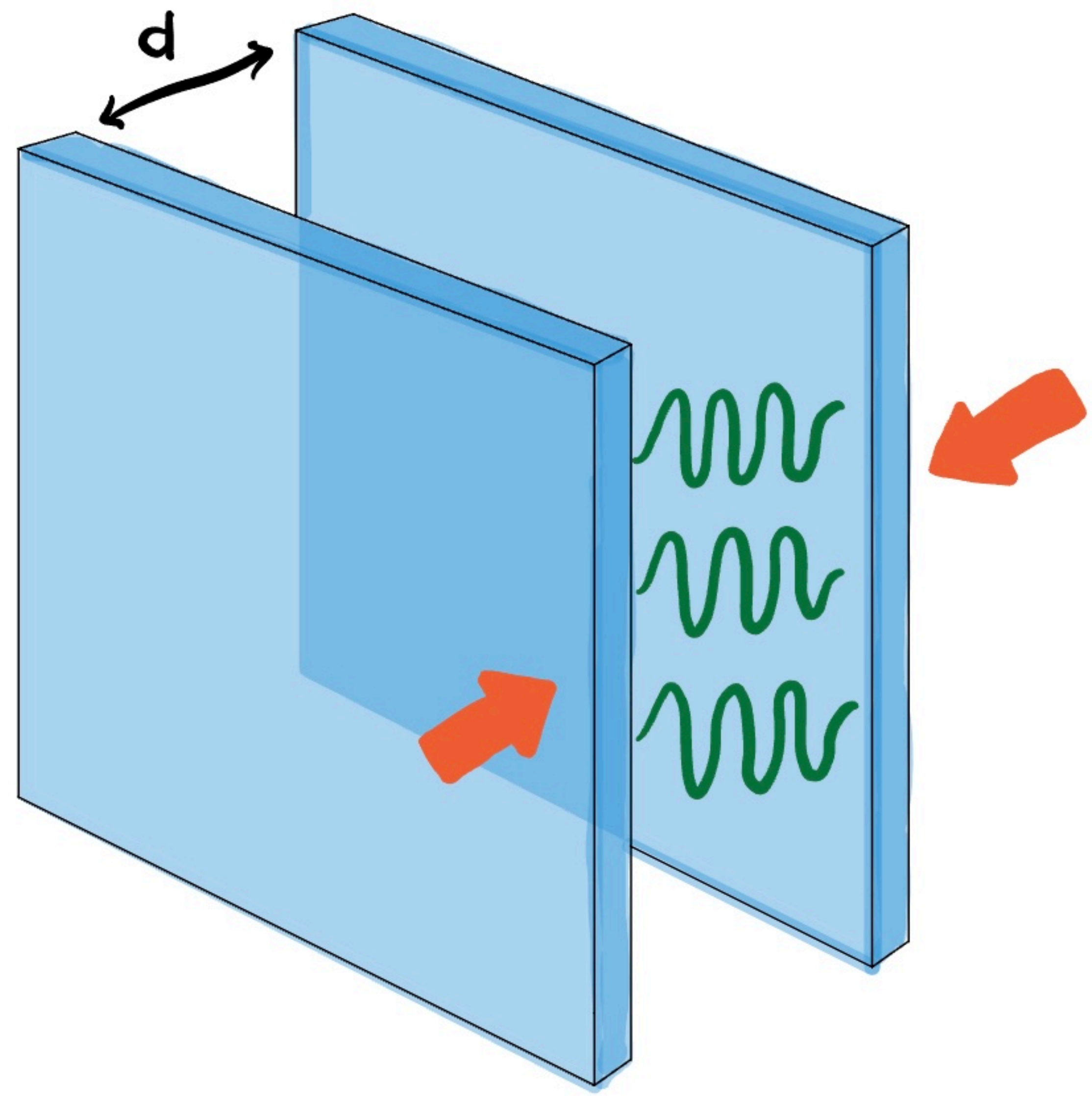
Casimir Effect (IV)

At resonance



$$d = k\,\frac{\lambda}{2}, \quad k \in \mathbb{N}$$

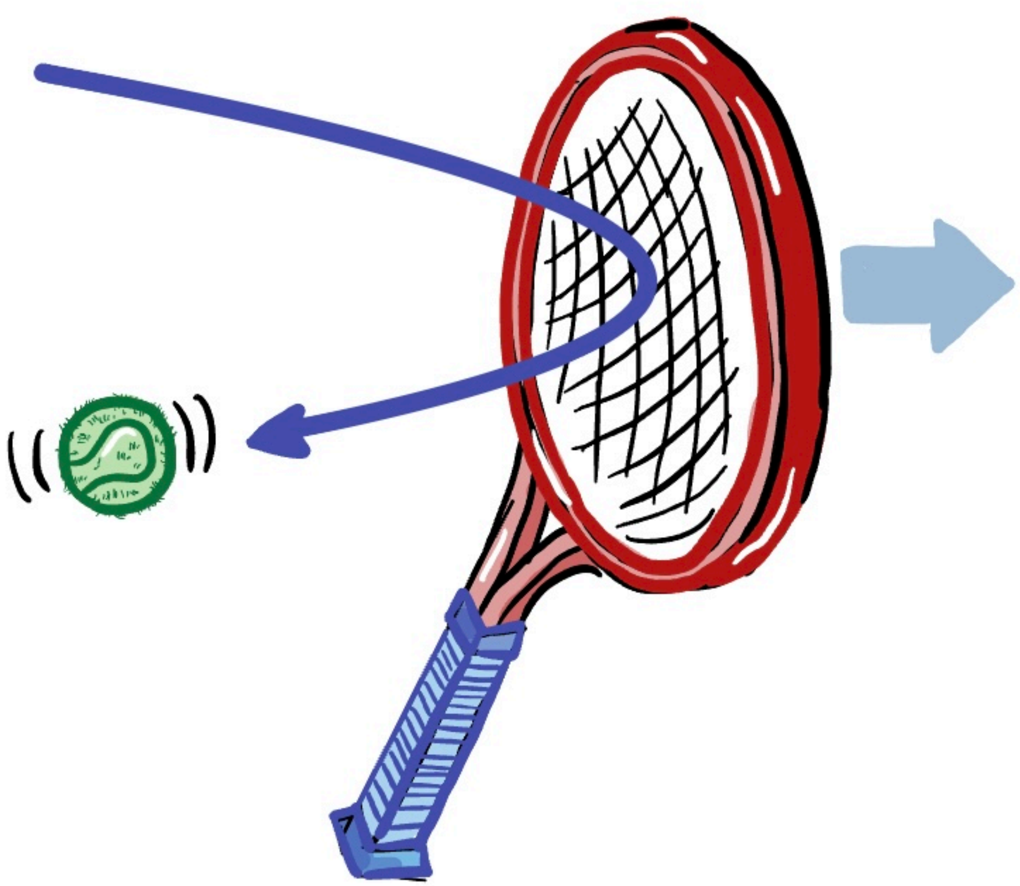
Out of resonance

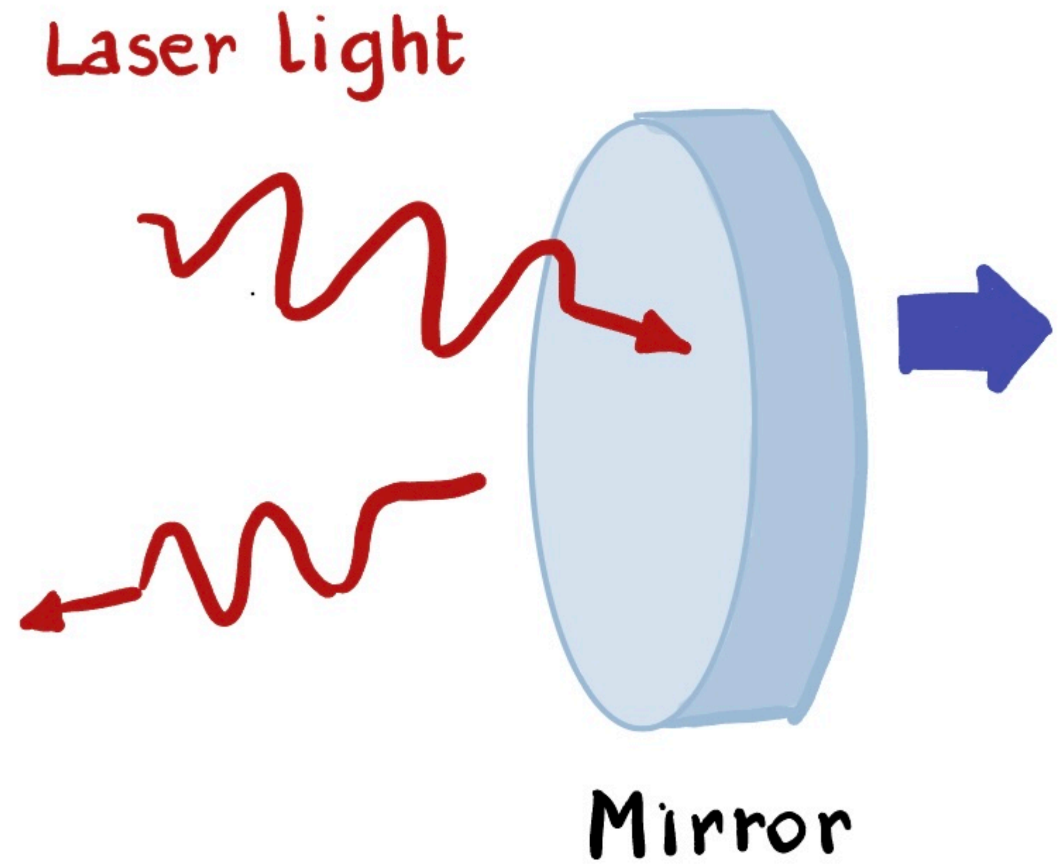


$$d \neq k\,\frac{\lambda}{2}, \quad k \in \mathbb{N}$$

Boundary Conditions

Classical systems & Quantum Fields



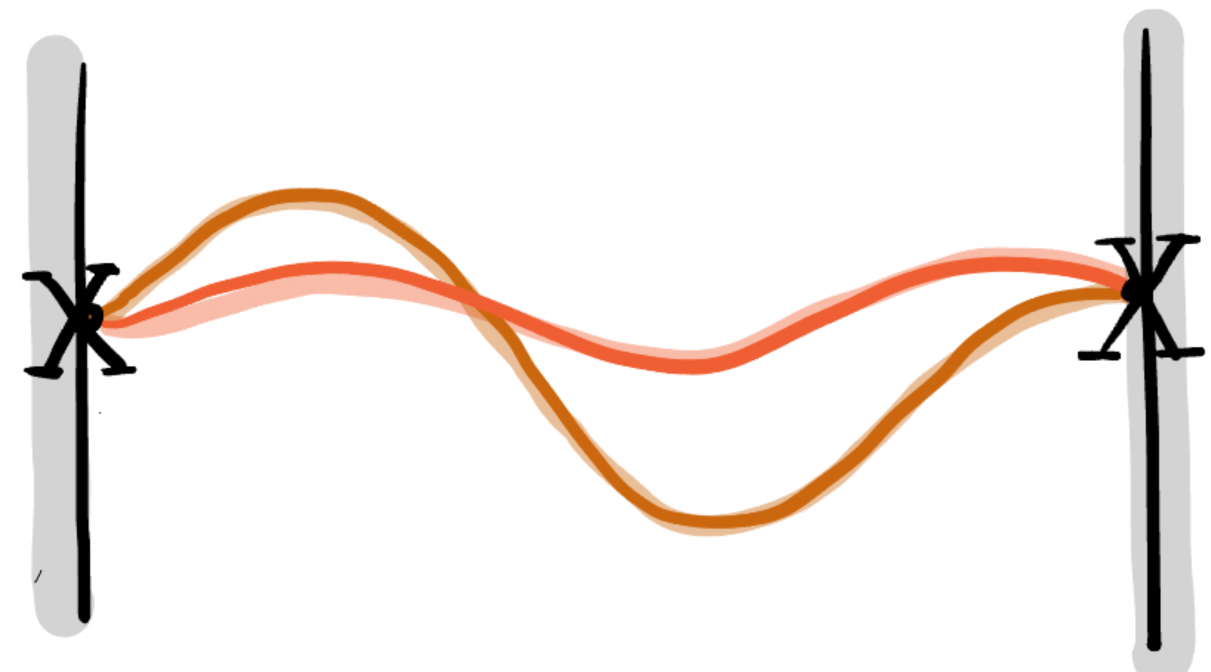


with external constraints

Dirichlet Boundary Conditions

P.G. Lejeune Dirichlet

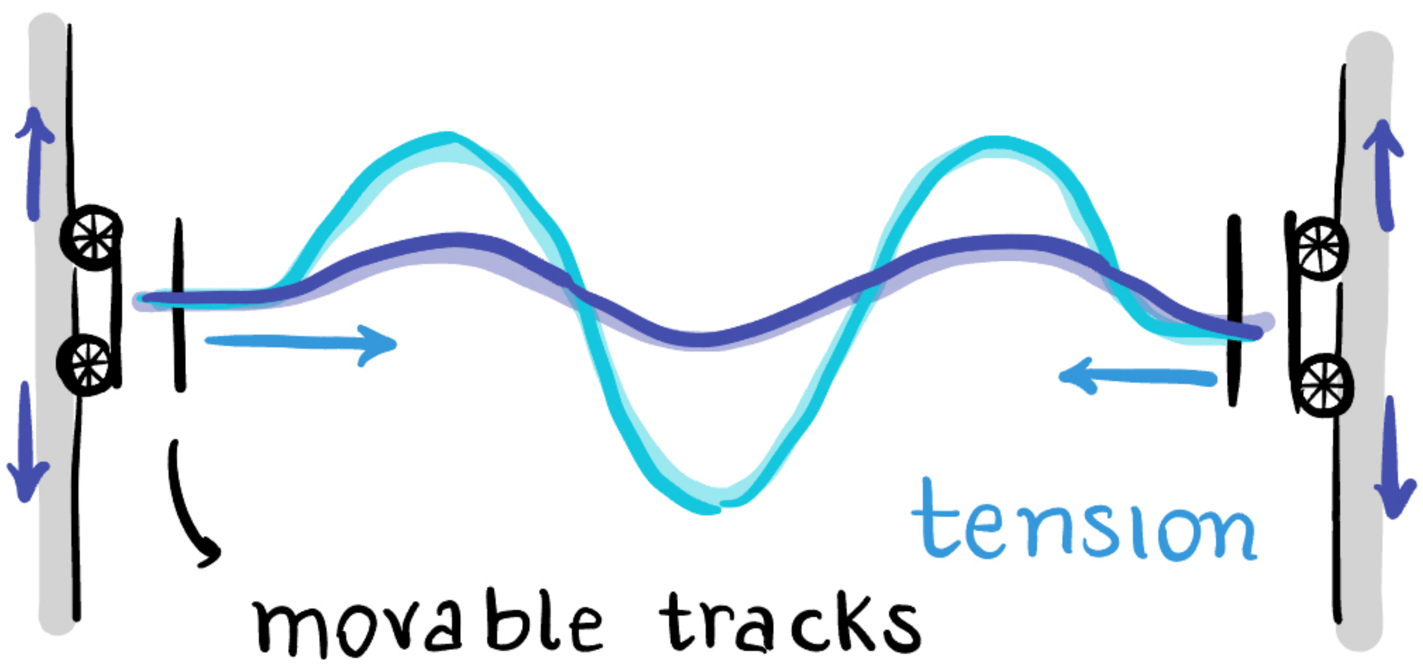
$$\boxed{\Psi\big|_{\partial} = \psi_0}$$



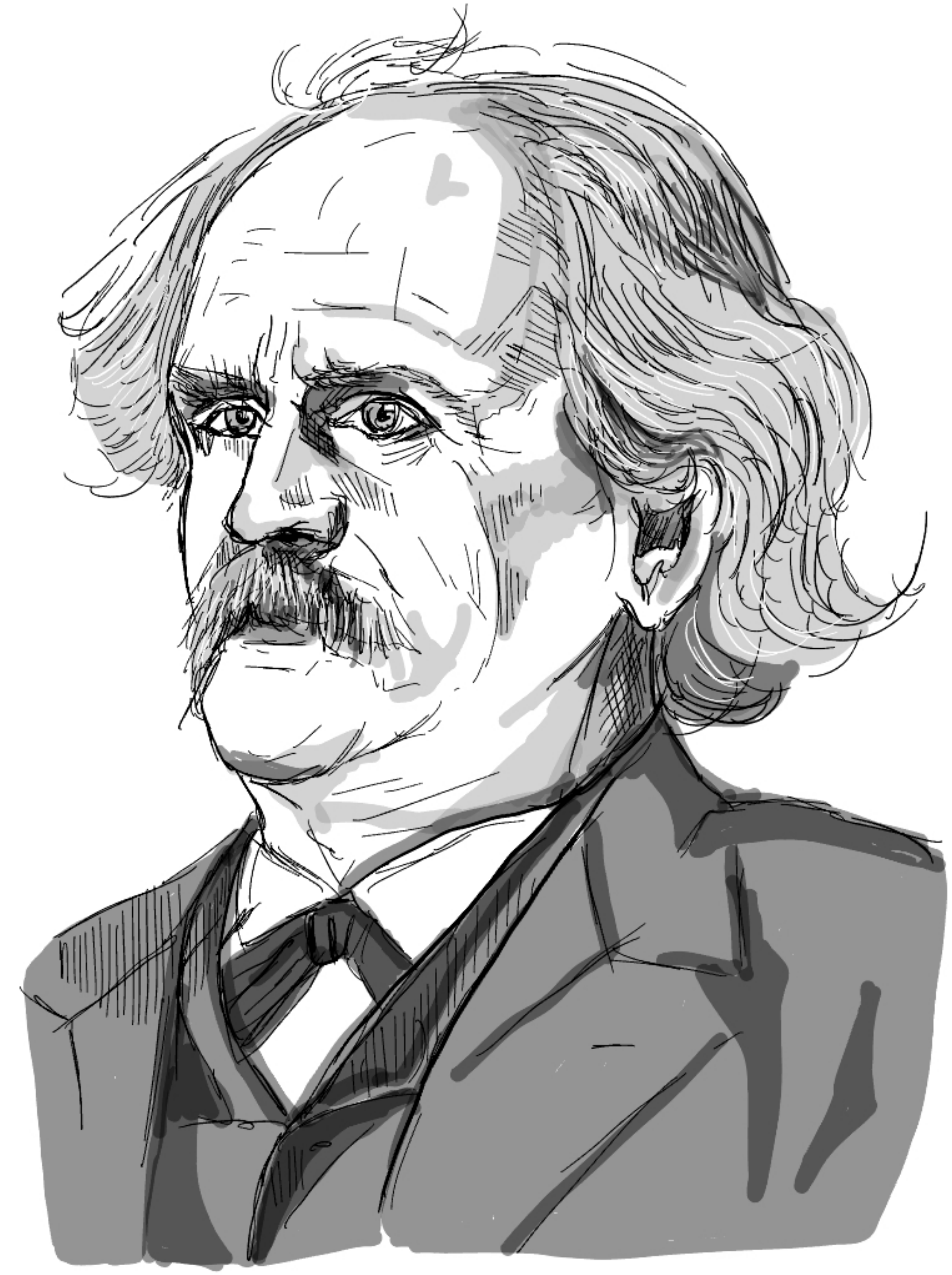
i.e., endpoint fixed

Neumann Boundary Conditions

$$\boxed{\nabla_n \psi \Big|_{\partial} = \psi_1}$$



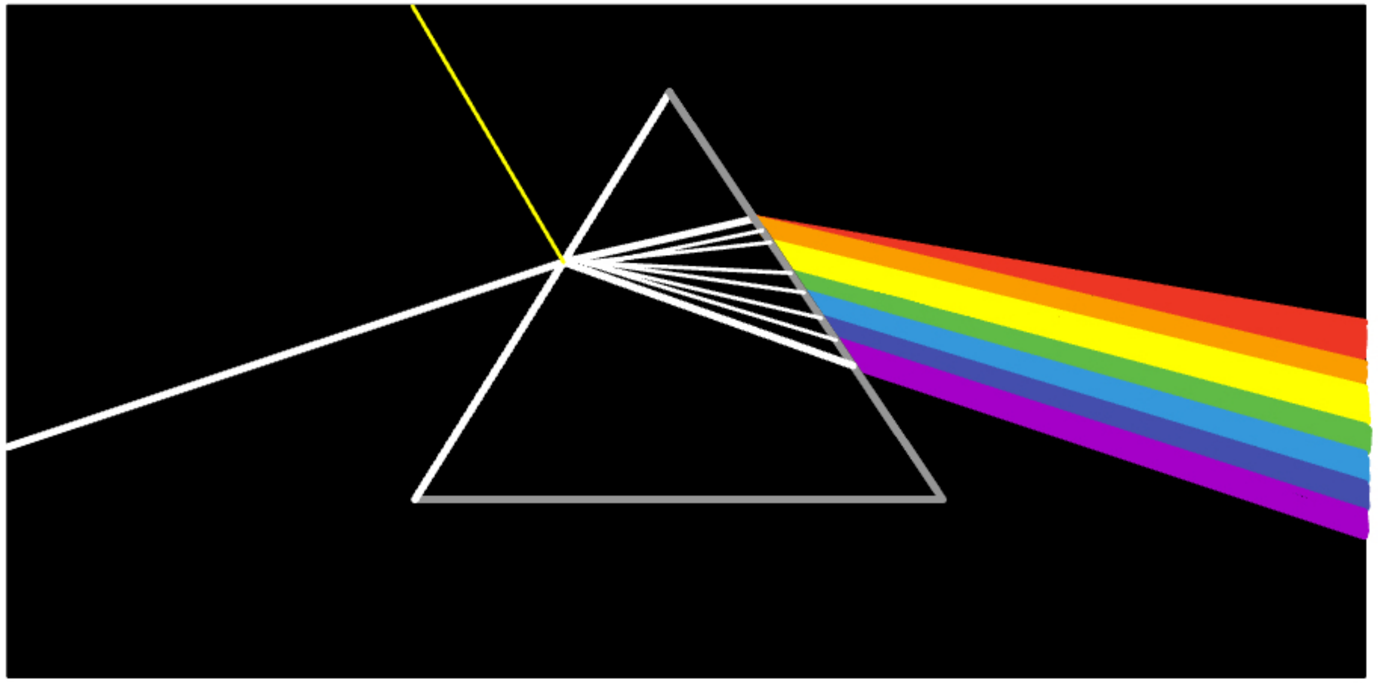
i.e., "slope" fixed at endpoints



Carl Neumann

Dirichlet & Neumann BCs are

REFLECTIVE



NO propagation along the Boundary

(and NO transmission across it)

... BUT

Real mirrors

are

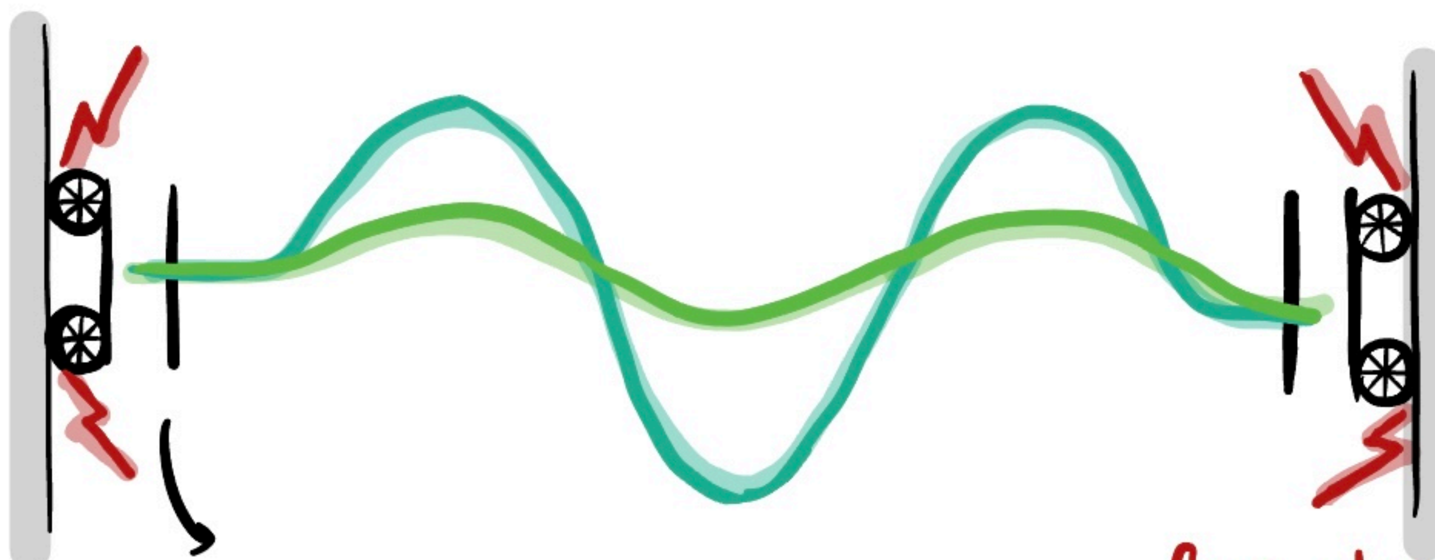
NOT

perfectly reflecting

Robin Boundary Conditions

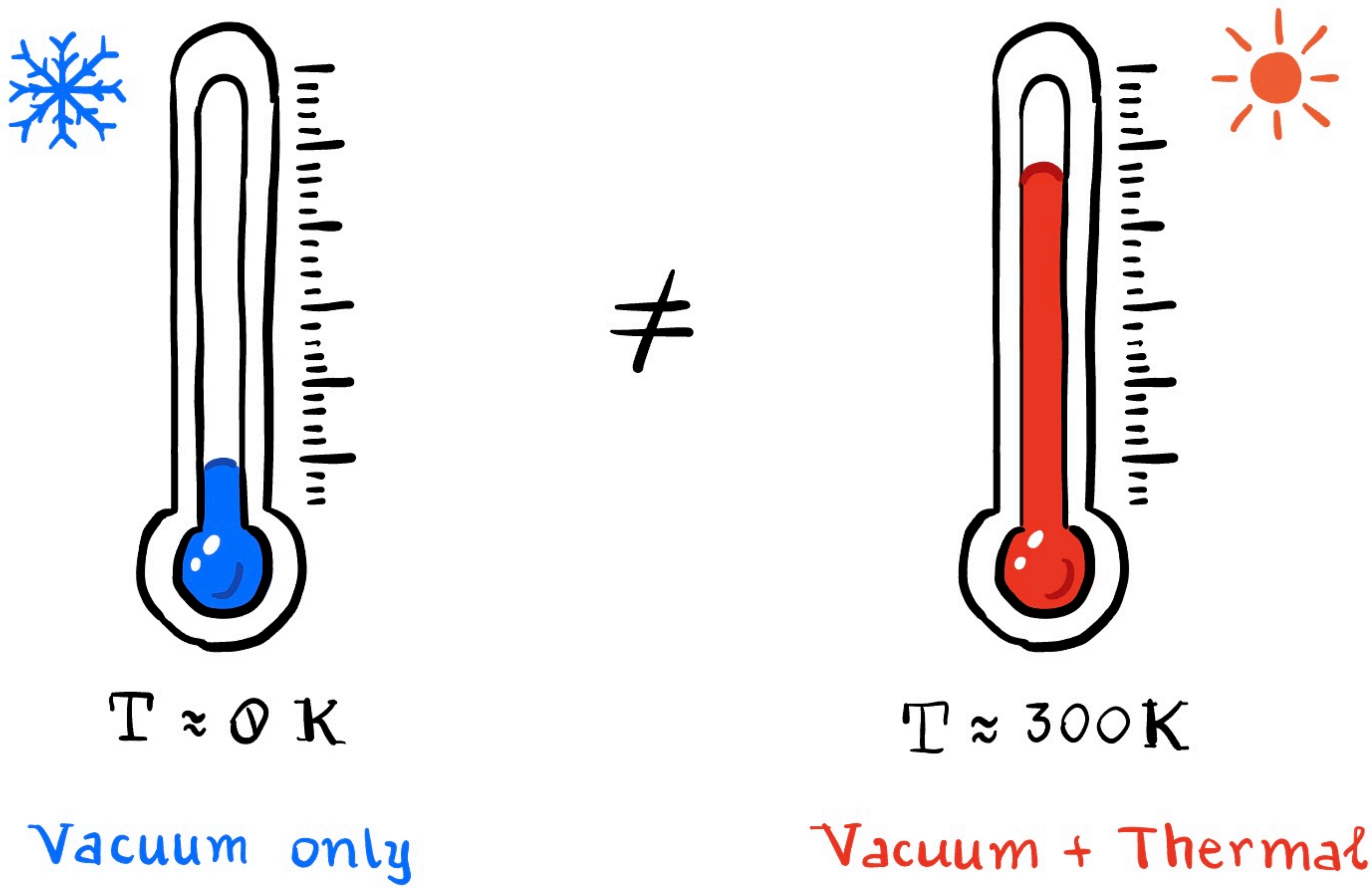
$$\boxed{\nabla_n \Psi \big|_\partial = \kappa \, \Psi \big|_\partial}$$

real constant



e.g., partially reflective/reactive surfaces

Thermal fluctuations **DO** contribute!



The project

Study Quantum fields on spacetimes with boundary

B. A. Juárez-Aubry

University of York

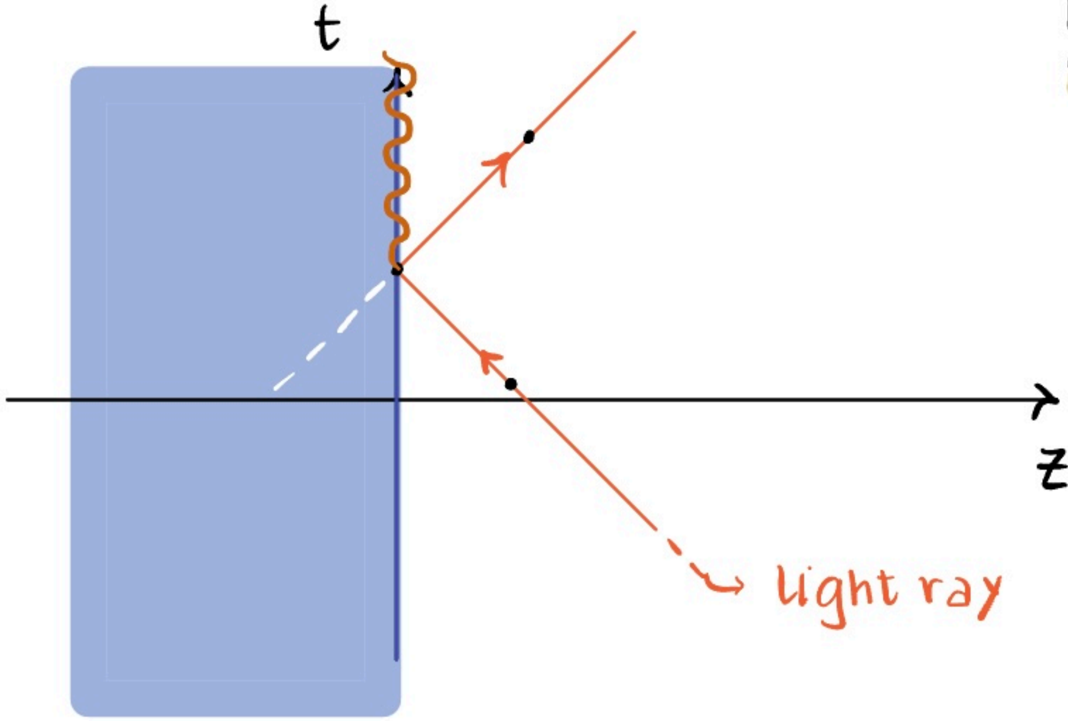
C. Dappiaggi

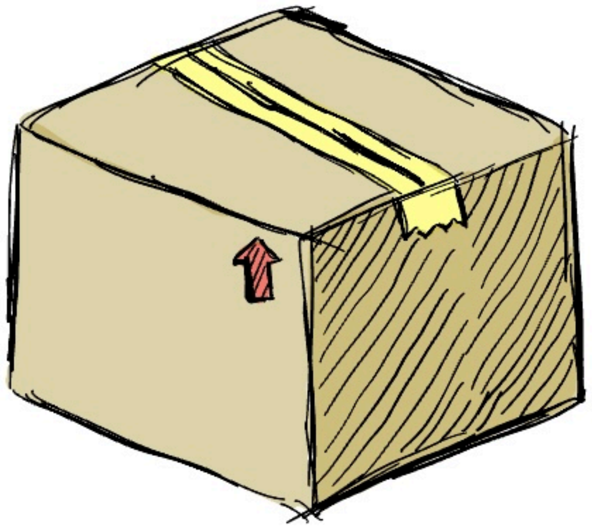
R. D. Singh

University of Pavia

The mathematical set up

Isolated system on half-Minkowski

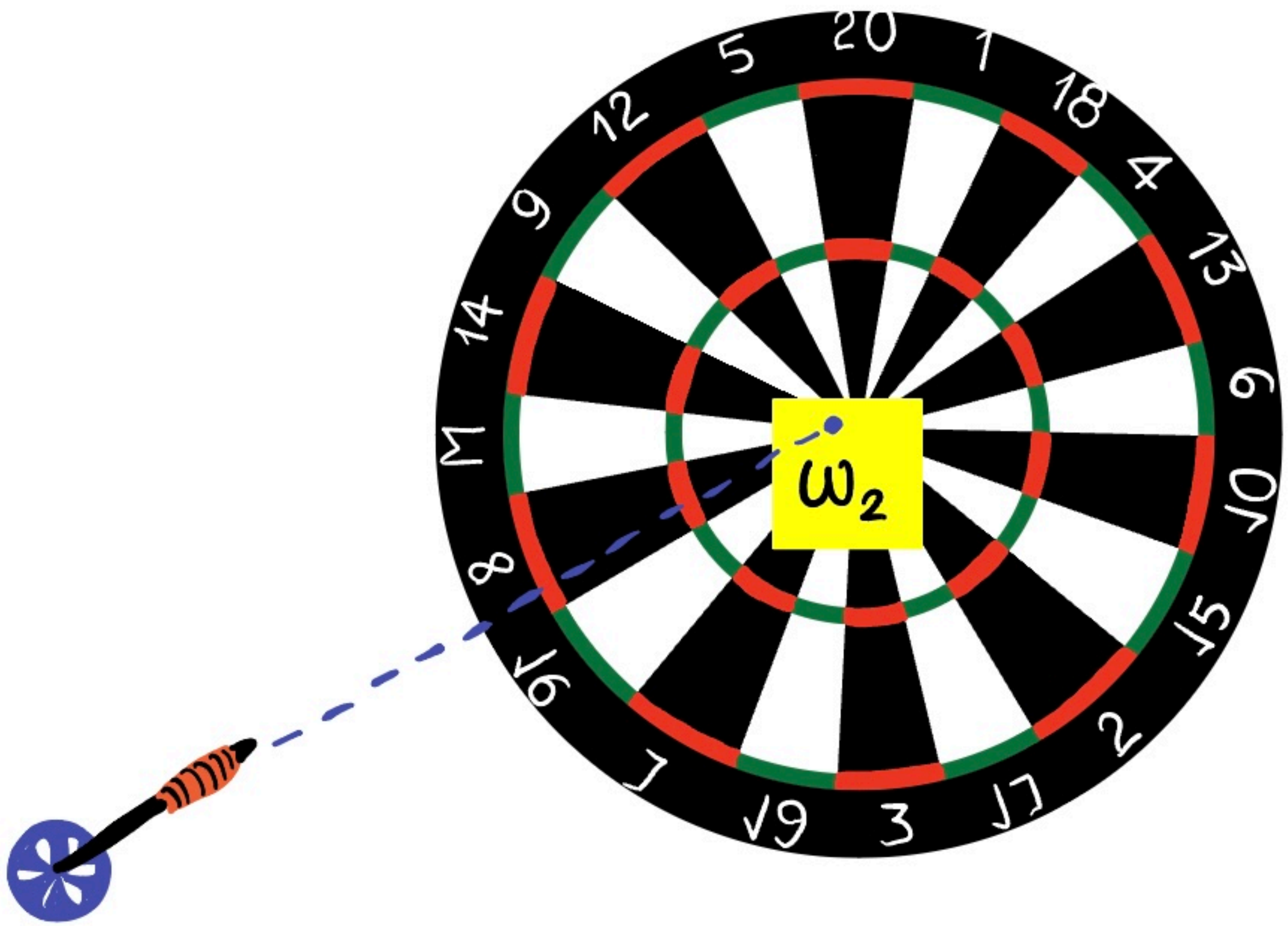




$$\begin{cases} \Box \varphi = 0 & \text{Wave equation} \\ \text{initial conditions} & \\ \partial_z \varphi \big|_{z=0} - \kappa \varphi \big|_{z=0} = 0 & \text{Robin BC} \end{cases}$$

The Goal

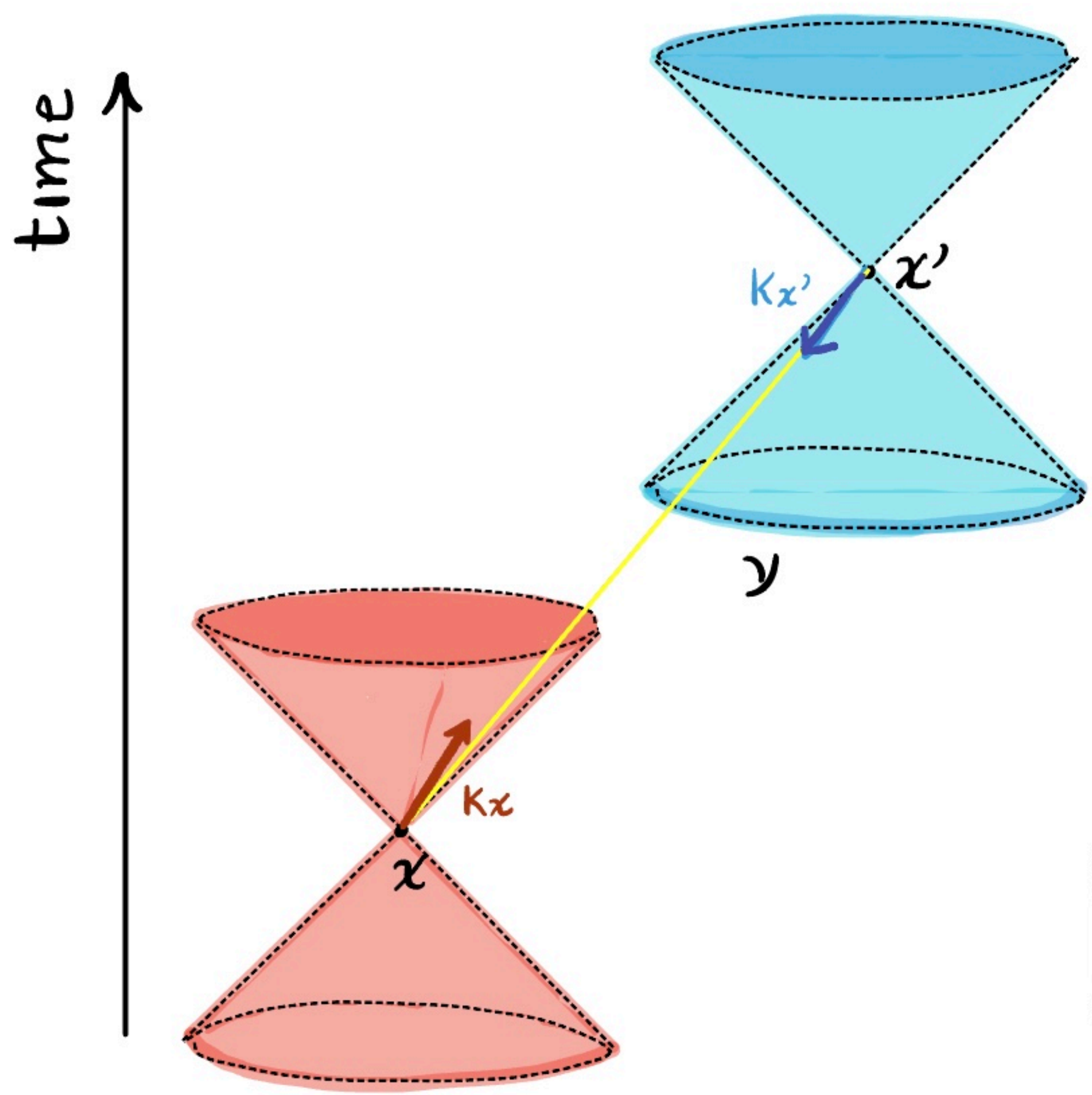
Construct the two-point correlation function

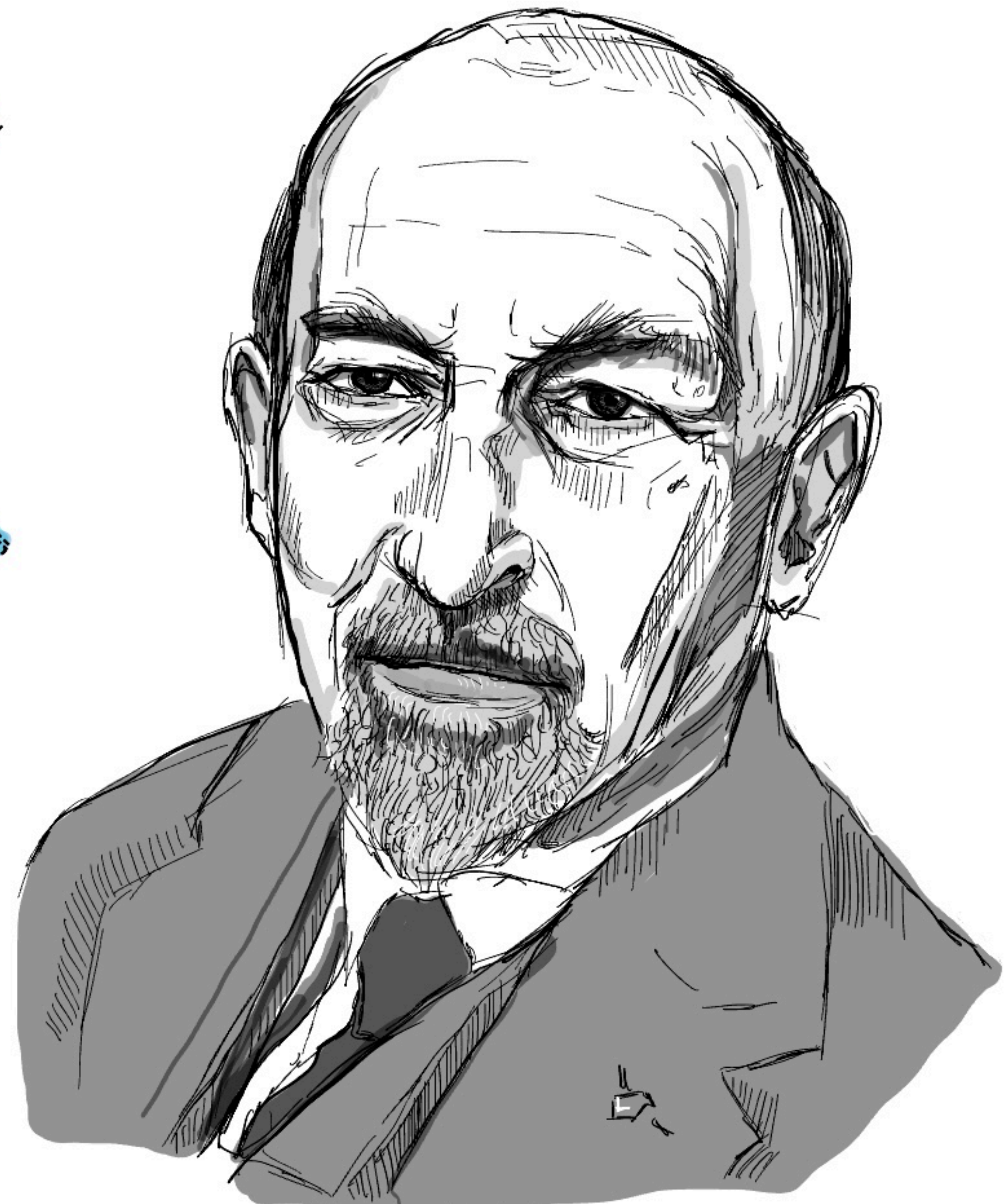


Yes, but **HOW**?

Propagation of singularities

PHYSICAL STATES are Hadamard states



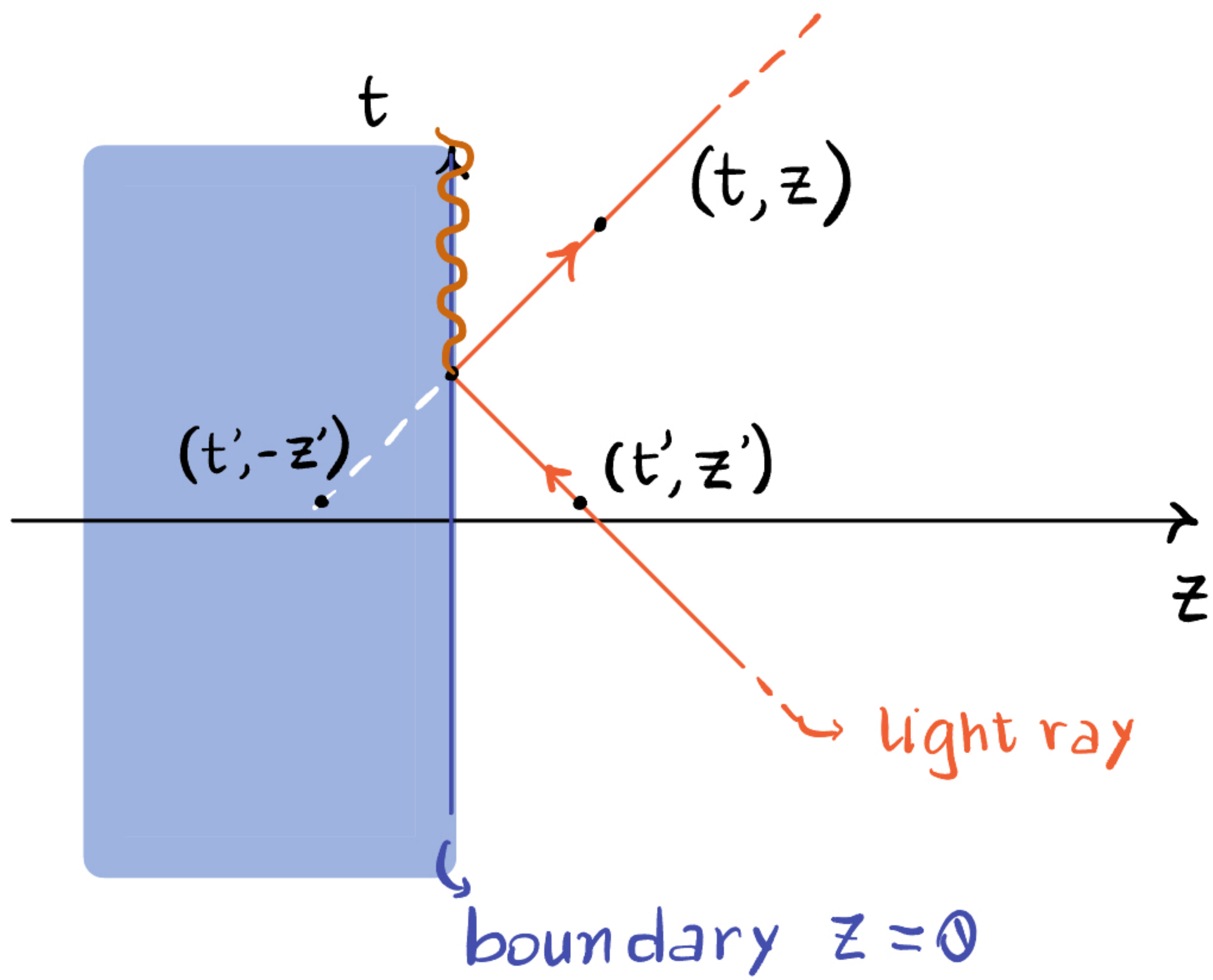


Jacques Hadamard

Two-point function in 4D (I)

Ansatz:

$$\omega_2 = \underbrace{U(\sigma)}_{\text{power series in } \sigma} \frac{1}{\sigma} + \underbrace{V(\sigma)}_{} \log(-\sigma) + \underbrace{U'(\sigma_-)}_{\text{power series in } \sigma_-} \frac{1}{\sigma_-} + \underbrace{V'(\sigma_-)}_{} \log(-\sigma_-)$$



Two-point function in 4D (II)

Exact solution $\longrightarrow$ Bondurant-Fulling's method

$$\omega_2 = \underbrace{\frac{1}{\sigma} + \frac{1}{\sigma_-}}_{\text{Reflection}} \underbrace{- \frac{2\kappa}{\sqrt{\sigma_\partial}}\, e^{-\kappa\left(\sqrt{\sigma_\partial} - z - z'\right)}}_{\text{Propagation along the boundary}}$$

- Recursion relations ✓
- Robin Boundary Conditions ✓

A brief recap

Robin conditions

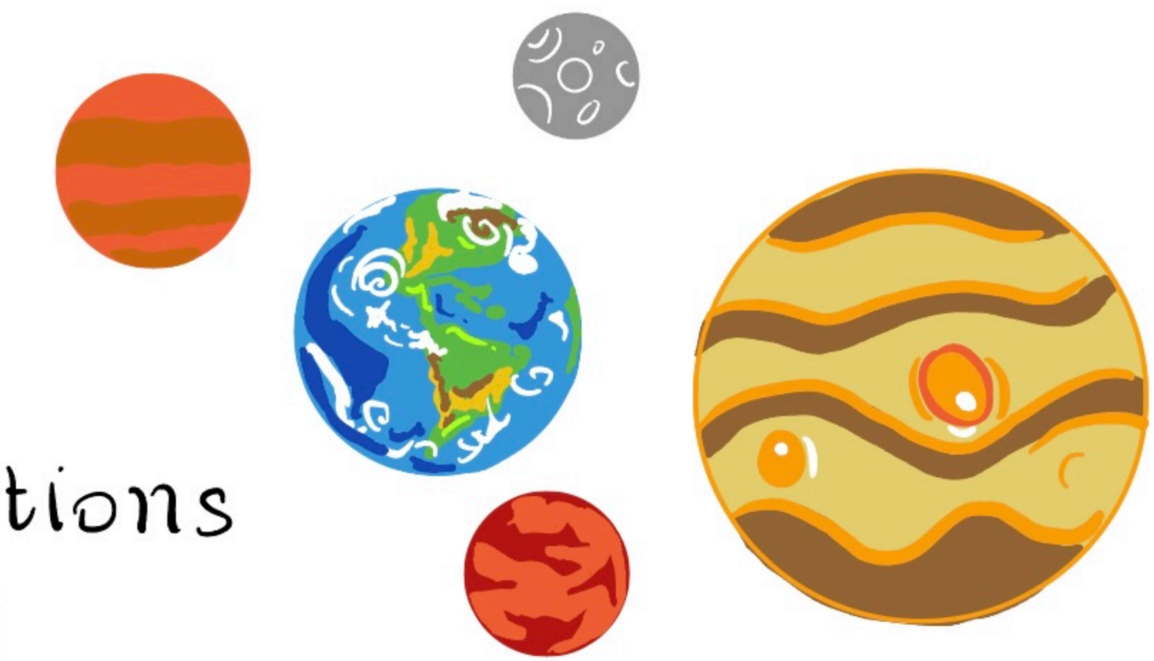
model

physical systems

interacting with the

BOUNDARY

Outlooks



1. Corners and Edges

2. Wentzell Boundary Conditions

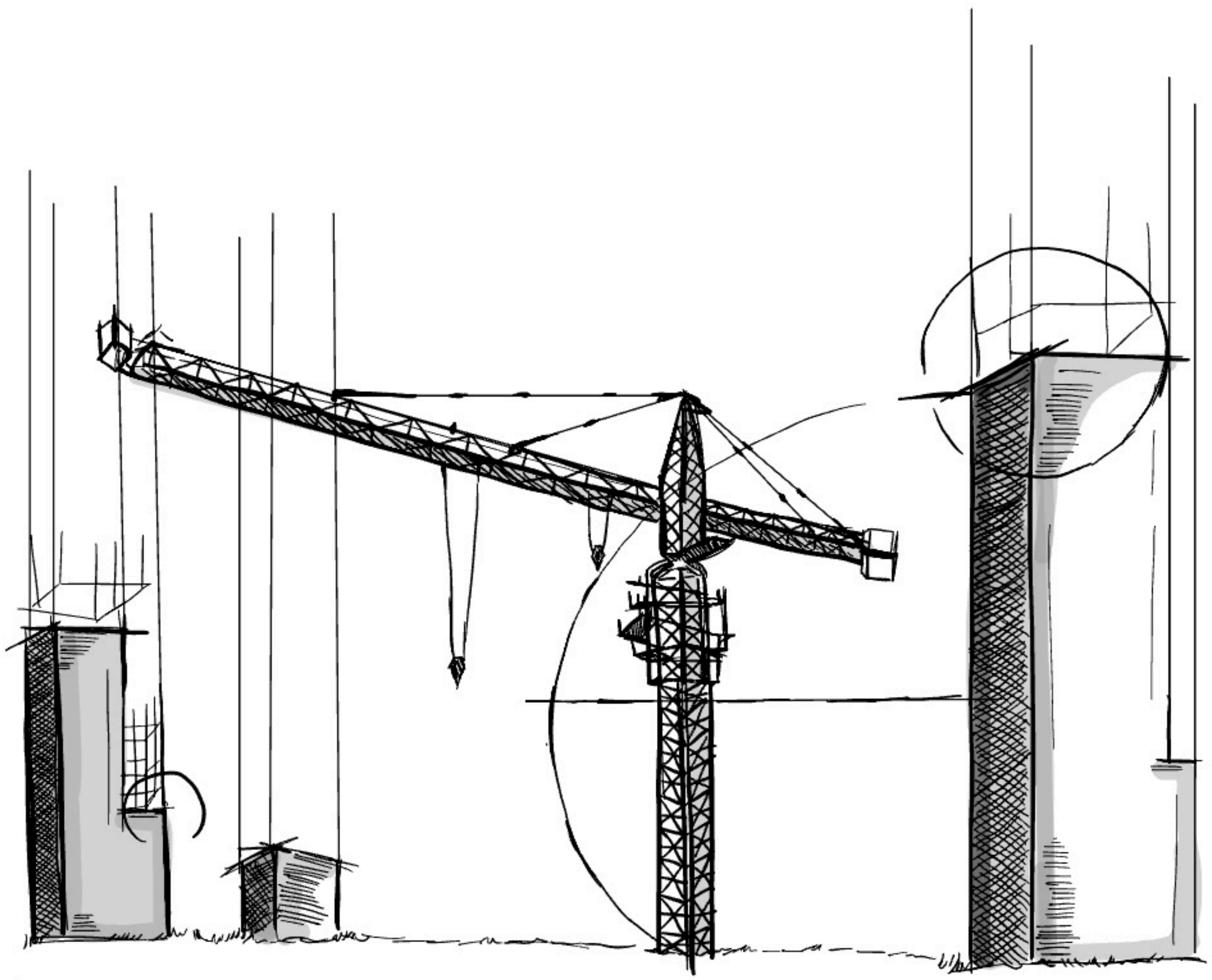
   $\hookrightarrow$ $\boxed{\text{AdS\textbackslash CFT correspondence}}$

3. Wetterich Equation:

   $\hookrightarrow$ with Robin Boundary Conditions

   $\hookrightarrow$ $\boxed{\text{Interplay between SPDEs and RG flow}}$

Ongoing Projects



- Stochastic Gravity and Thermal Fluctuations
  - with Claudio Dappiaggi and Paolo Meda (UNITN)

- Hadamard States for linearized Yang-Mills theories
  - with Simone Murro (UNIGE)

  Grant Progetto Giovani INdAM 2025