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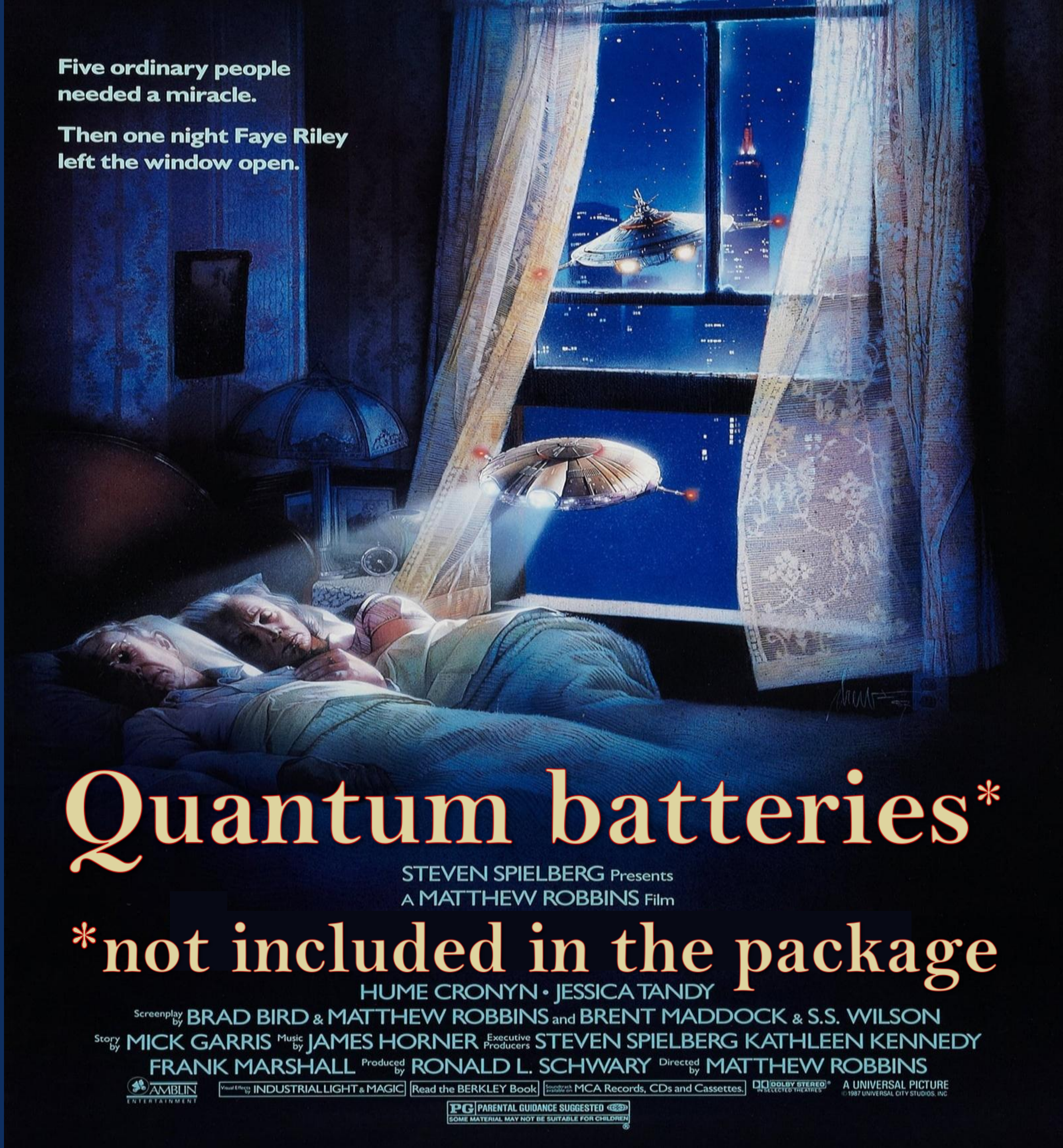
Speaker:
Davide Rinaldi

Supervisors:
Prof. Dario Gerace
Prof. Giacomo Guarnieri

Coming soon at the
End Of Year Seminars

17th September 2025



Quantum batteries roadmap

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Quantum batteries roadmap

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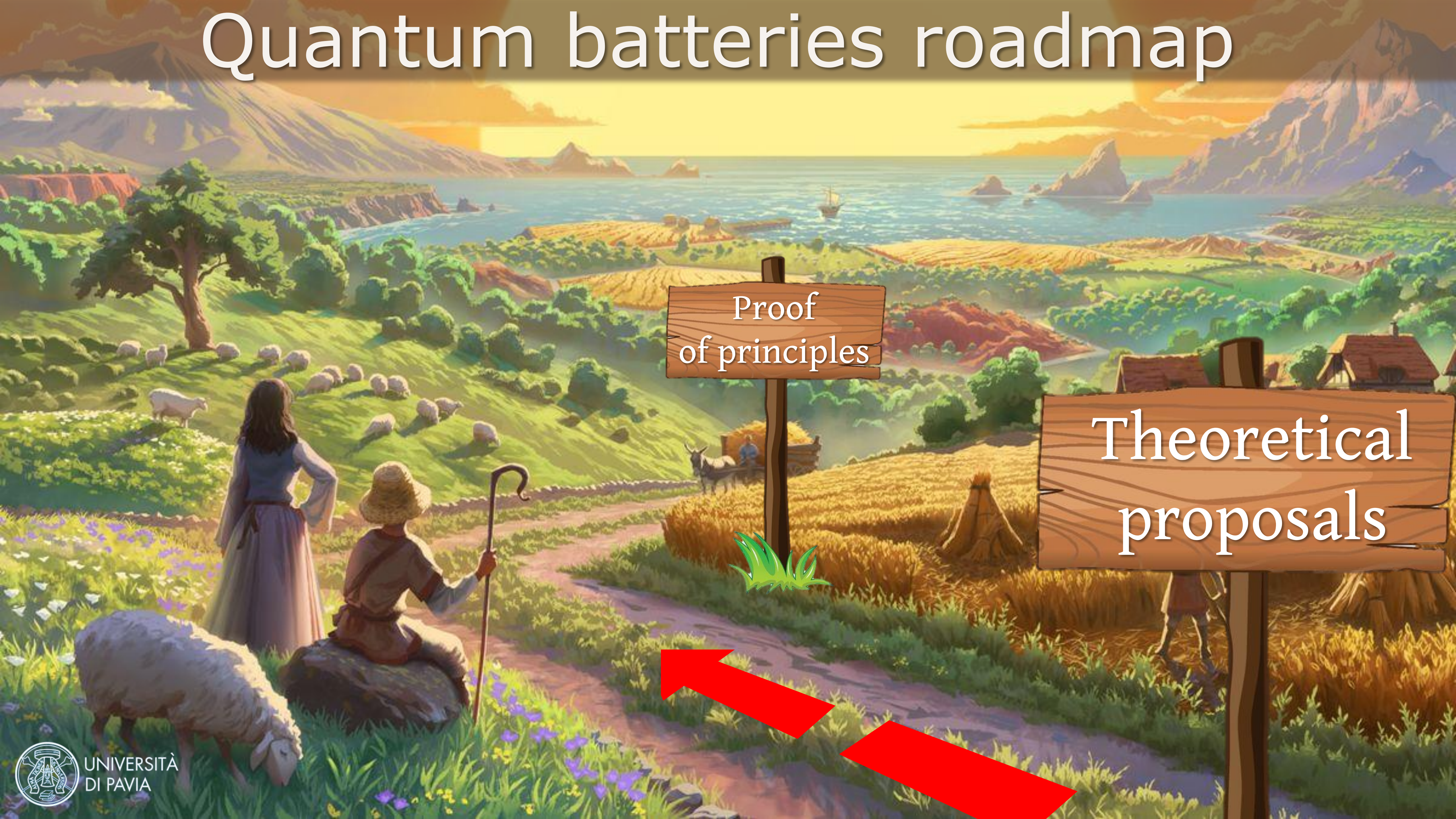

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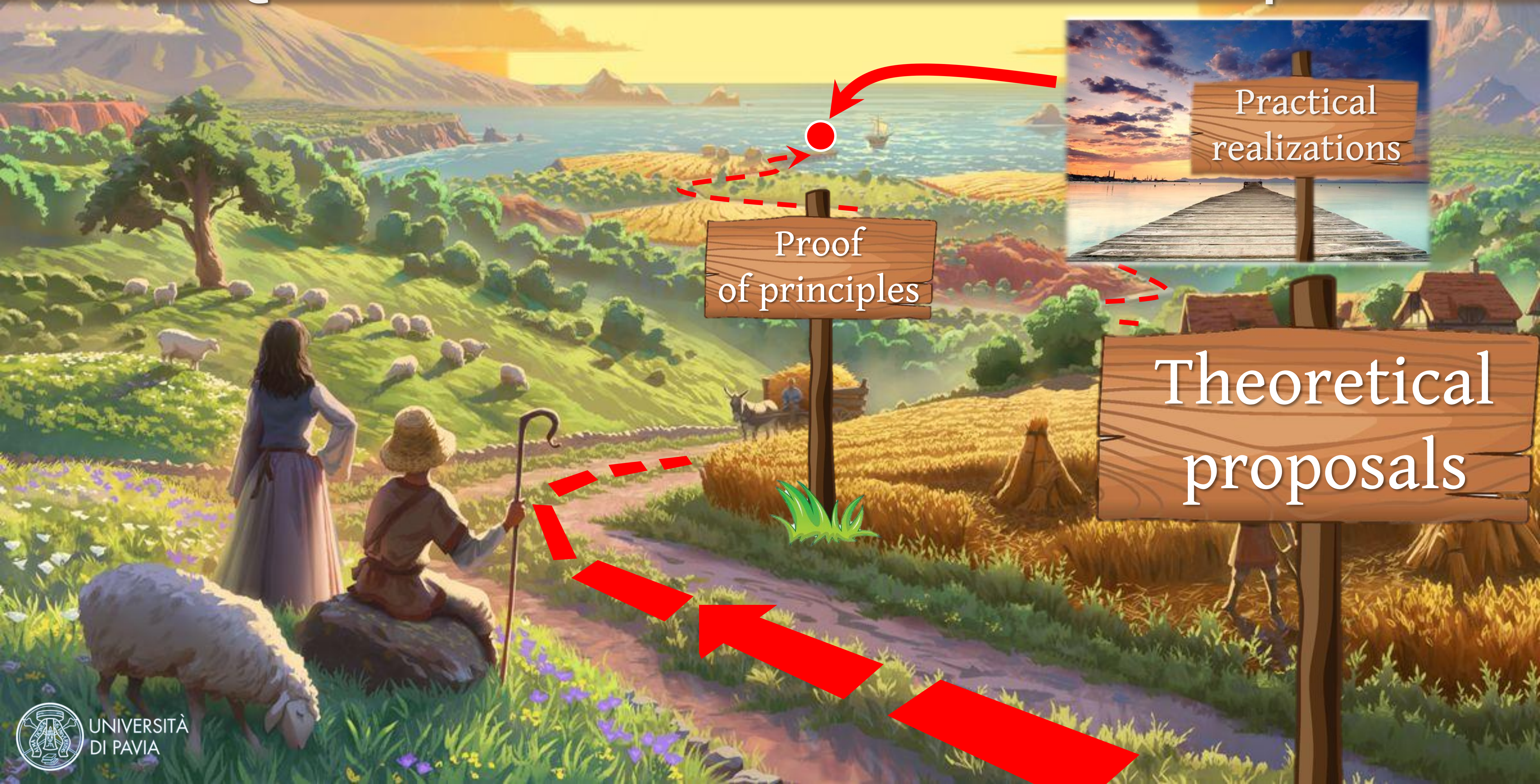

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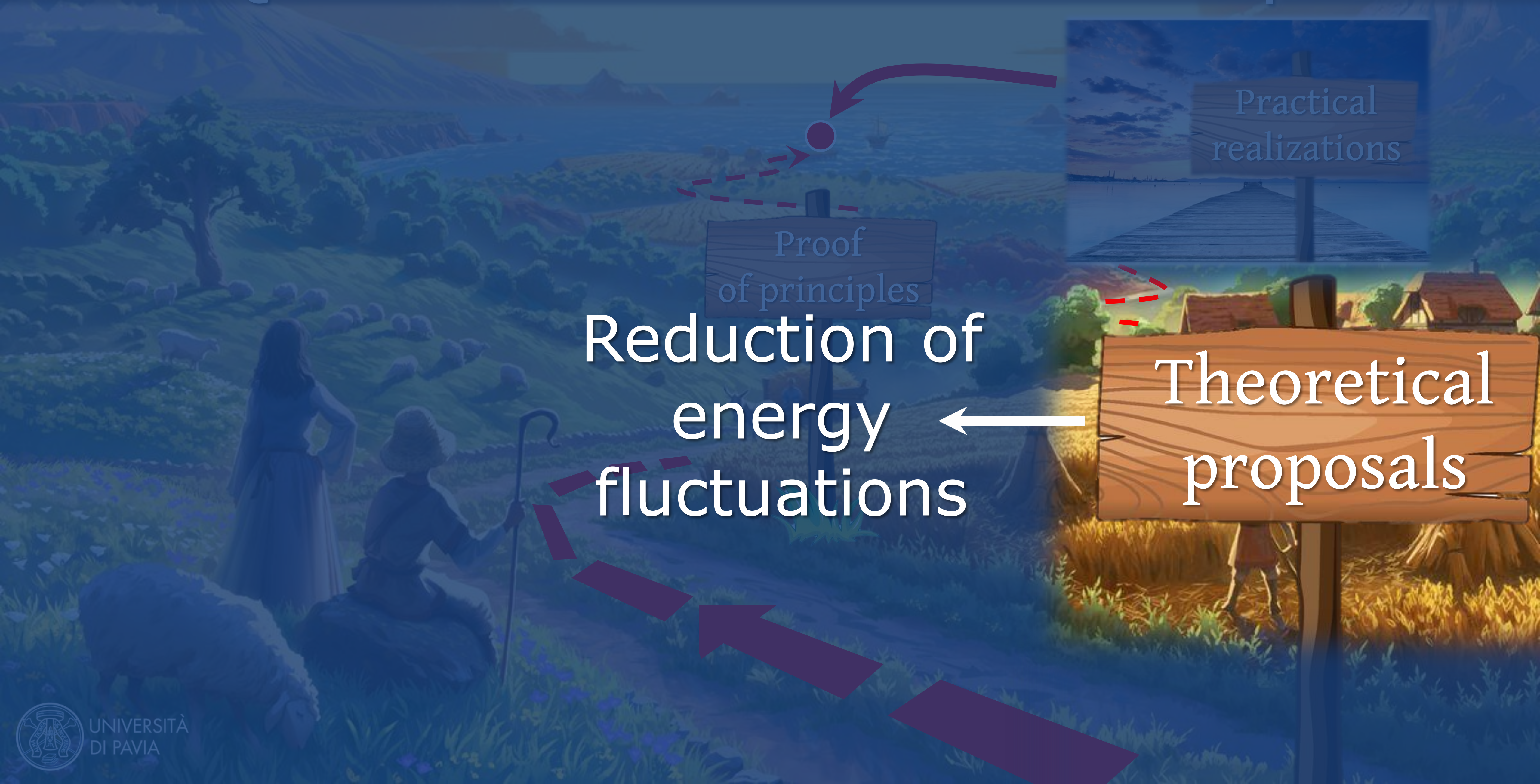

Reduction of energy fluctuations

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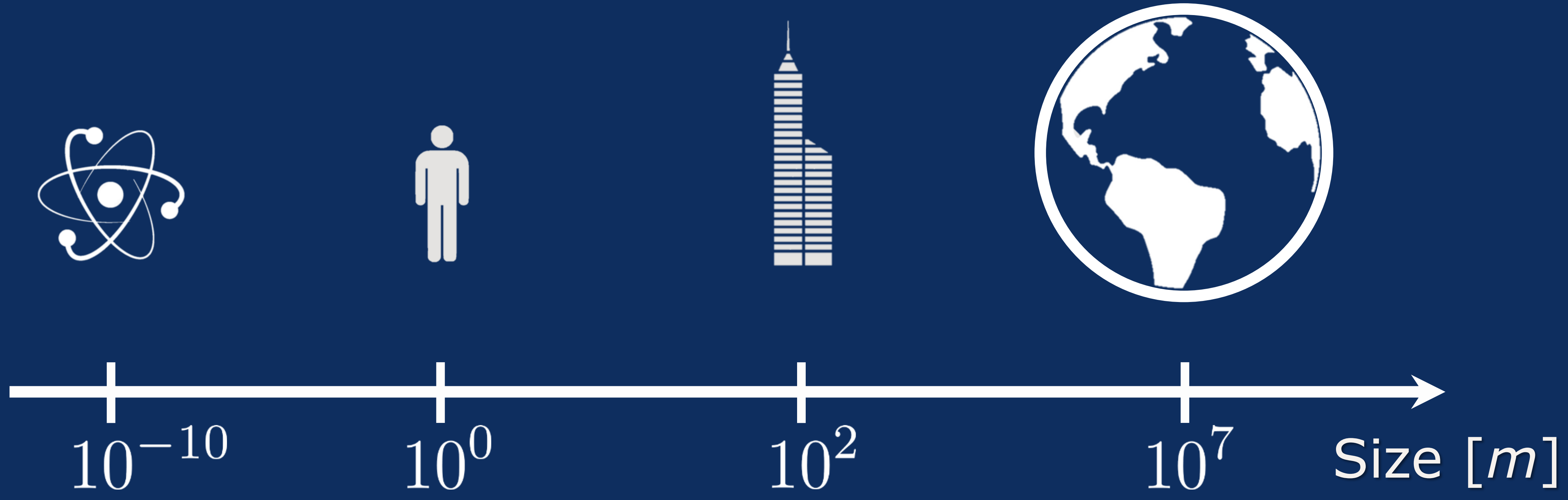
Energy fluctuations

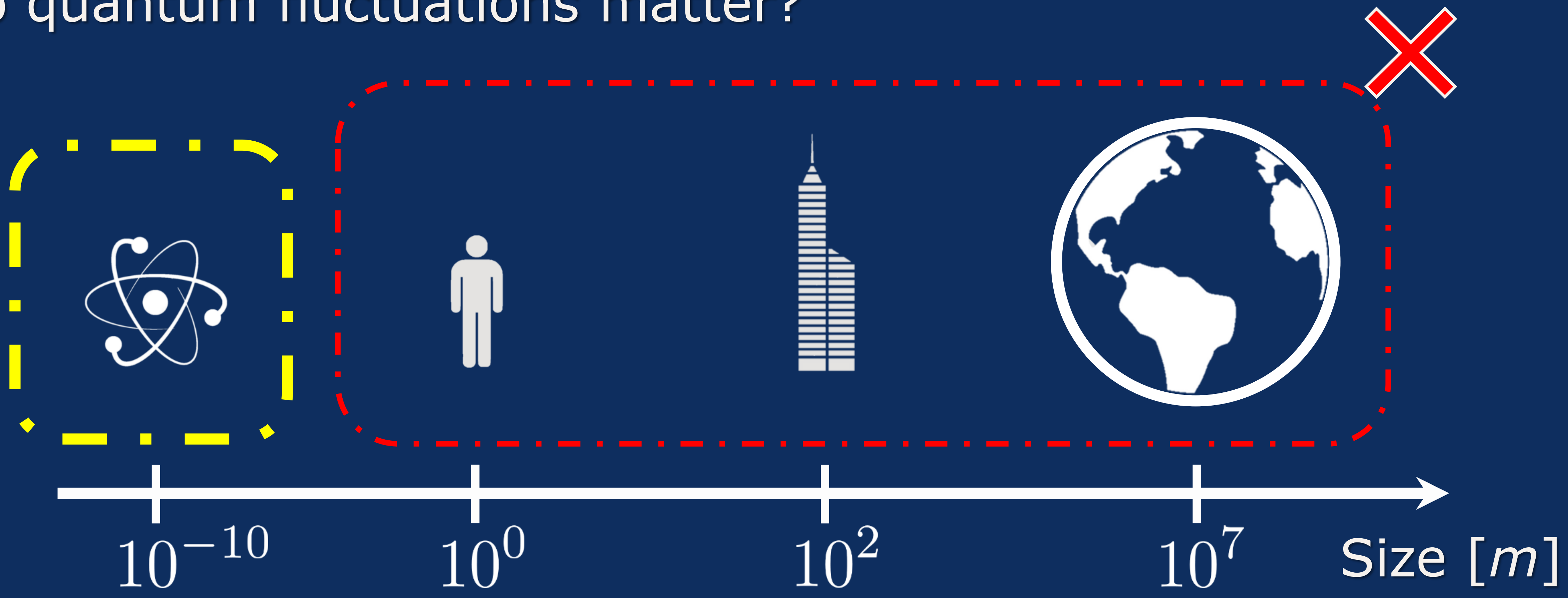
Energy fluctuations

Do quantum fluctuations matter?



Energy fluctuations

Do quantum fluctuations matter?



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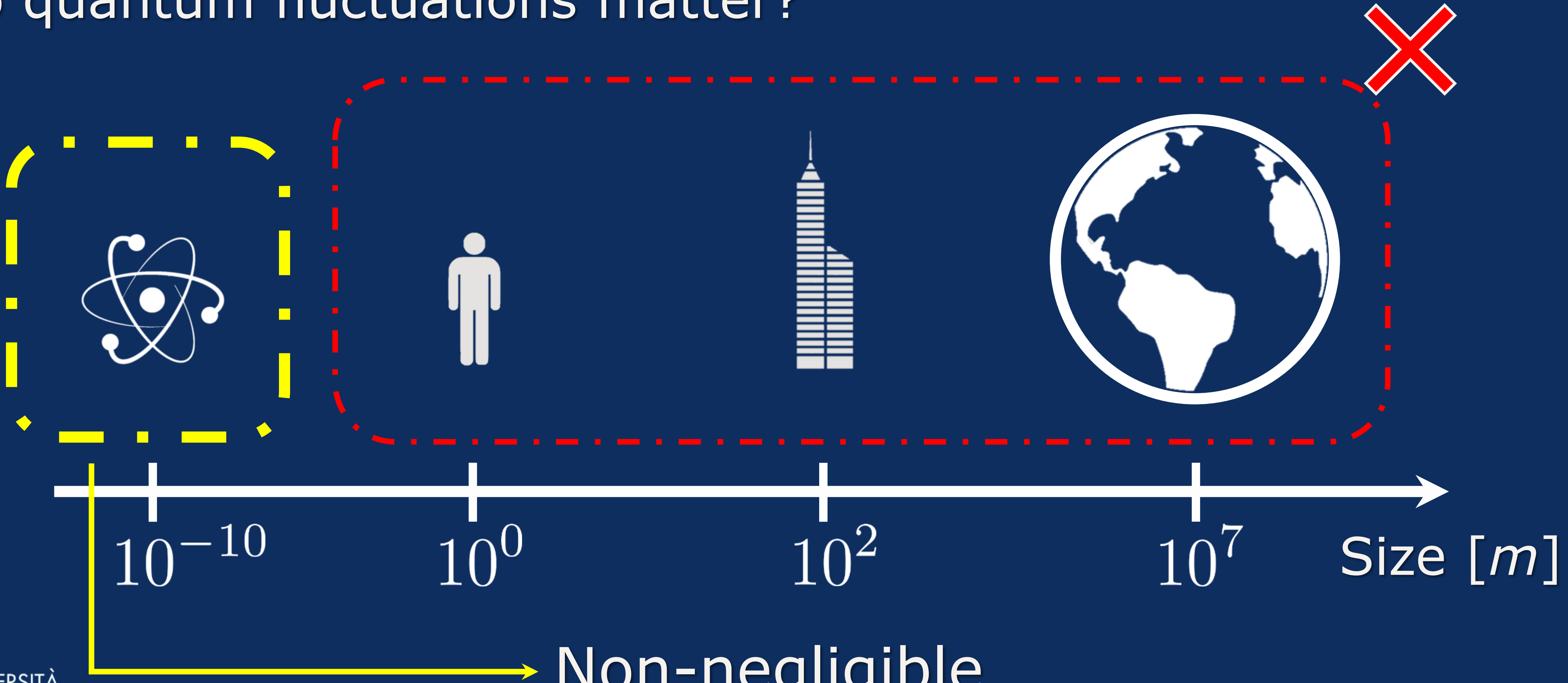

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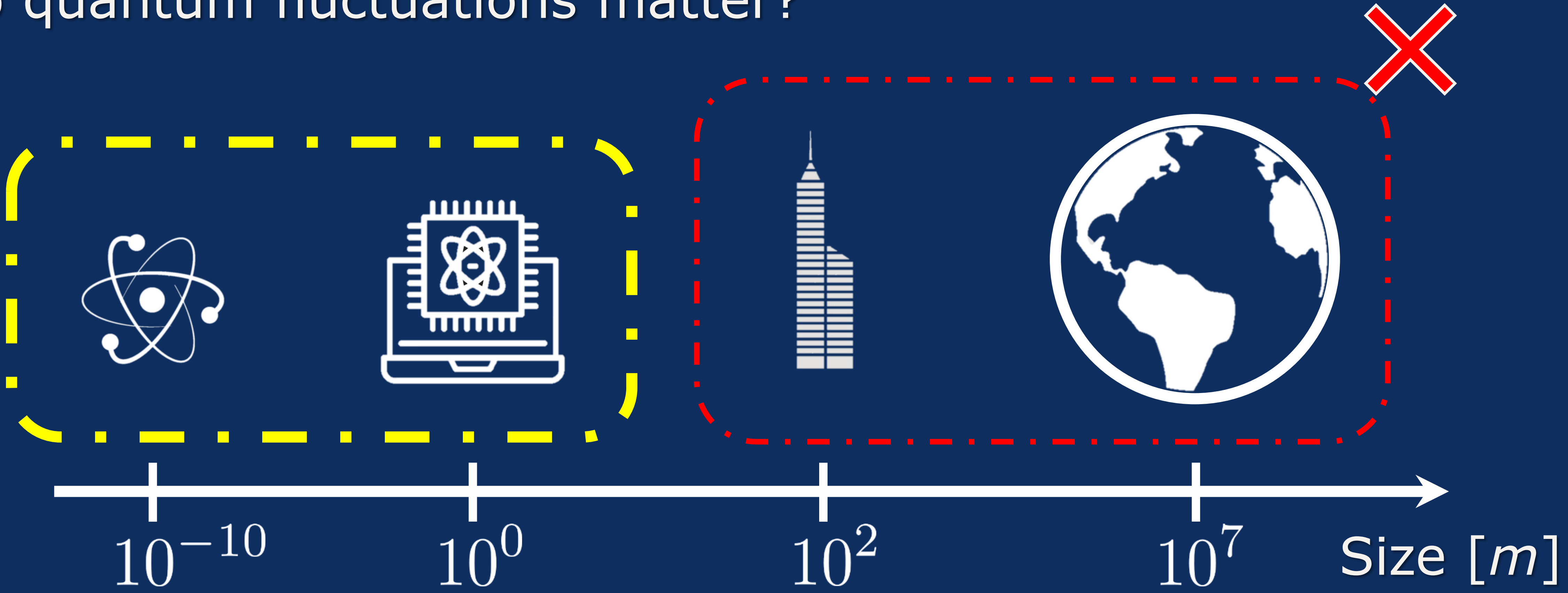

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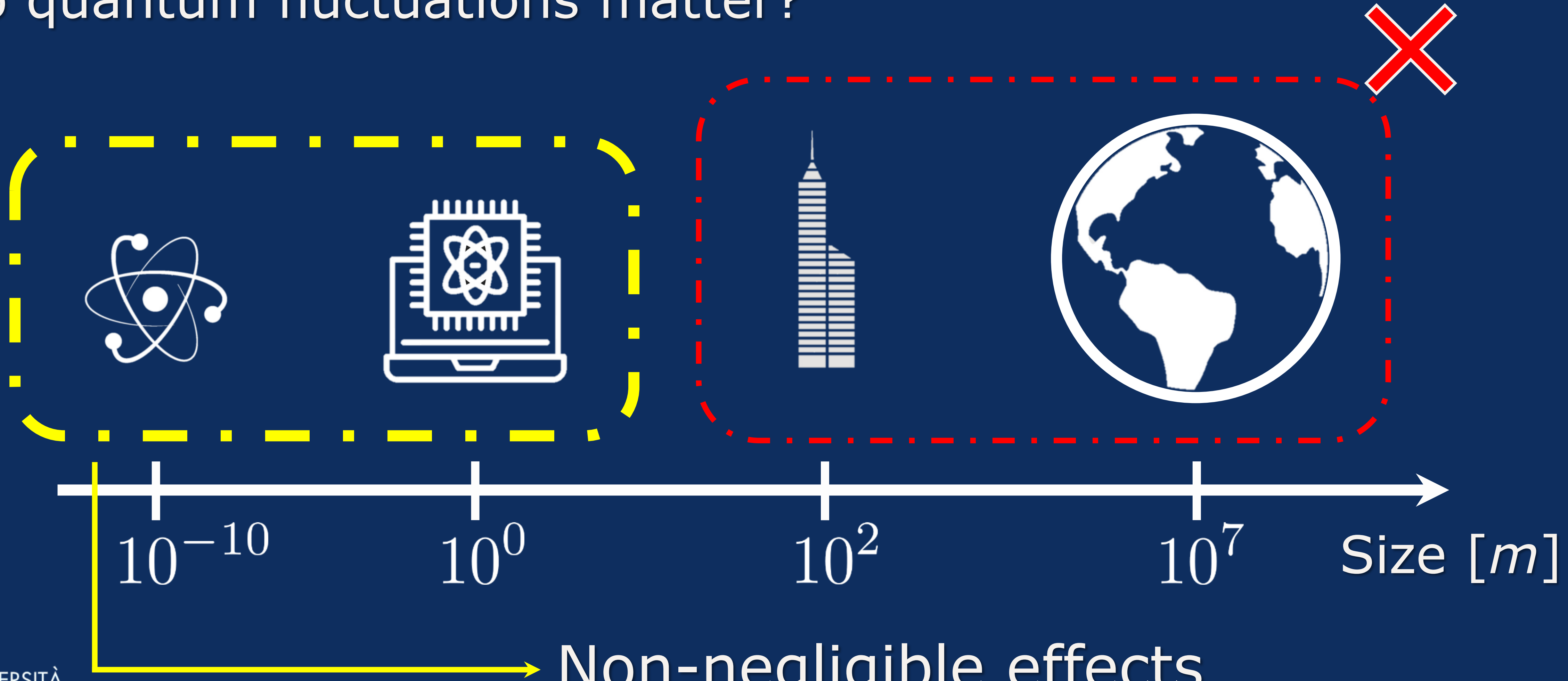

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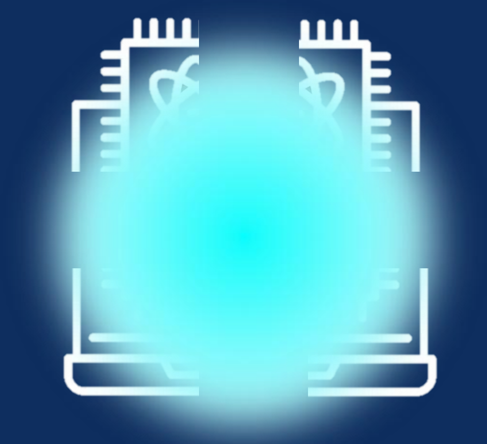
Energy fluctuations

Can we trust the process?

Energy fluctuations

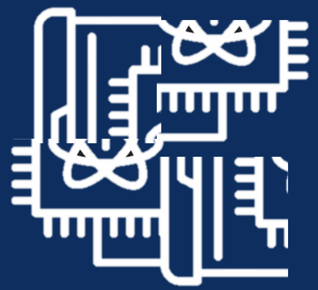
Can we trust the process?



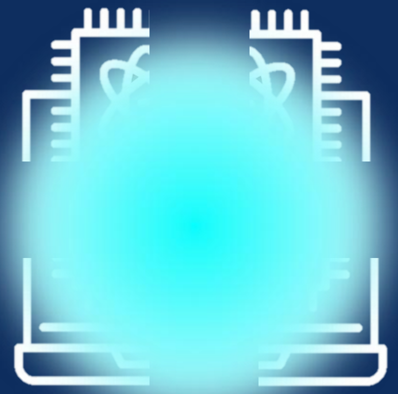
Problem

Energy fluctuations

Can we trust the process?



Sol. 1: low variance, wrong result



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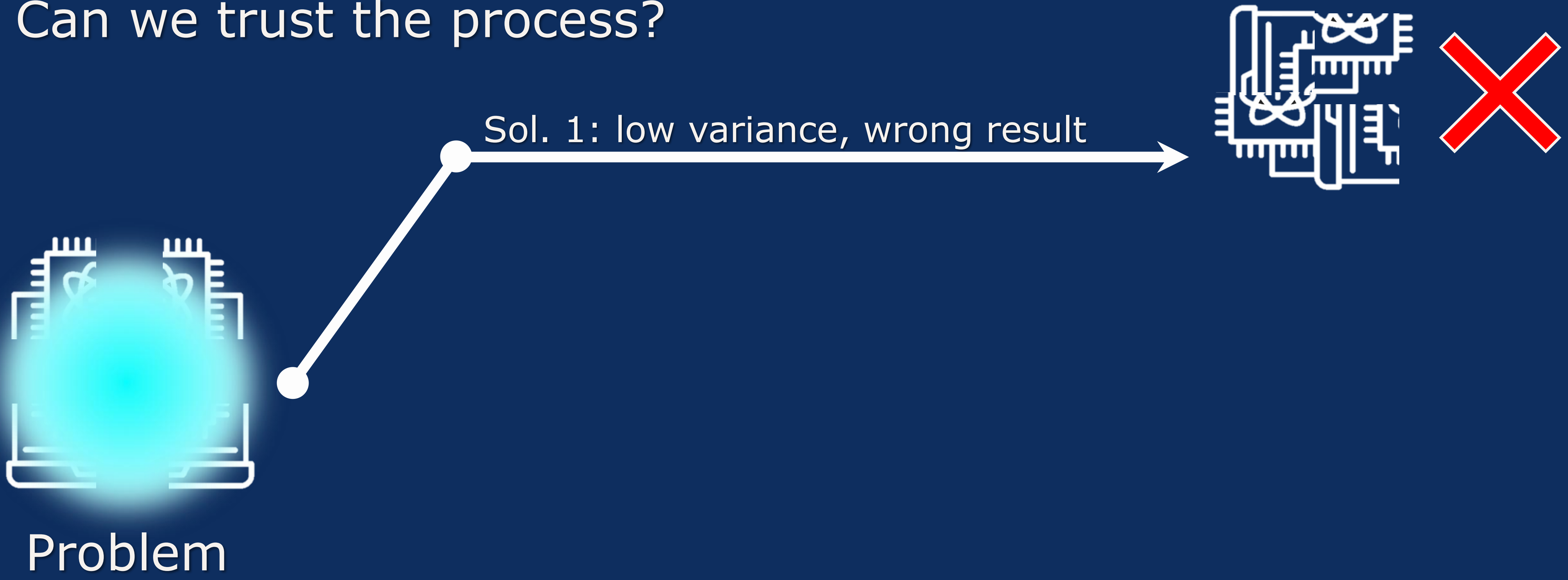

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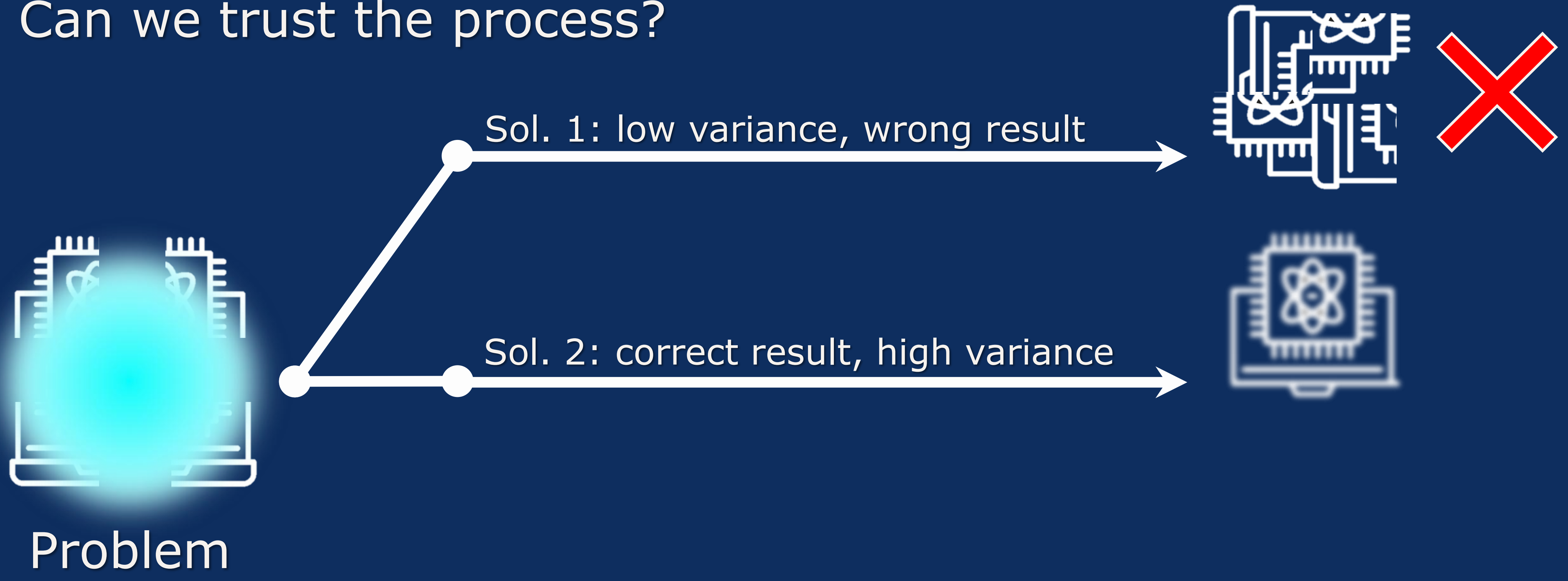

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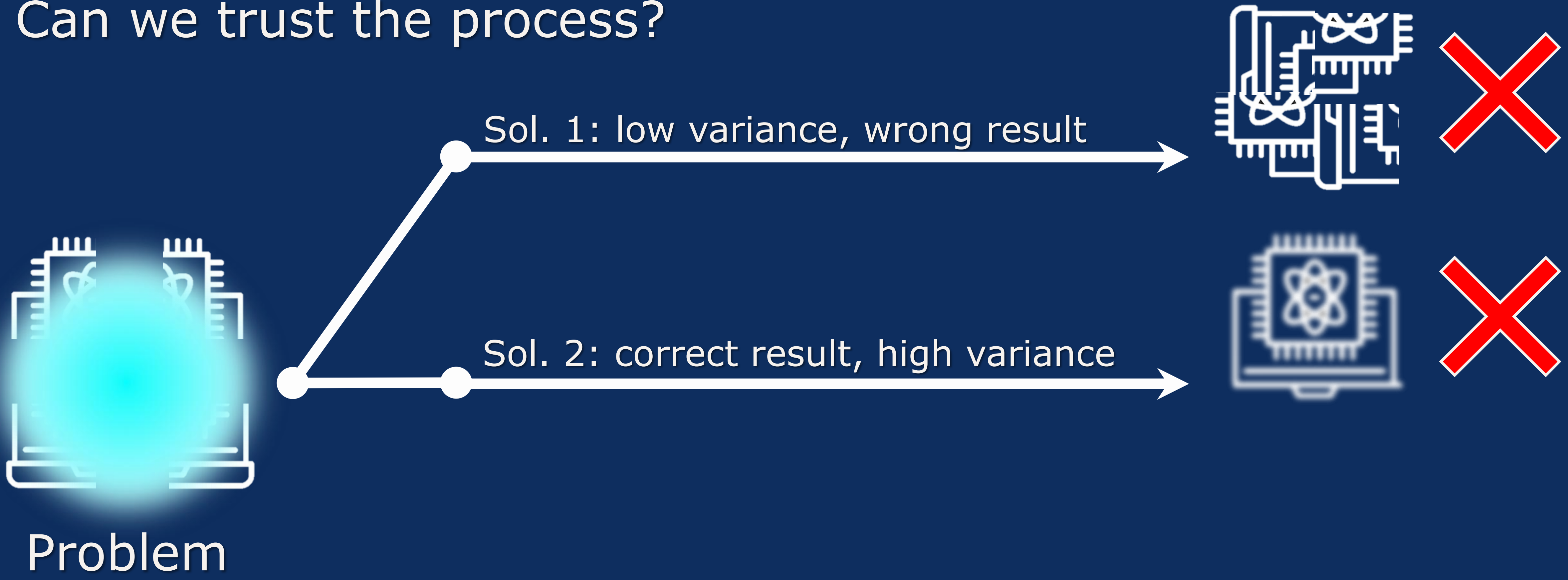

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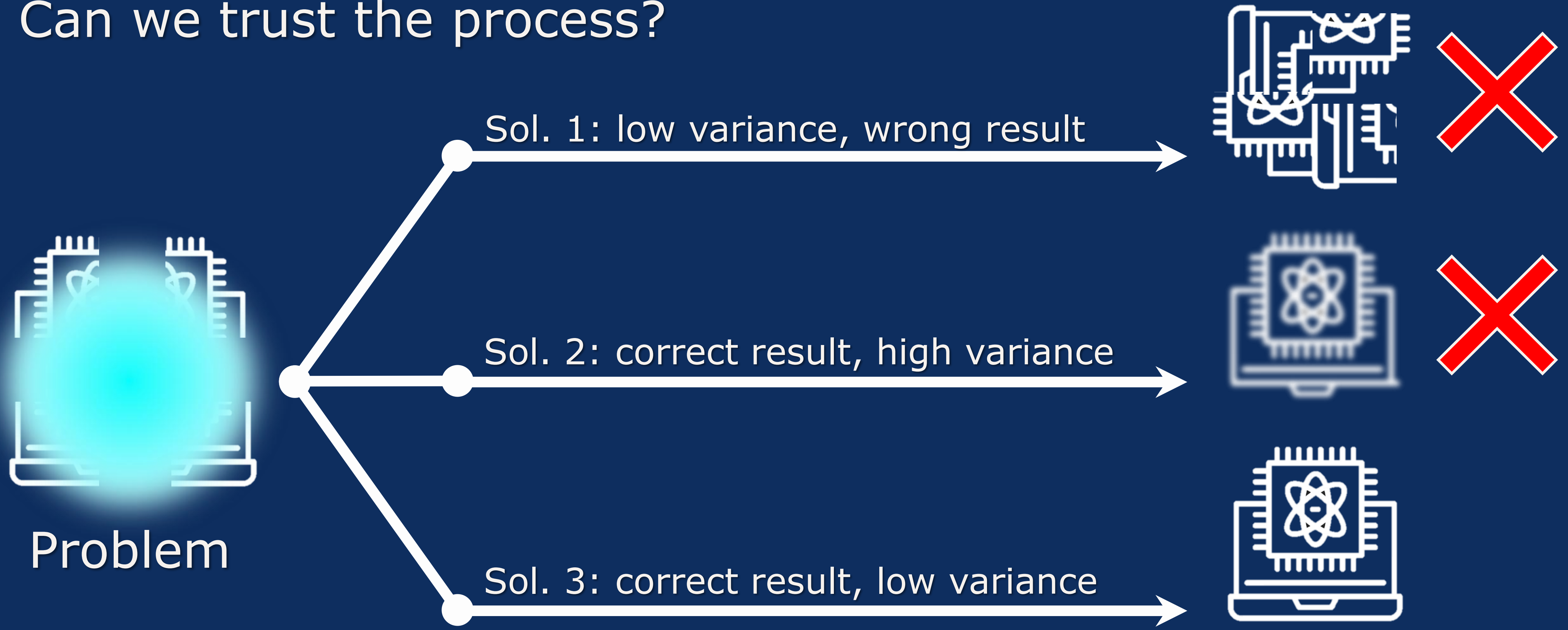

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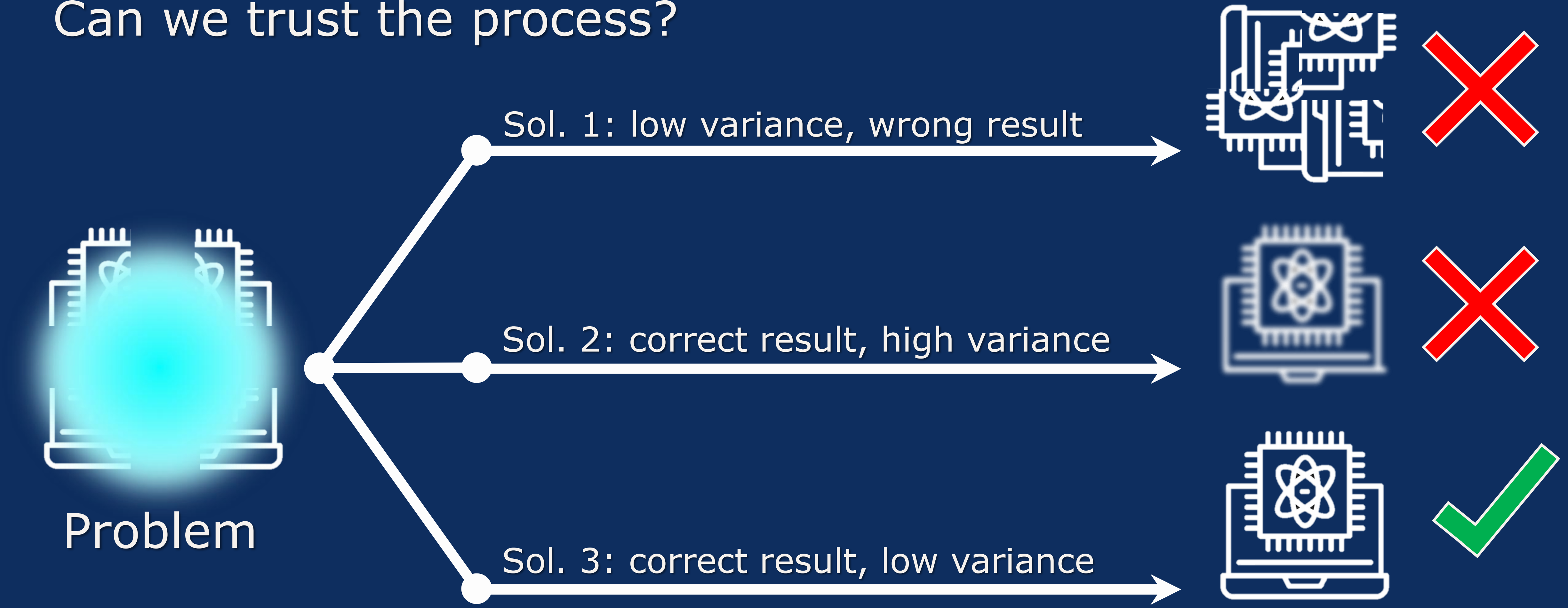

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A matter of thermodynamics

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Thermodynamics

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Thermodynamics

Thermodynamics

Thermodynamics

Thermodynamics

SYSTEM

S

Thermodynamics

SYSTEM

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Thermodynamics

SYSTEM

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ENVIRONMENT

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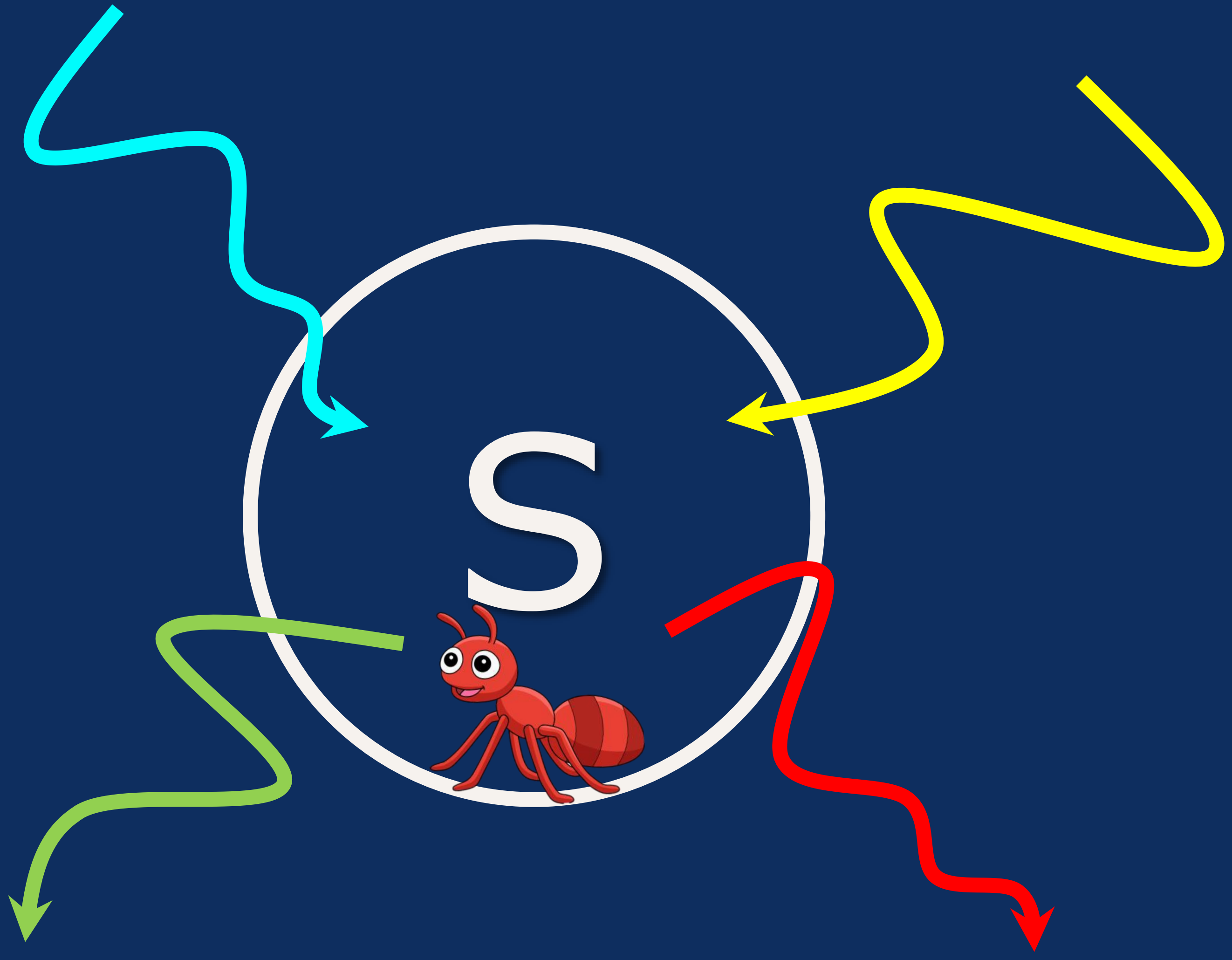
Thermodynamics

Thermodynamics

Thermodynamics

Thermodynamics

Thermodynamics



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Thermodynamics

Thermodynamics

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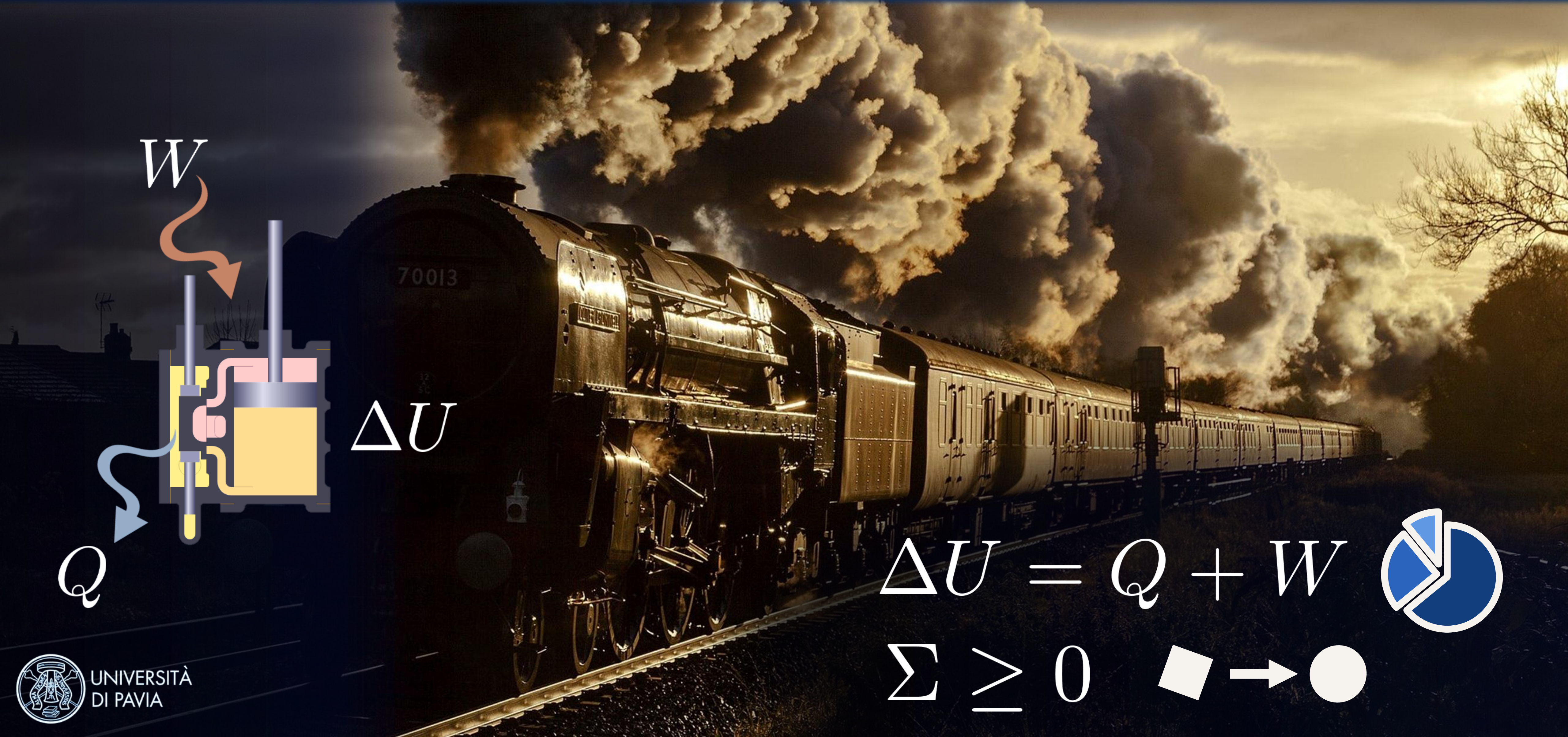
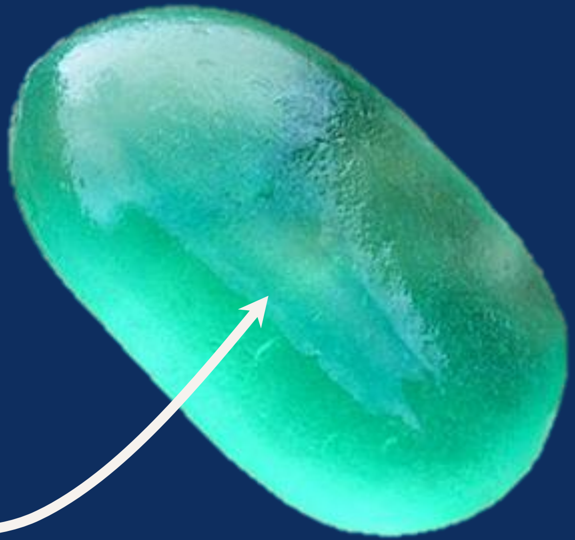
Thermodynamics

Thermodynamics



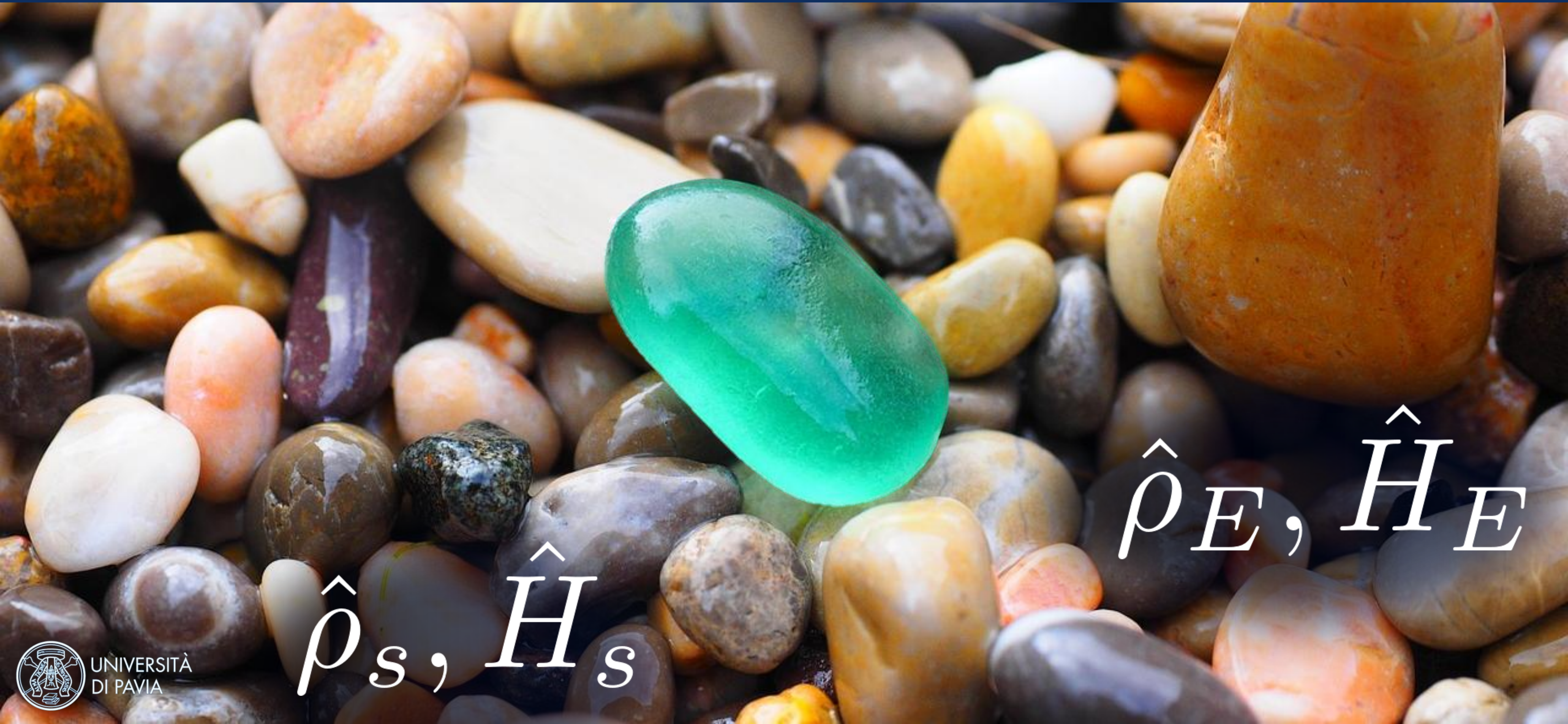
Quantum Thermodynamics

Quantum Thermodynamics

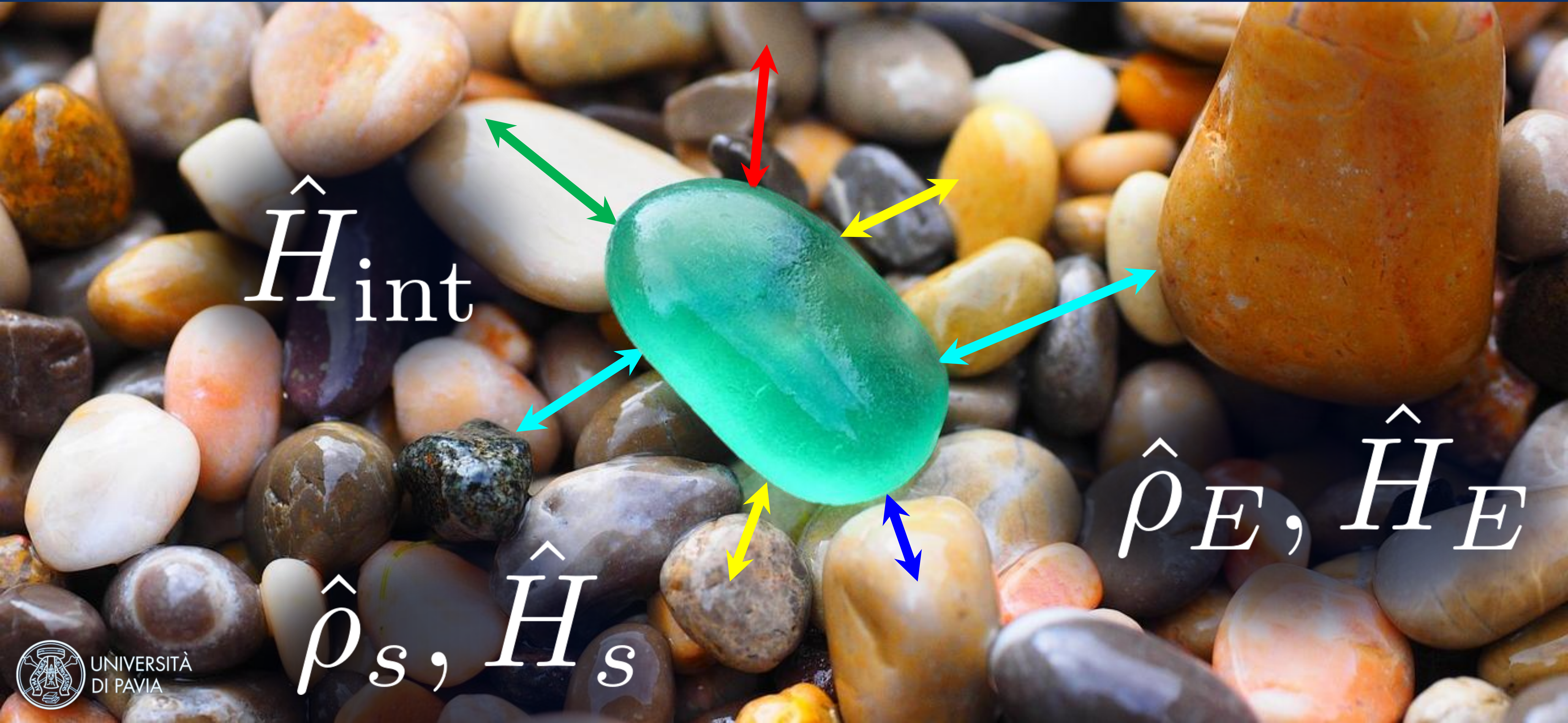


$$\hat{\rho}_S, \hat{H}_S$$

Quantum Thermodynamics



Quantum Thermodynamics



Quantum Thermodynamics

Quantum Thermodynamics

How to **test** it?

Quantum Thermodynamics

How to **test** it?

Can we exploit its results for **practical applications**?

Quantum Thermodynamics

How to **test** it?

Can we exploit its results for **practical applications**?

How does it fit into the **development of quantum technologies**?

Quantum batteries

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Quantum batteries

Energetics of a quantum object

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R. Alicki, M. Fannes, Phys. Rev. E, 2013, 87.4: 042123.

Quantum batteries

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Energetics of a quantum object

$$\hat{H}_{\mathrm{sys}} = \hat{H} = \sum_n \varepsilon_n \left|\varepsilon\right\rangle \left\langle\varepsilon\right|$$

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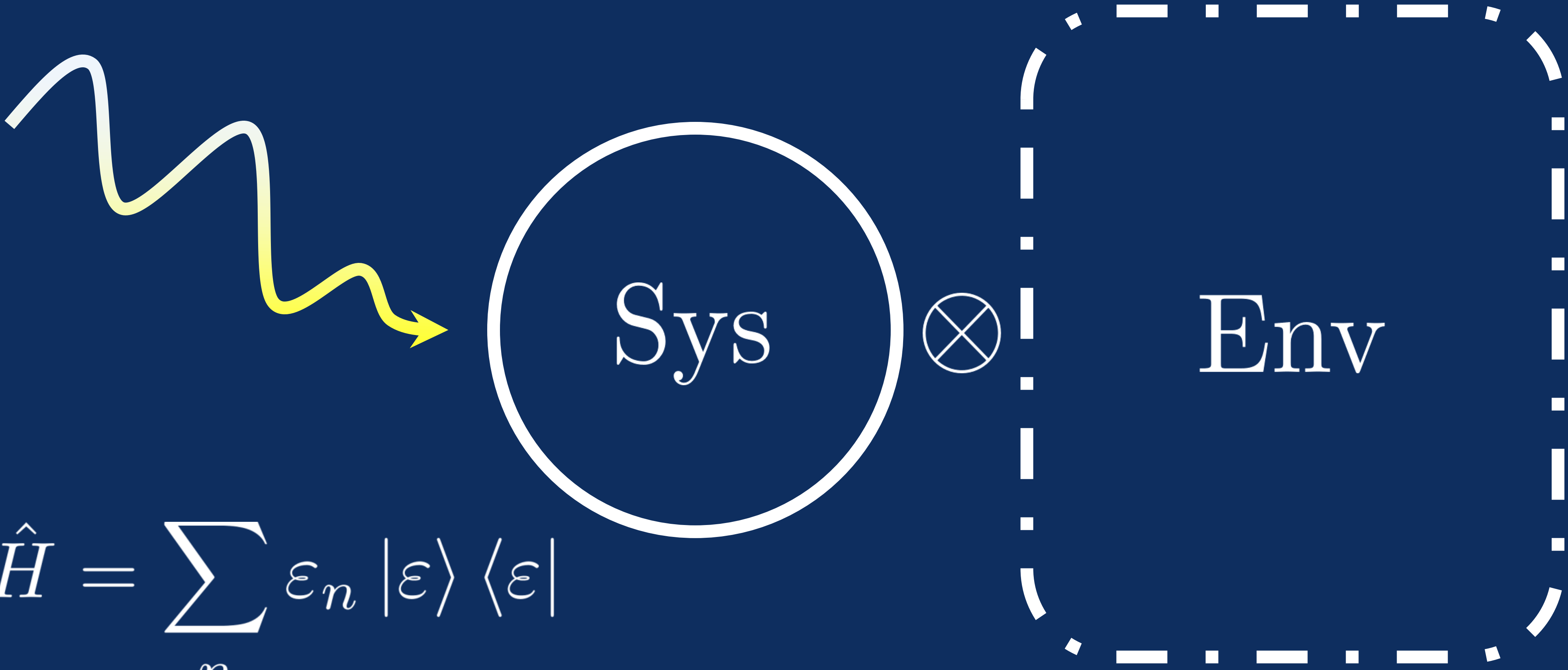

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Quantum batteries

Injecting, storing, delivering energy

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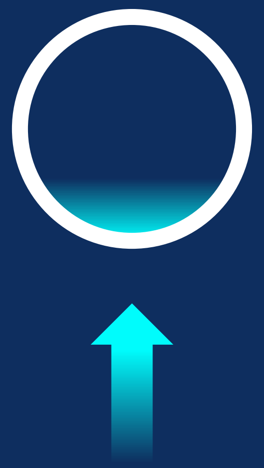
F. Campaioli *et al.*, Reviews of Modern Physics, 2024, 96.3: 031001.

Quantum batteries

Injecting, storing, delivering energy



Energy injection via
unitary operations

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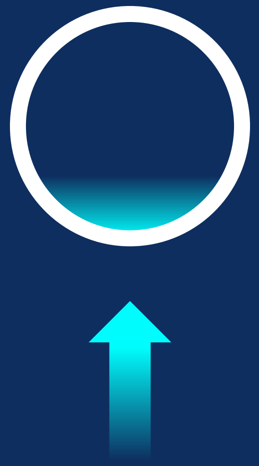
Quantum batteries

Injecting, storing, delivering energy



Energy injection via unitary operations

Stabilization

F. Campaioli *et al.*, Reviews of Modern Physics, 2024, 96.3: 031001.

Quantum batteries

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Quantum batteries

Energy injected in the system

Quantum batteries

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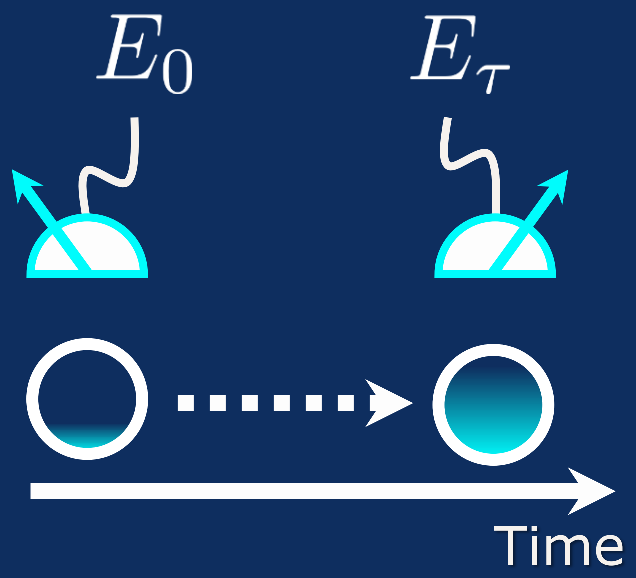

Quantum batteries

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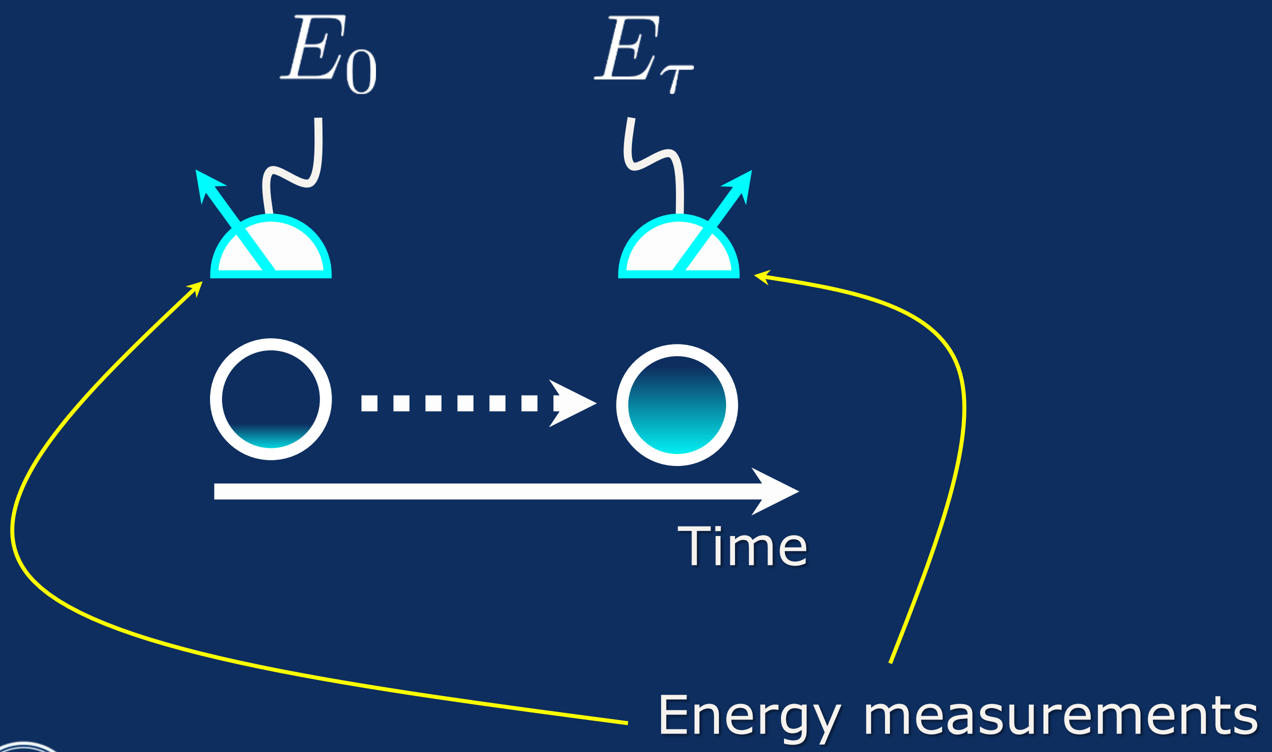

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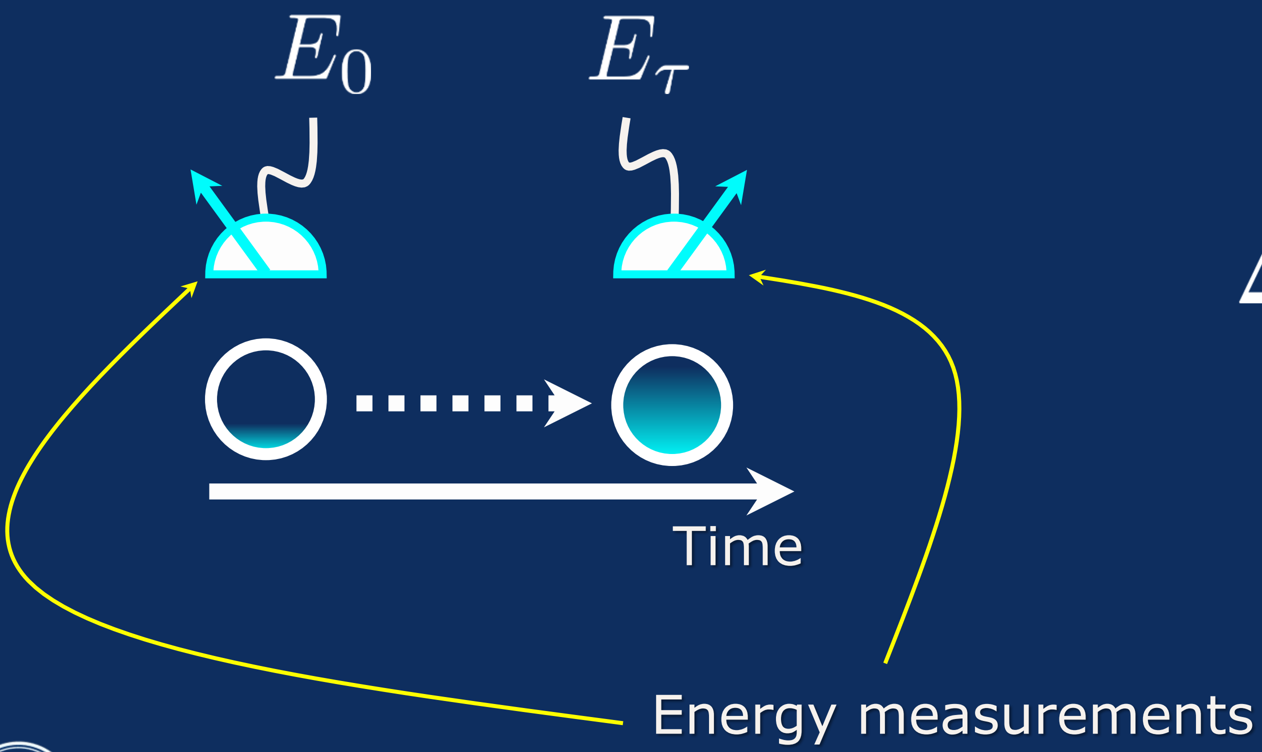

$$\Delta E_\tau = E_\tau - E_0$$

Quantum batteries

Quantum batteries

Test bed for Quantum Thermodynamics

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'Useful' energy management

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Highly-controlled structures

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Highly-controlled structures

Decoherence and **noise** mechanisms

Platform **efficiency**

S. Elghaayda *et al*., Advanced Quantum Technologies, 2025, 2400651.

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Weren't we talking about

energy fluctuations?

Energy fluctuations

How to estimate them?

Energy fluctuations

How to estimate them?

We need:

Energy fluctuations

How to estimate them?

We need:

A stochastic variable
representing an energy difference: ΔE_τ

Energy fluctuations

How to estimate them?

We need:

A stochastic variable
representing an energy difference: ΔE_τ

The probability distribution that governs it: $p(\Delta E_\tau)$

Energy fluctuations

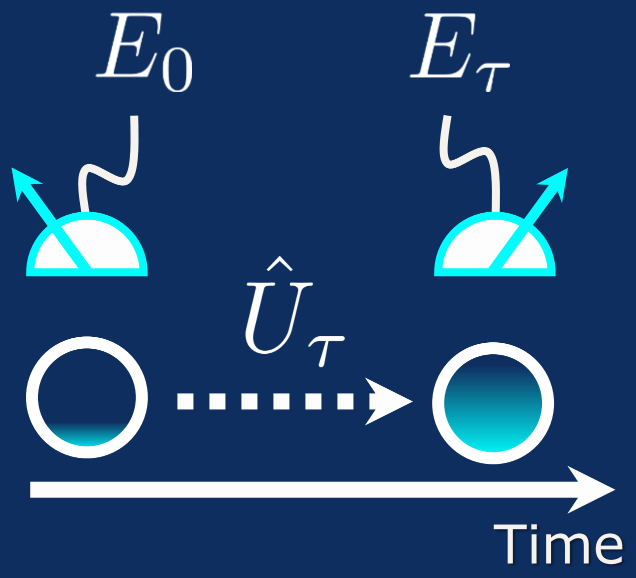
Tool: **Full Counting Statistics (FCS)**

M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

Energy fluctuations

Tool: **Full Counting Statistics (FCS)**

Two-point measurement
(TPM) scheme



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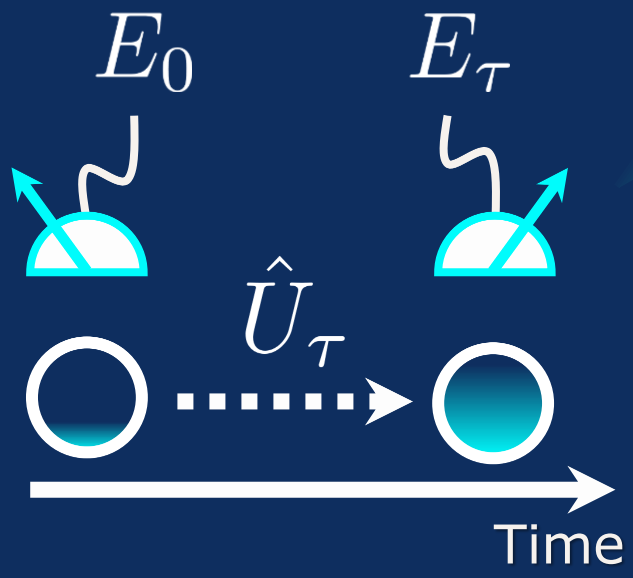

Generating function

$$G(\chi)$$

$$\updownarrow$$

$$p(\Delta E_\tau)$$

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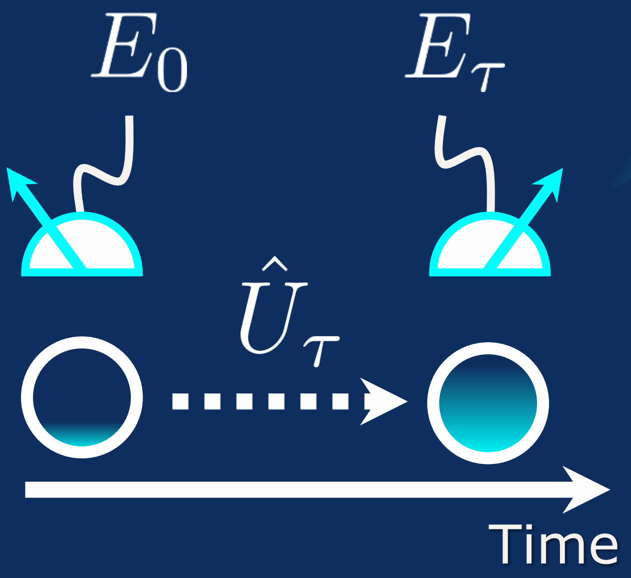


Generating function

$$G(\chi)$$

$$\updownarrow$$

$$p(\Delta E_\tau)$$

Moments and cumulants

$$\langle \Delta E_\tau \rangle$$

$$\mathrm{var}(\Delta E_\tau)$$

...and higher orders

M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

Energy fluctuations

The variance

$$\mathrm{var}(\Delta E_\tau)$$

Energy fluctuations

The variance

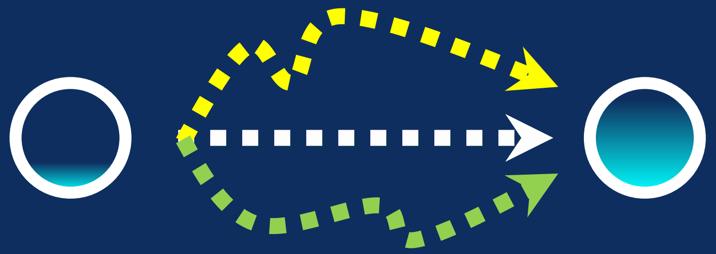
$$\mathrm{var}(\Delta E_\tau)$$

Initial and **final** states matter

Energy fluctuations

The variance

$$\mathrm{var}(\Delta E_\tau)$$



Initial and **final** states matter

All the in-between **processes** are considered

Energy fluctuations

Quantity and quality: the Signal-to-Noise Ratio (SNR)

Energy fluctuations

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$$\mathrm{SNR}(\Delta E_\tau) = \frac{\langle \Delta E_\tau \rangle^2}{\mathrm{var}(\Delta E_\tau)}$$

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Quantity and quality: the Signal-to-Noise Ratio (SNR)

$$\mathrm{SNR}(\Delta E_\tau) = \frac{\langle \Delta E_\tau \rangle^2}{\mathrm{var}(\Delta E_\tau)}$$

Signal:
squared mean
injected energy

Energy fluctuations

Quantity and quality: the Signal-to-Noise Ratio (SNR)

$$\mathrm{SNR}(\Delta E_\tau) = \frac{\langle \Delta E_\tau \rangle^2}{\mathrm{var}(\Delta E_\tau)}$$

Signal:
squared mean
injected energy

Noise:
variance of the
mean injected energy

A Jaynes-Cummings quantum battery

The Jaynes-Cummings model

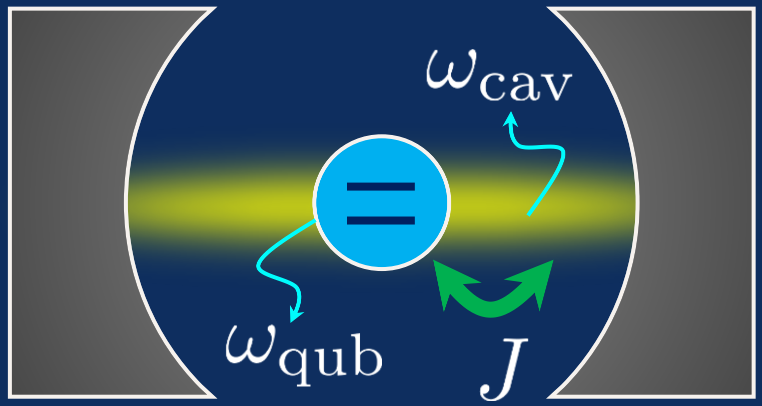
Jaynes-Cummings (JC) in rotating wave approximation (RWA)

S. Stenholm, Physics Reports 6, 1 (1973).

The Jaynes-Cummings model

Jaynes-Cummings (JC) in rotating wave approximation (RWA)

$$\hat{H} = \hbar\omega_{\mathrm{qub}}\hat{\sigma}_+\hat{\sigma}_- + \hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a} + \hbar J(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$



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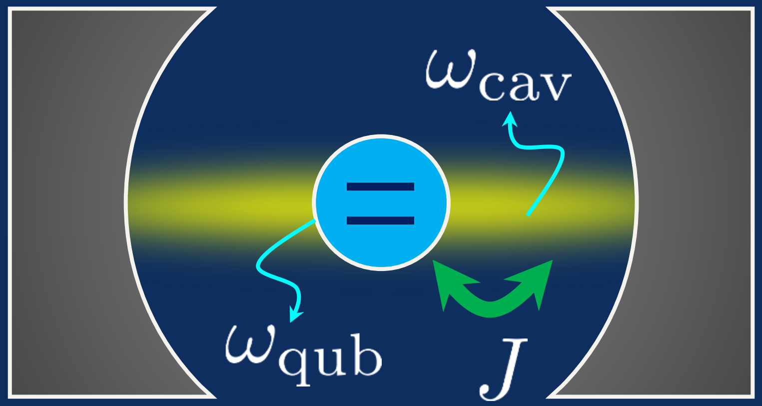
Qubit



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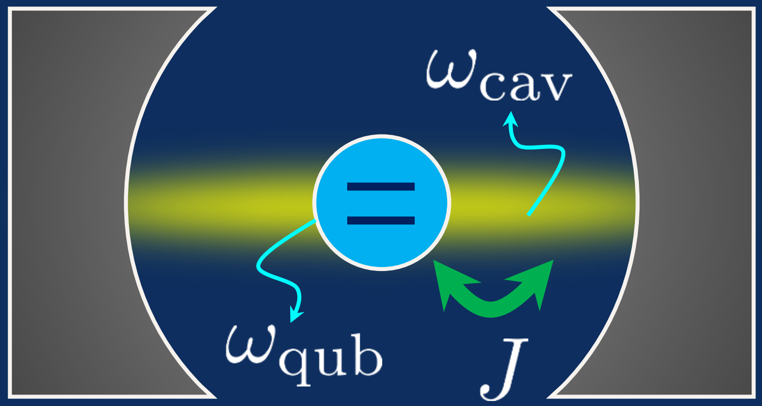
Qubit　　　　Cavity



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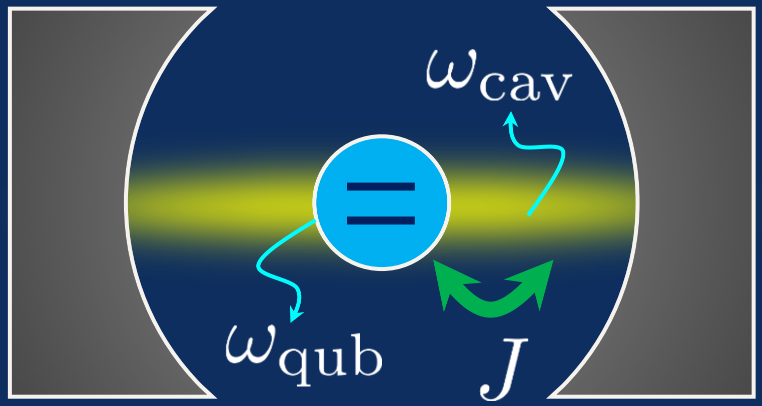
$$\hat{H} = \underbrace{\hbar\omega_{\mathrm{qub}}\hat{\sigma}_+\hat{\sigma}_-}_{\text{Qubit}} + \underbrace{\hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a}}_{\text{Cavity}} + \underbrace{\hbar J(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)}_{\text{Interaction}}$$



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S. Stenholm, Physics Reports 6, 1 (1973).

The Jaynes-Cummings model

Evolution

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The Jaynes-Cummings model

Evolution

$$\hat{U}_\tau = \begin{pmatrix} \hat{U}_{00} & \hat{U}_{01} \\ \hat{U}_{10} & \hat{U}_{11} \end{pmatrix}$$

S. Stenholm, Physics Reports 6, 1 (1973).

The Jaynes-Cummings model

Evolution

Cavity operators

$$\hat{U}_\tau = \begin{pmatrix} \hat{U}_{00} & \hat{U}_{01} \\ \hat{U}_{10} & \hat{U}_{11} \end{pmatrix}$$

JC evolution operator:
qubit + cavity

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The Jaynes-Cummings model

Evolution

Cavity operators

$$\hat{U}_\tau = \begin{pmatrix} \hat{U}_{00} & \hat{U}_{01} \\ \hat{U}_{10} & \hat{U}_{11} \end{pmatrix}$$

Analytically known

JC evolution operator:
qubit + cavity

S. Stenholm, Physics Reports 6, 1 (1973).

Result 1:

Fluctuations are reduced by quantum and non-Gaussian cavity states

$$|\mathrm{Q}\rangle$$

Results

Goal: use the JC interaction to obtain the transition:

$$|g\rangle\langle g| \longrightarrow |e\rangle\langle e|$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

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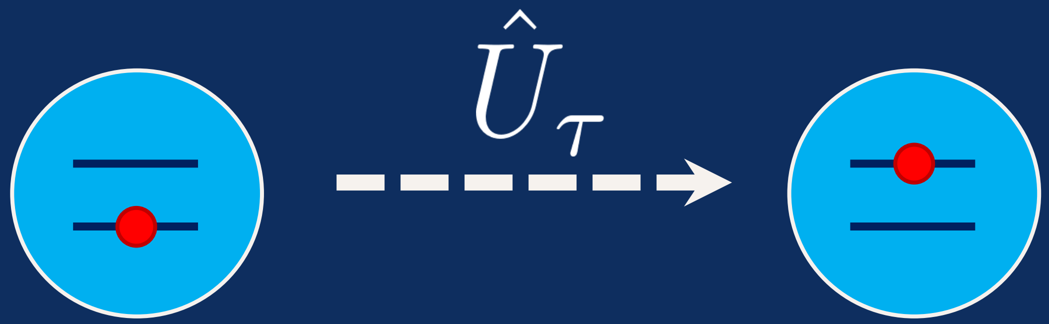

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Results

Trick:

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Results

Trick:

$$\hat{U}_\tau$$

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Results

Trick:

$$\hat{U}_{\tau} \longrightarrow \hat{U}_{\chi,\tau}$$

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$$\hat{U}_{\tau} \longrightarrow \hat{U}_{\chi,\tau} \longrightarrow \hat{\rho}_{\chi,\tau}$$

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Physical Review A, 2025, 112.1: 012205.

Results

Trick:

$$\hat{U}_\tau \longrightarrow \hat{U}_{\chi,\tau} \longrightarrow \hat{\rho}_{\chi,\tau}$$

$$\longrightarrow G(\chi) = \mathrm{Tr}\{\hat{\rho}_{\chi,\tau}\}$$

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Results

Key quantity: the SNR for the qubit injected energy

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Results

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$$\mathrm{SNR}(\Delta E_\tau) = \frac{J^2 N \mathcal{S}^2(\tau)}{1 - J^2 N \mathcal{S}^2(\tau)}$$

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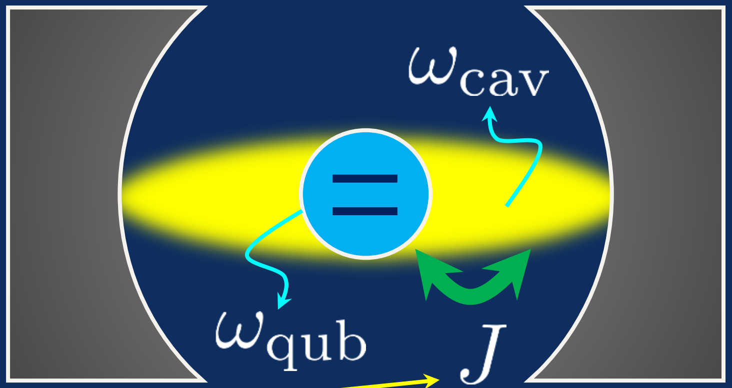
Results

Key quantity: the SNR for the qubit injected energy

$$\mathrm{SNR}(\Delta E_\tau) = \frac{J^2 N \mathcal{S}^2(\tau)}{1 - J^2 N \mathcal{S}^2(\tau)}$$



D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

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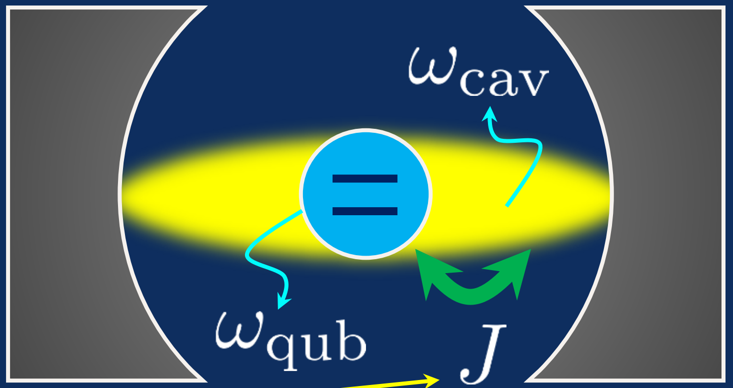

$$\hat{\rho}_0^{\mathrm{qub}} = |g\rangle\langle g|$$

$$\hat{\rho}_0^{\mathrm{cav}} = |N\rangle\langle N|$$

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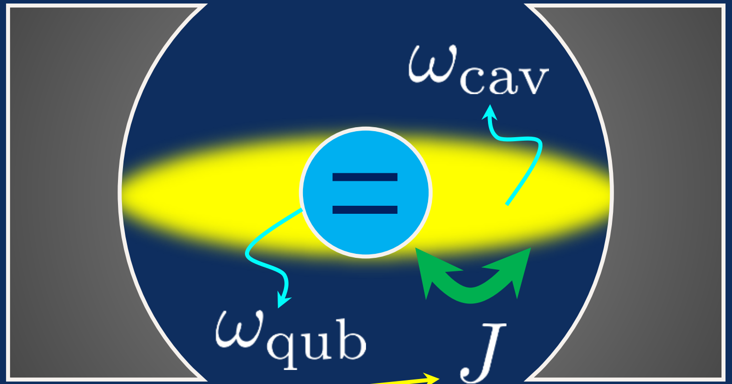

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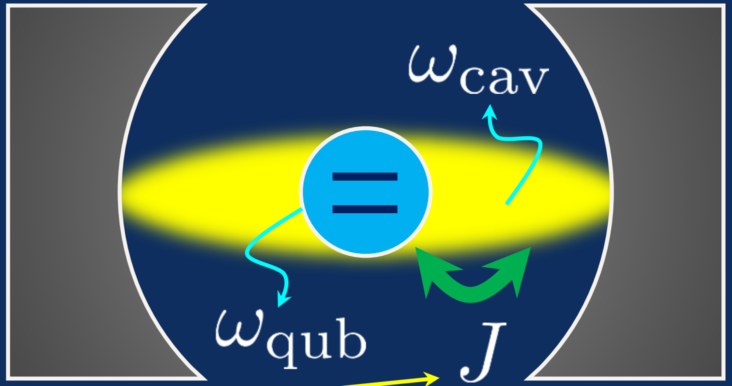

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$$\Delta\omega = \omega_{\mathrm{qub}} - \omega_{\mathrm{cav}}$$

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Results

Which cavity state allows for the best performance?

$$\hat{\rho}_0 = \hat{\rho}_0^{\mathrm{qub}} \otimes \hat{\rho}_0^{\mathrm{cav}}$$

$$\hat{\rho}_0^{\mathrm{qub}} \downarrow |g\rangle\langle g| \qquad \hat{\rho}_0^{\mathrm{cav}} \downarrow \; ?$$

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Results

Which cavity state allows for the best performance?

$$e^{-\beta \hat{H}_{\mathrm{cav}}}/\mathrm{Tr}\{e^{-\beta \hat{H}_{\mathrm{cav}}}\}$$

Thermal: ‘classical’

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$$|\alpha\rangle\langle\alpha|$$

Coherent: ‘classical’

$$|\alpha,\xi\rangle\langle\alpha,\xi|$$

Squeezed coherent: quantum Gaussian

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$e^{-\beta \hat{H}_{\mathrm{cav}}} / \mathrm{Tr}\{e^{-\beta \hat{H}_{\mathrm{cav}}}\}$ **Thermal**: ‘classical’

$\left|\alpha\right\rangle \left\langle\alpha\right|$ **Coherent**: ‘classical’

$\left|\alpha, \xi\right\rangle \left\langle\alpha, \xi\right|$ **Squeezed coherent**: quantum Gaussian

$\left|N\right\rangle \left\langle N\right|$ **Fock**: quantum non-Gaussian

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Results

Which cavity state allows for the best performance?

Given the same number of initial photons

$$e^{-\beta \hat{H}_{\mathrm{cav}}}/\mathrm{Tr}\{e^{-\beta \hat{H}_{\mathrm{cav}}}\}$$

Thermal: ‘classical’

$$|\alpha\rangle\langle\alpha|$$

Coherent: ‘classical’

$$|\alpha,\xi\rangle\langle\alpha,\xi|$$

Squeezed coherent: quantum Gaussian

$$|N\rangle\langle N|$$

Fock: quantum non-Gaussian

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Results

Correctness: the process leads to the target state

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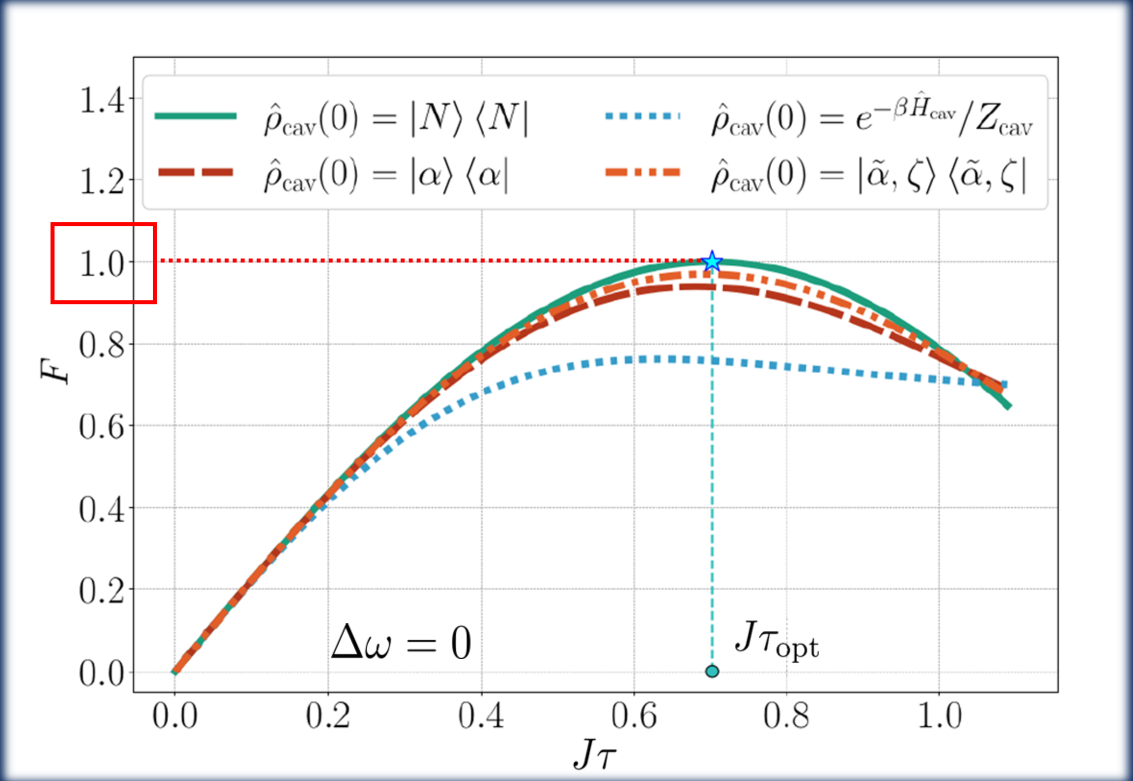

Fidelity: $F = F(\hat{\rho}_\tau^{\rm qub}, |e\rangle\langle e|)$

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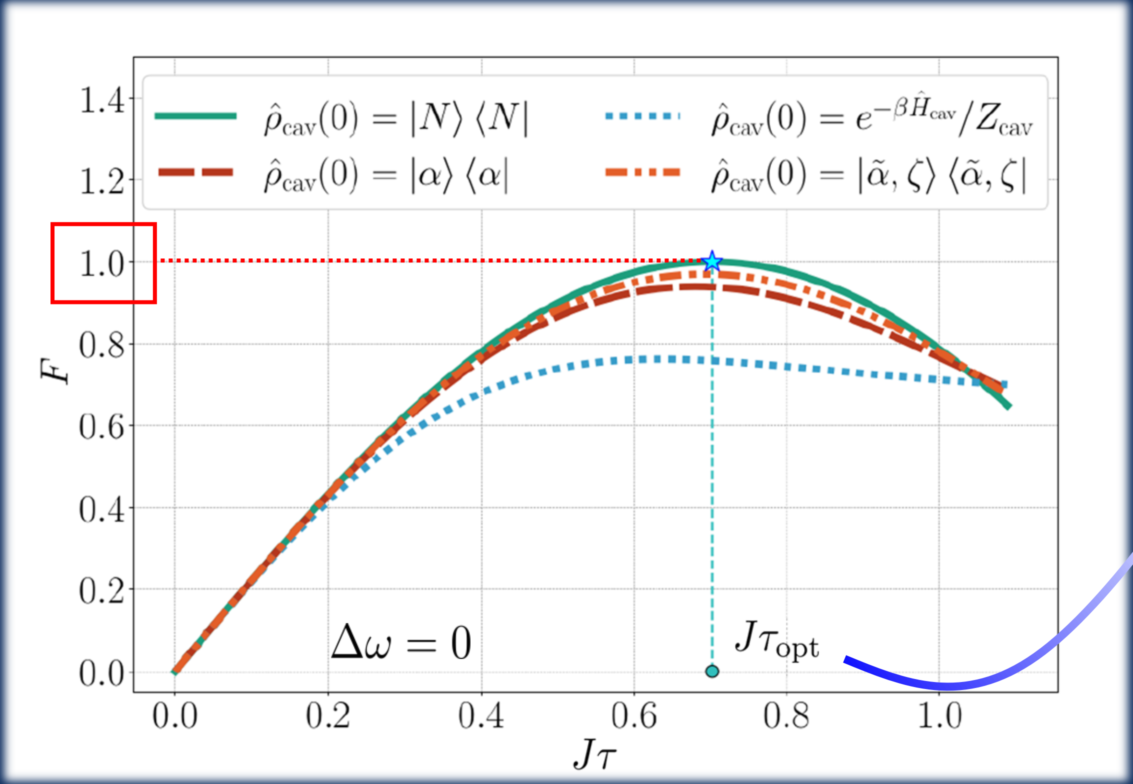

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At time $J\tau_{\mathrm{opt}}$, the fidelity is maximized

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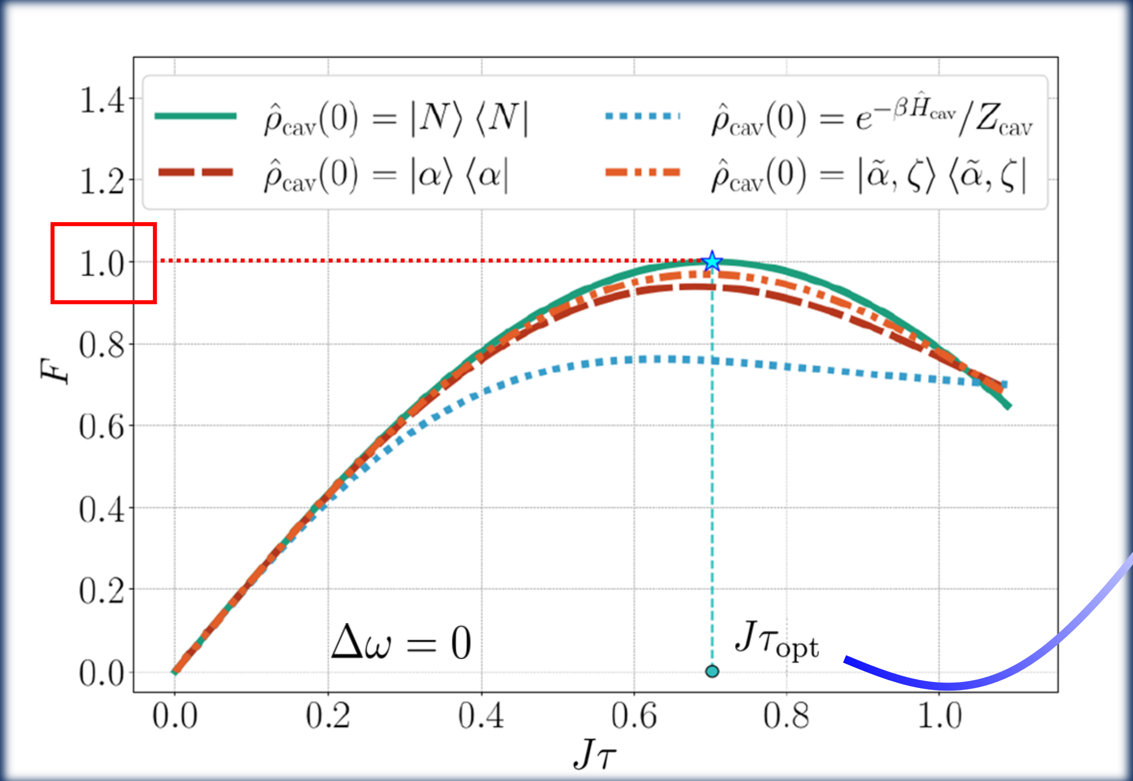

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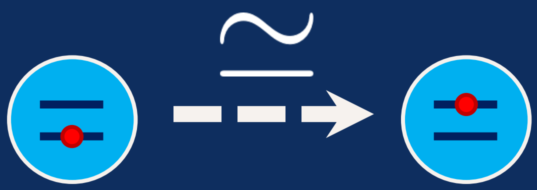

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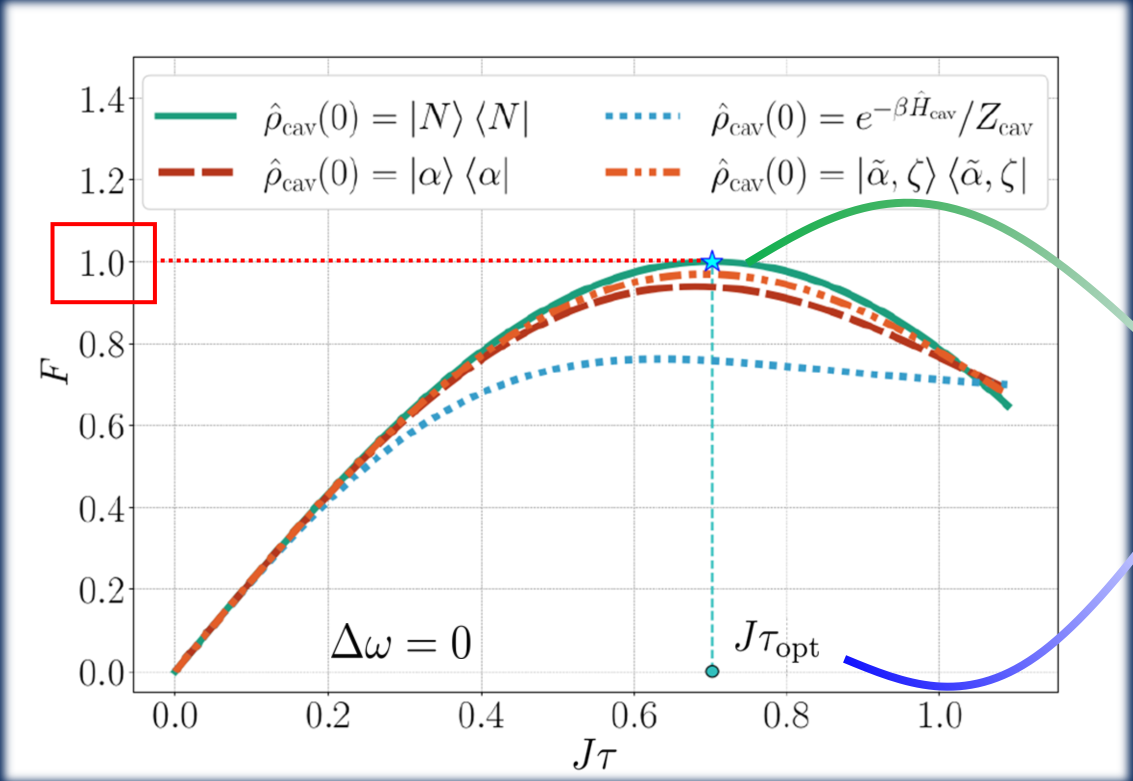

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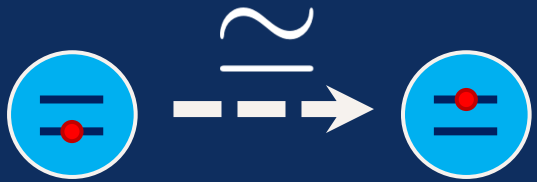

Fock state cavity: $F_{\rm max} = 1$

if $\Delta\omega = 0$

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Reliability: the SNR is maximized

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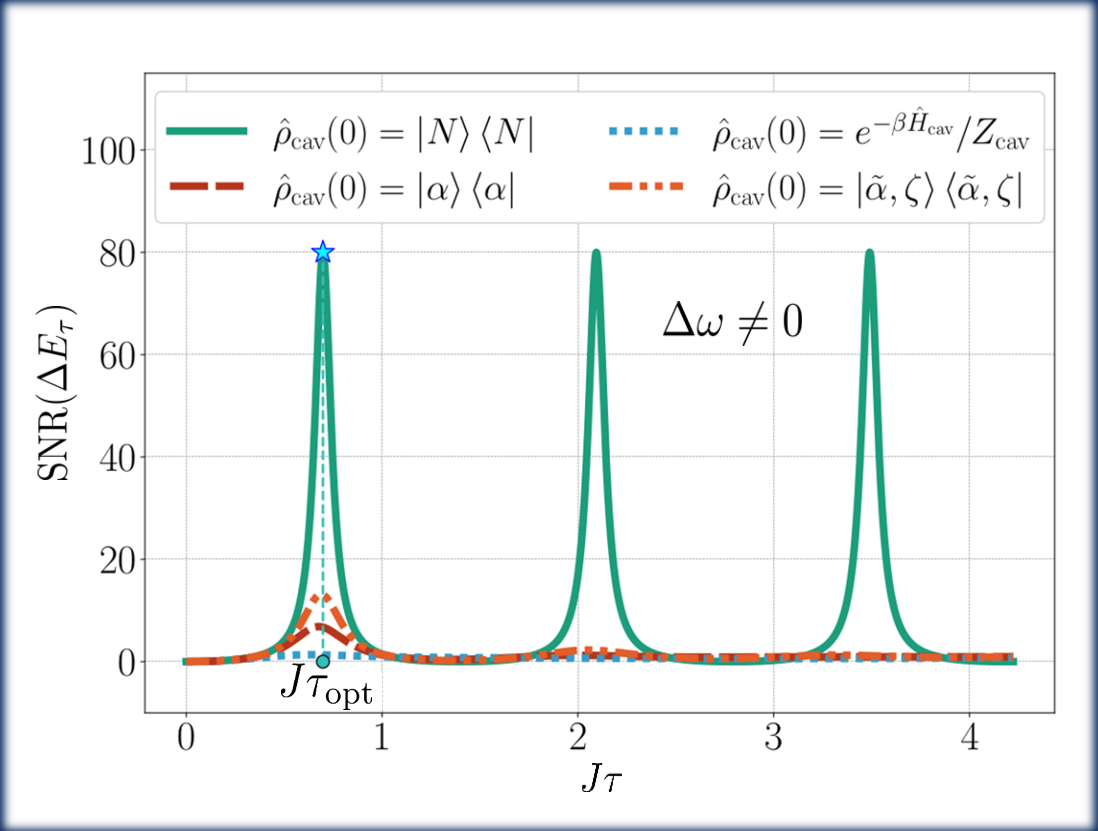

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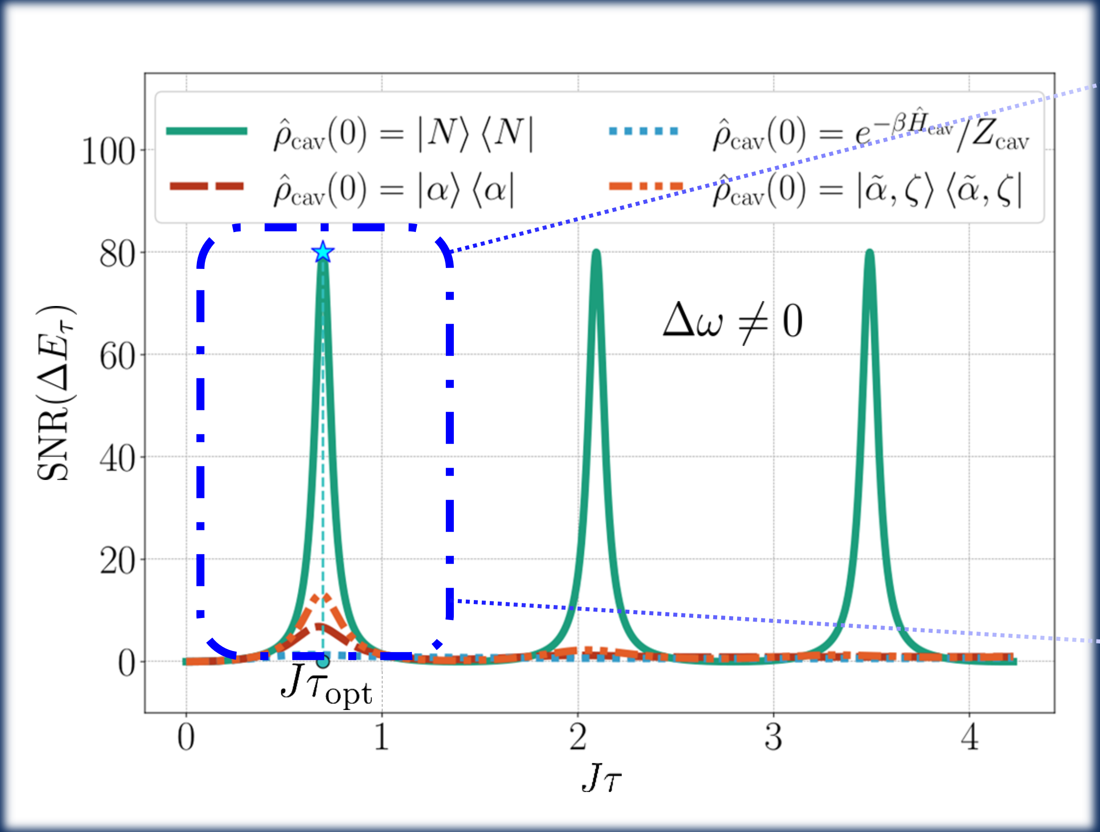

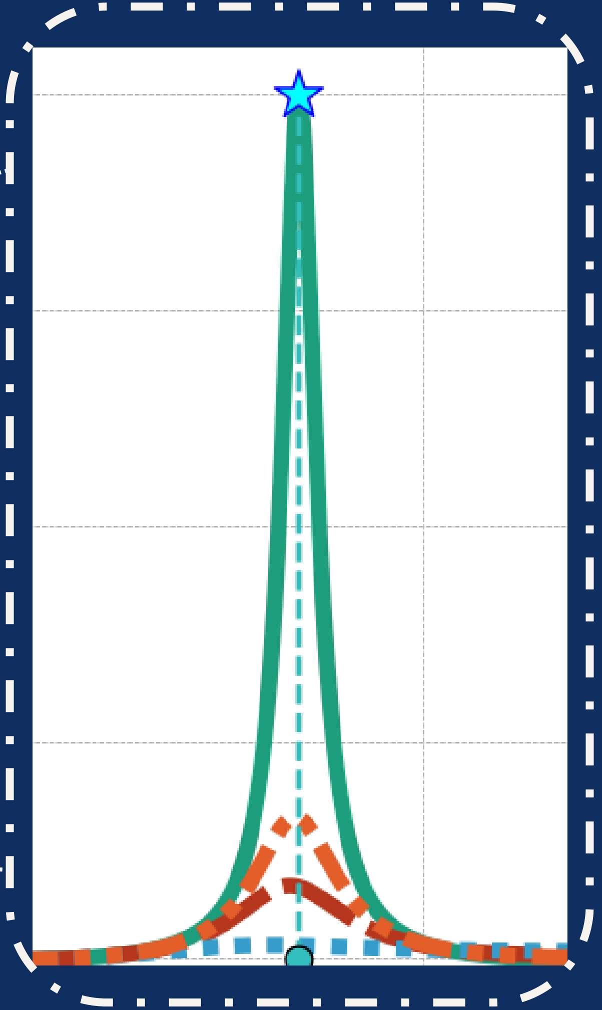

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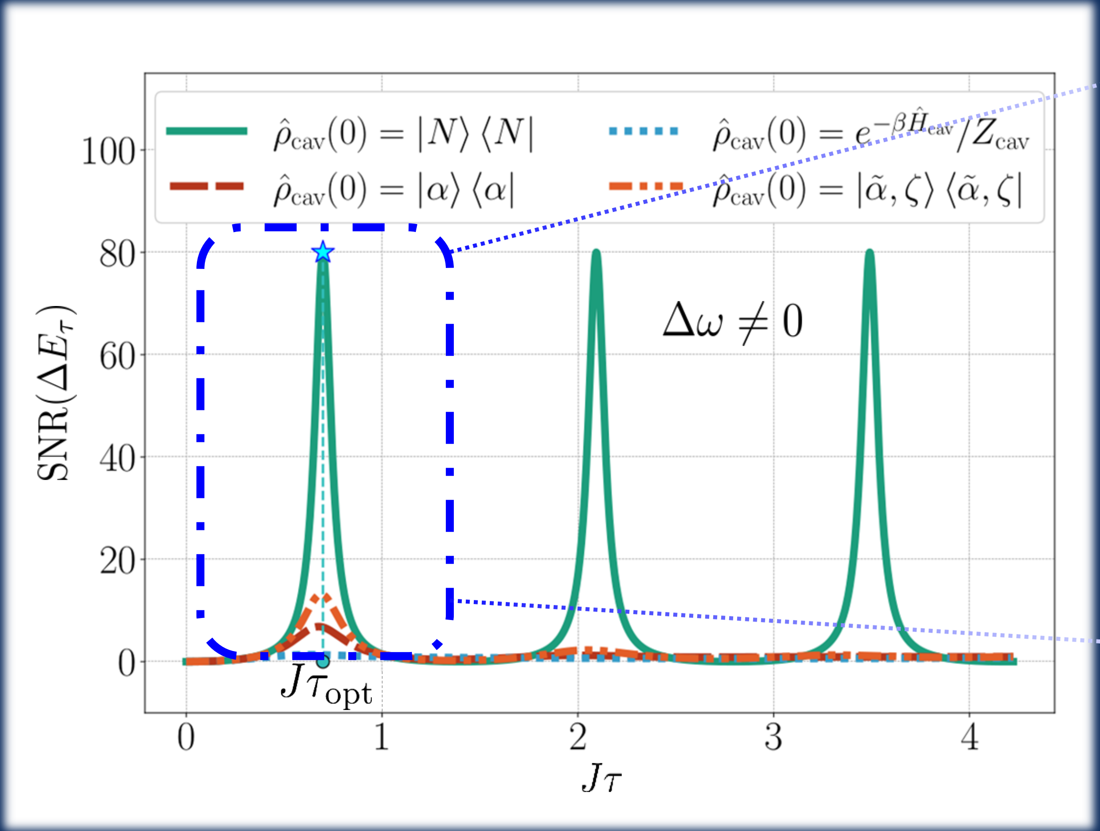

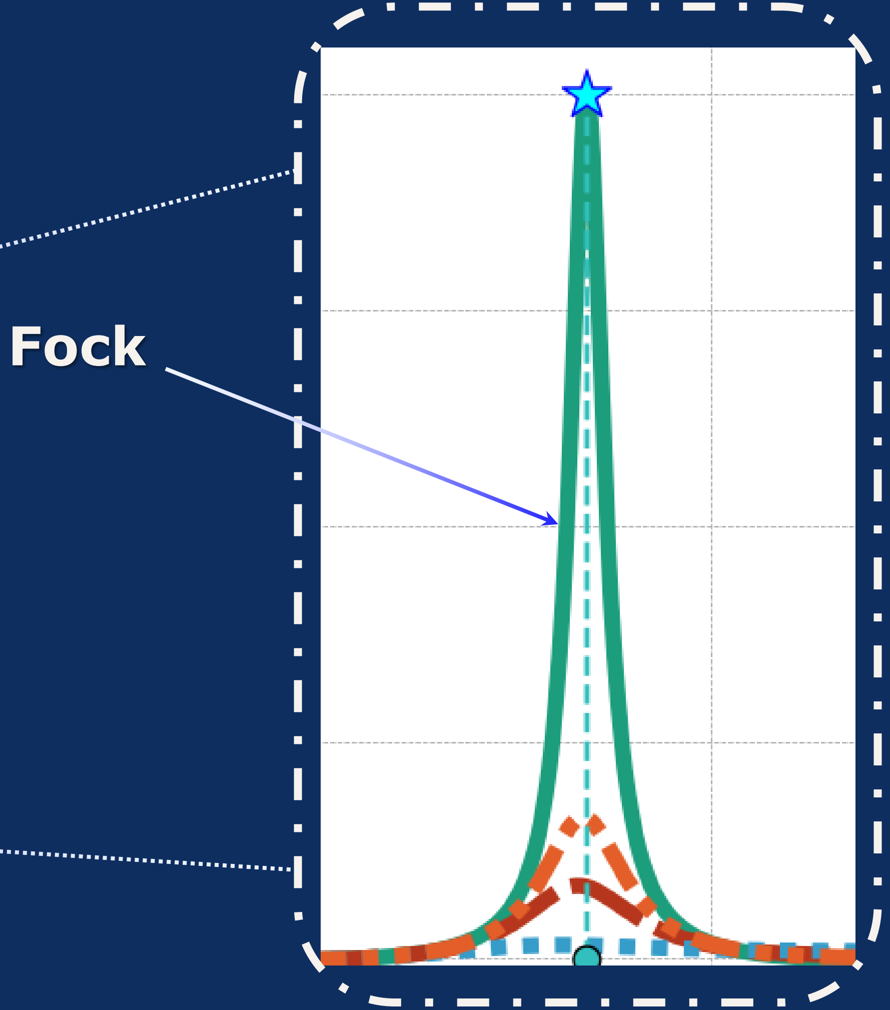

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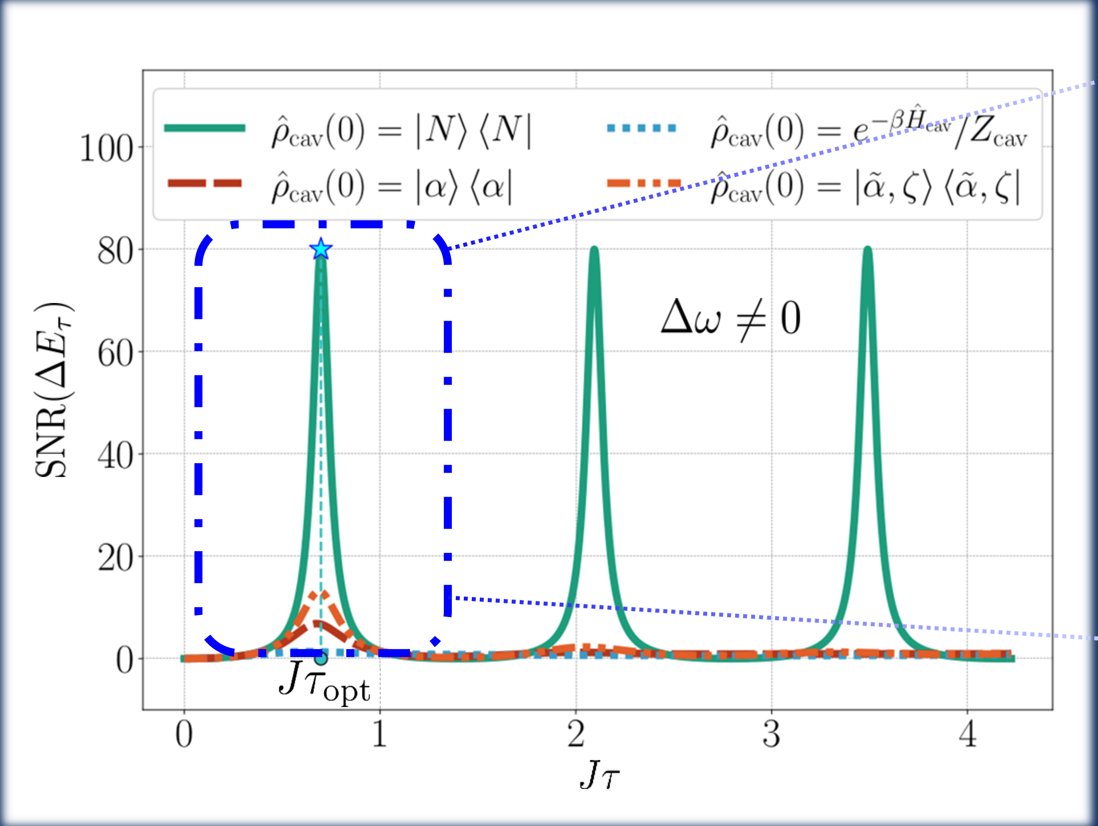

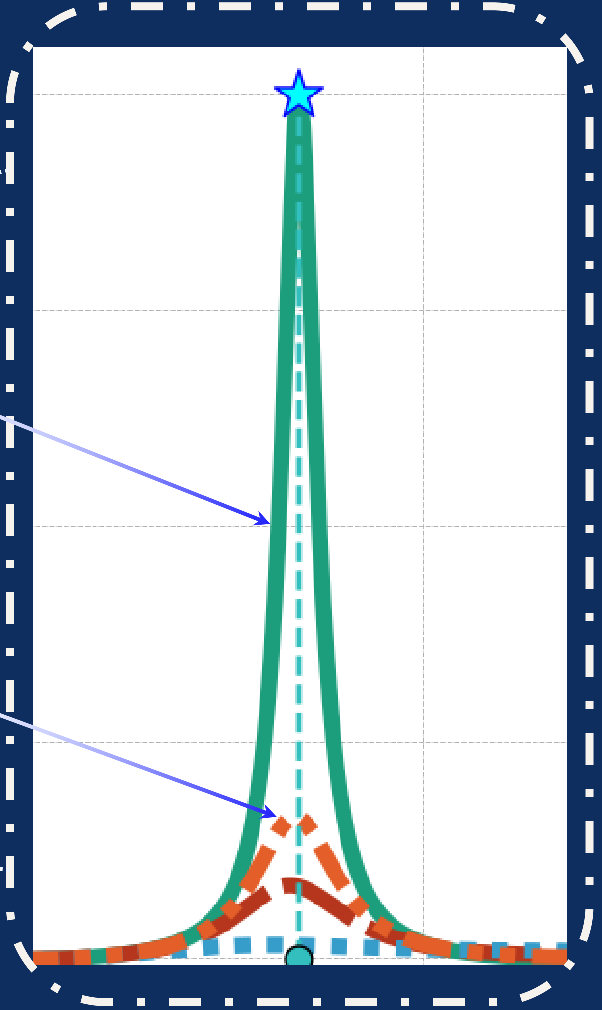

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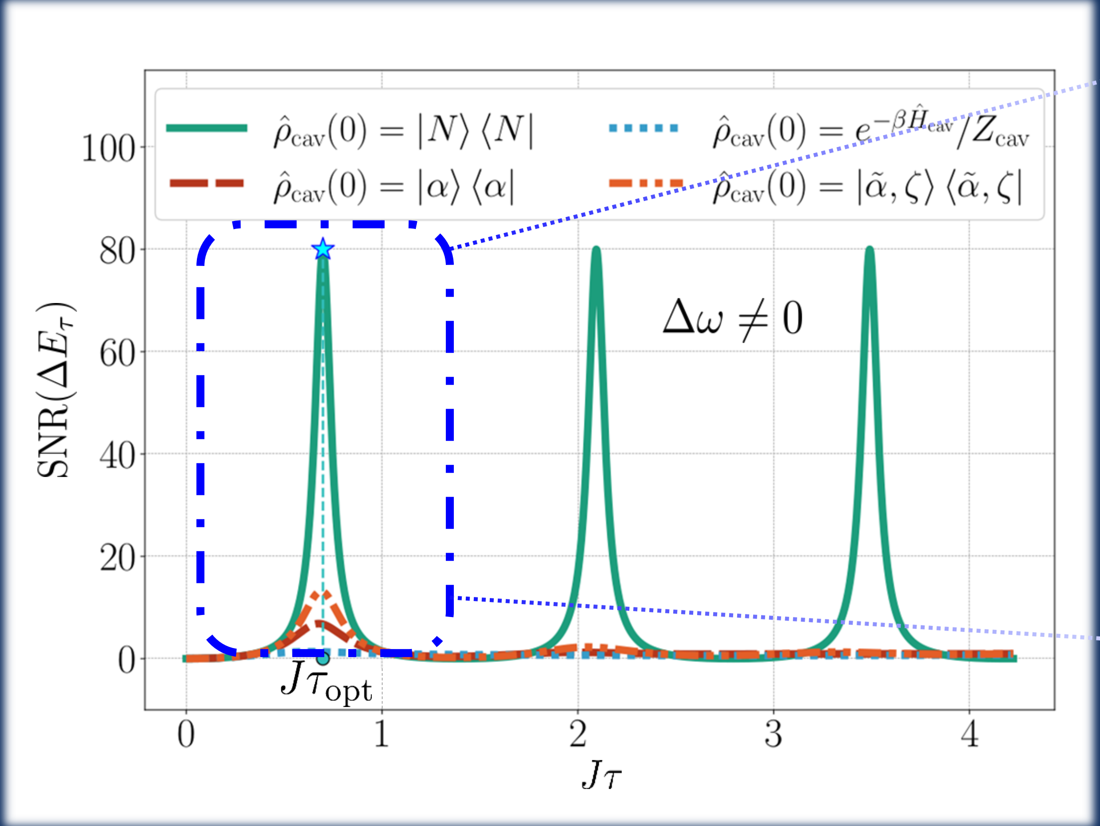

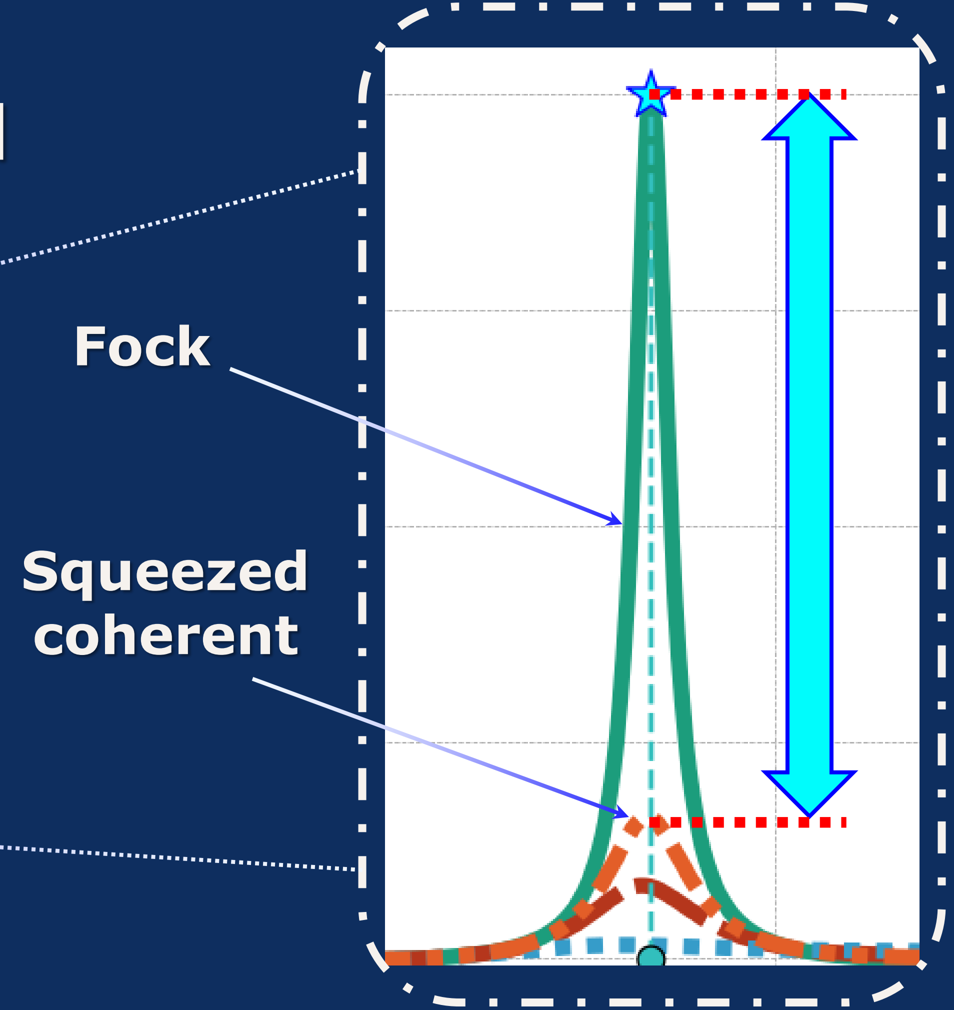

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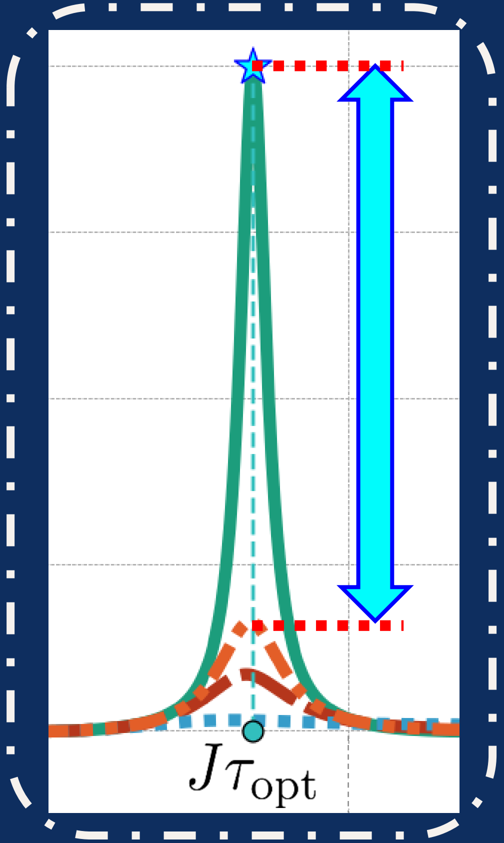
Results

A *quantum non-Gaussian* cavity state performs better



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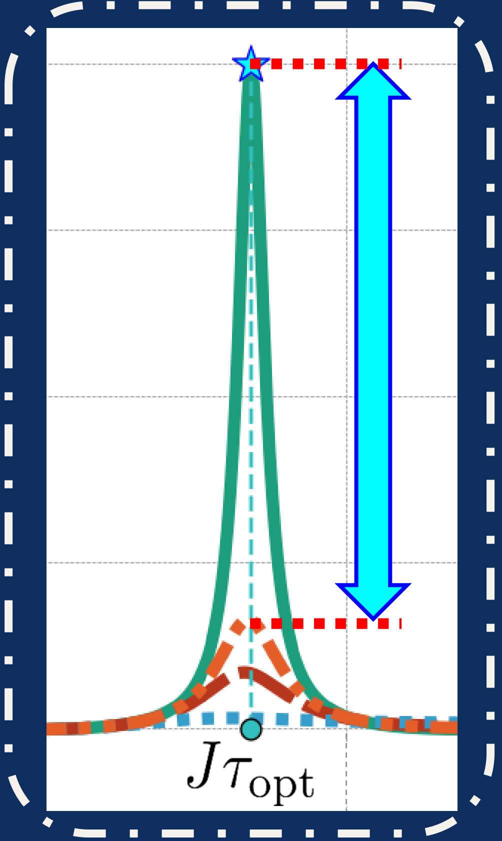

Fock state fidelity: $F_{\mathrm{max}} = 1$ if $\Delta\omega = 0$

Other states: $F_{\mathrm{max}} < 1$

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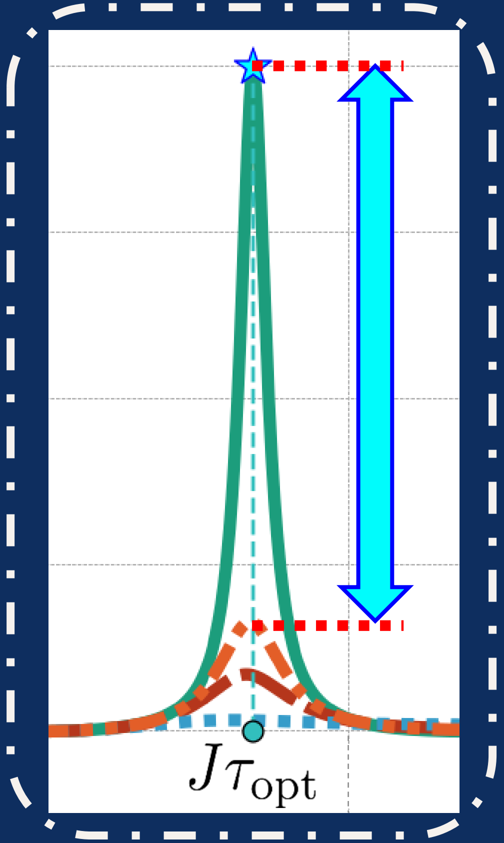

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→ Correct outcome

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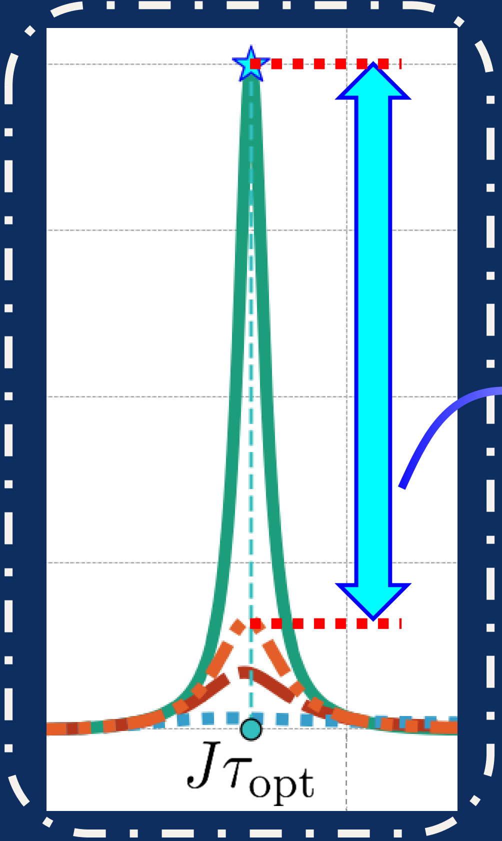

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Other states: $F_{\max} < 1$

$\longrightarrow$ Correct outcome

Efficiency advantage:

Fluctuations are reduced
thanks to the quantum non-Gaussianity

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Physical Review A, 2025, 112.1: 012205.

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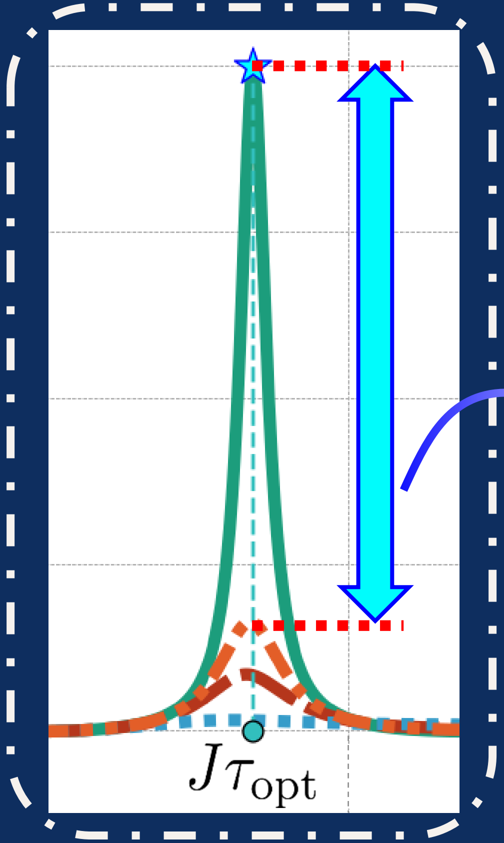

Fock state fidelity: $F_{\max} = 1$ if $\Delta\omega = 0$

Other states: $F_{\max} < 1$

⟶ Correct outcome

Efficiency advantage:

Fluctuations are reduced thanks to the quantum non-Gaussianity

⟶ Reliable outcome

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Result 2:

The advantage is robust under noise

Noise

Non-perfect state preparation

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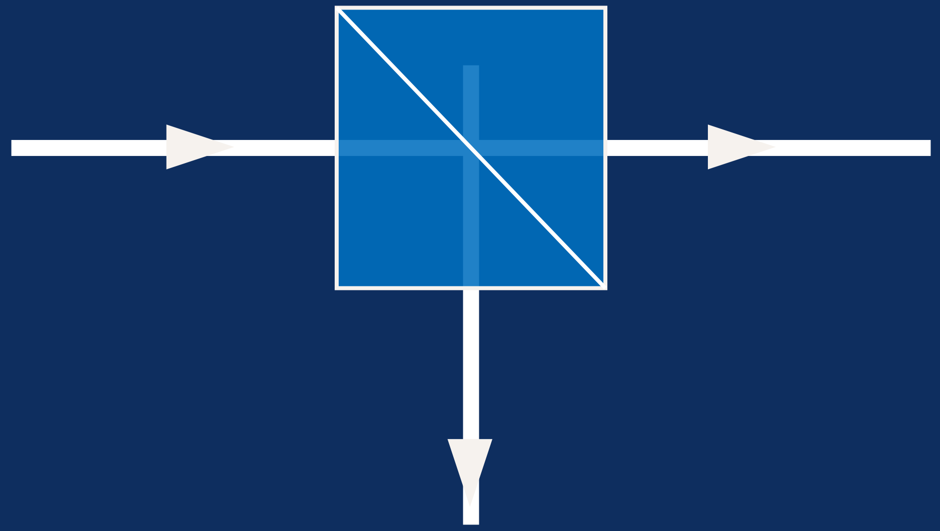
Noise

Non-perfect state preparation

Beam splitter



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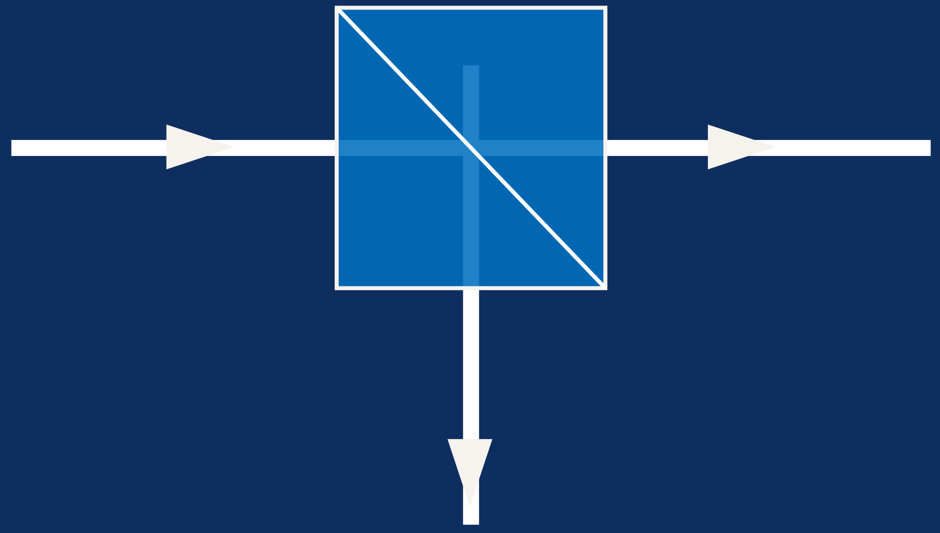
Noise

Non-perfect state preparation

Beam splitter

$$|N\rangle \langle N|$$



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Noise

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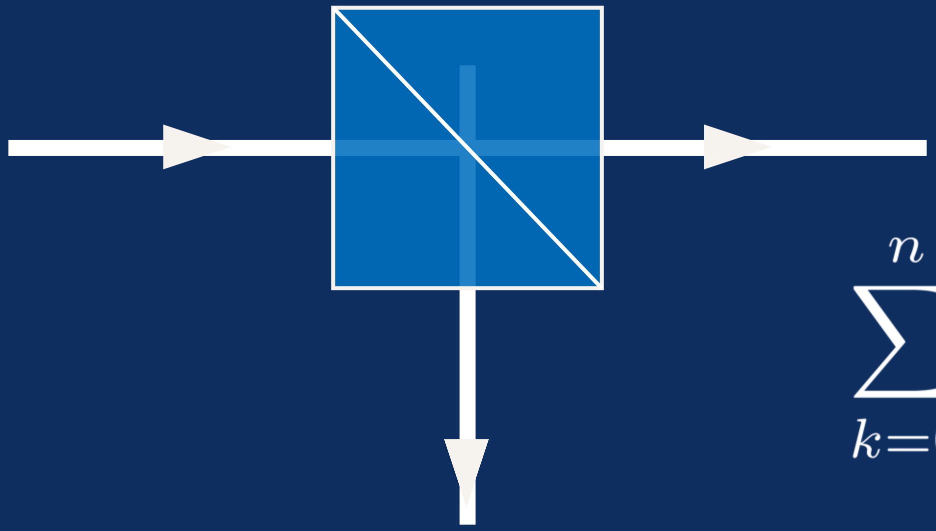

Attenuated state

$$\sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k}|k\rangle\langle k|$$

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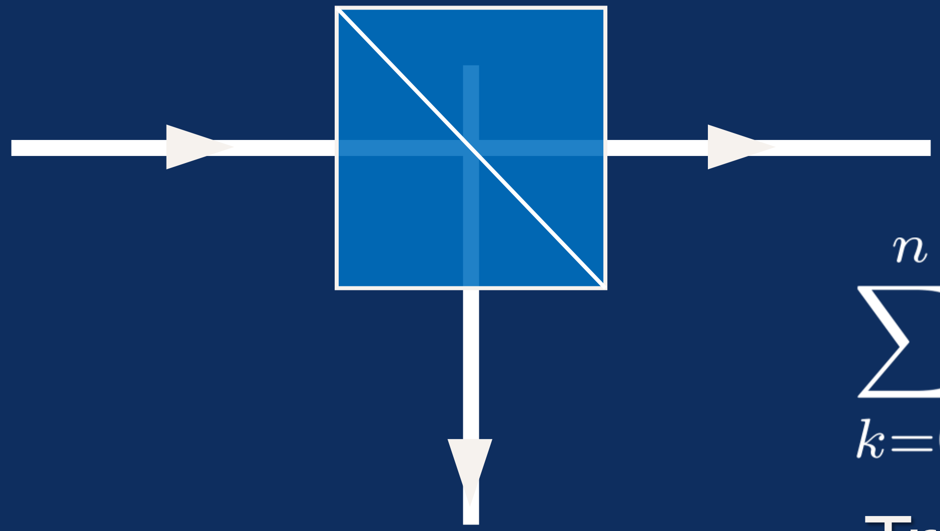
Noise

Non-perfect state preparation

Beam splitter

$$|N\rangle\langle N|$$



Attenuated state

$$\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} |k\rangle\langle k|$$

Transmissivity: $p = t^2$

Reflectivity: $1 - p = r^2$

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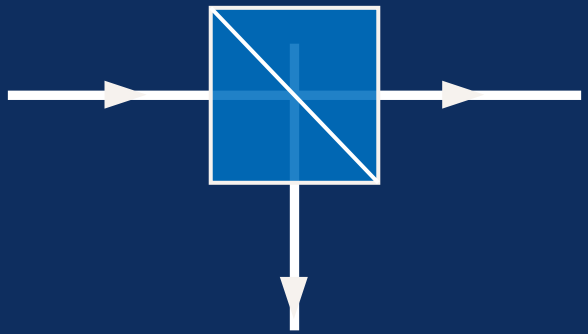
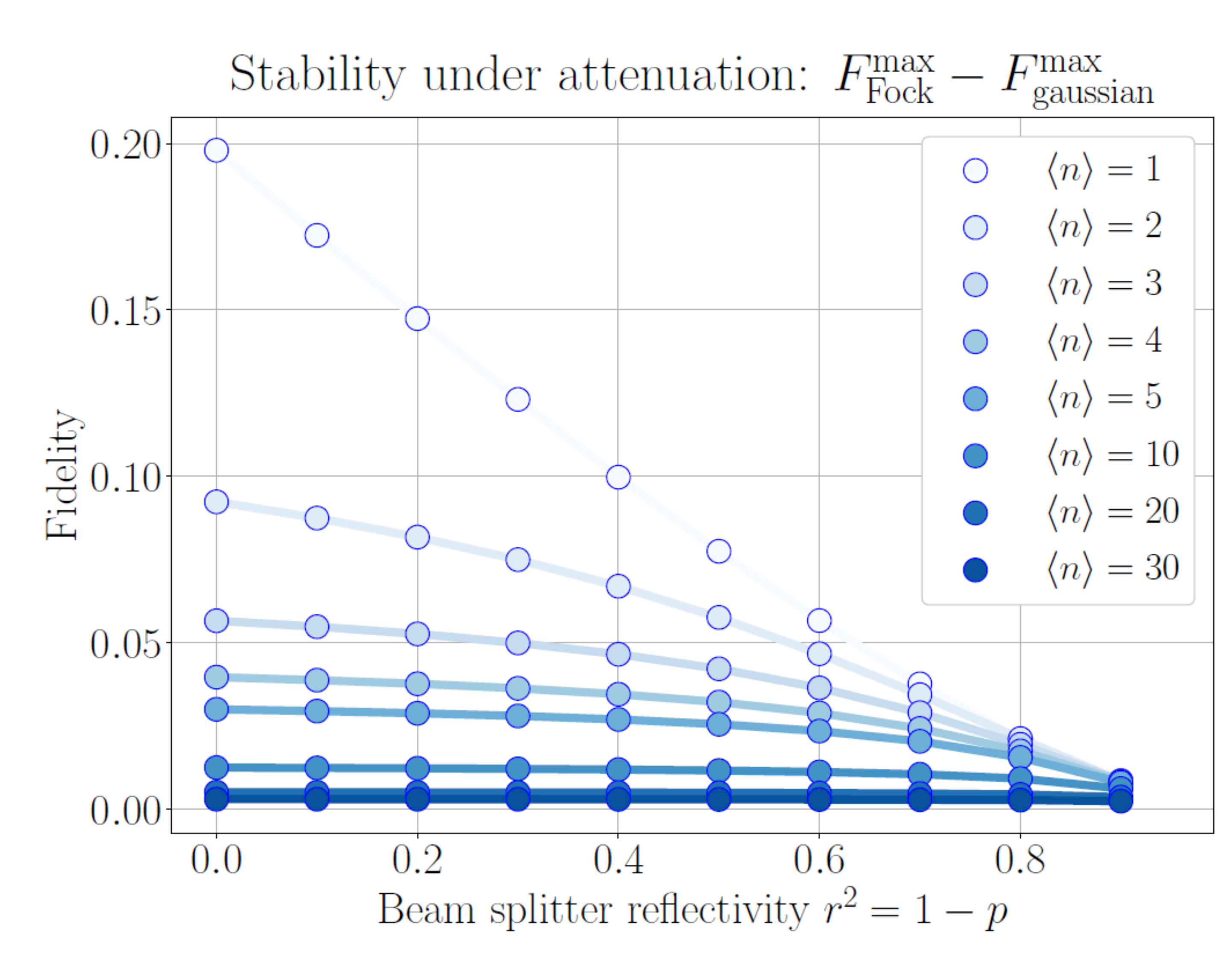
Noise





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Noise

Presence of thermal photons

From A. Ritboon, L. Slodička, and R. Filip, 2022 Quantum Sci. Technol. 7 015023

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

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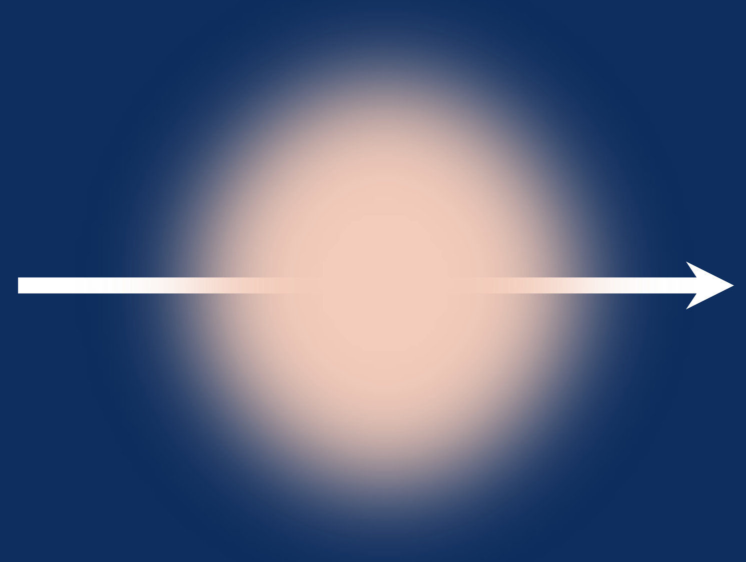

$$\sum_m p_m(\bar{n}_{\mathrm{th}}) |m\rangle \langle m|$$

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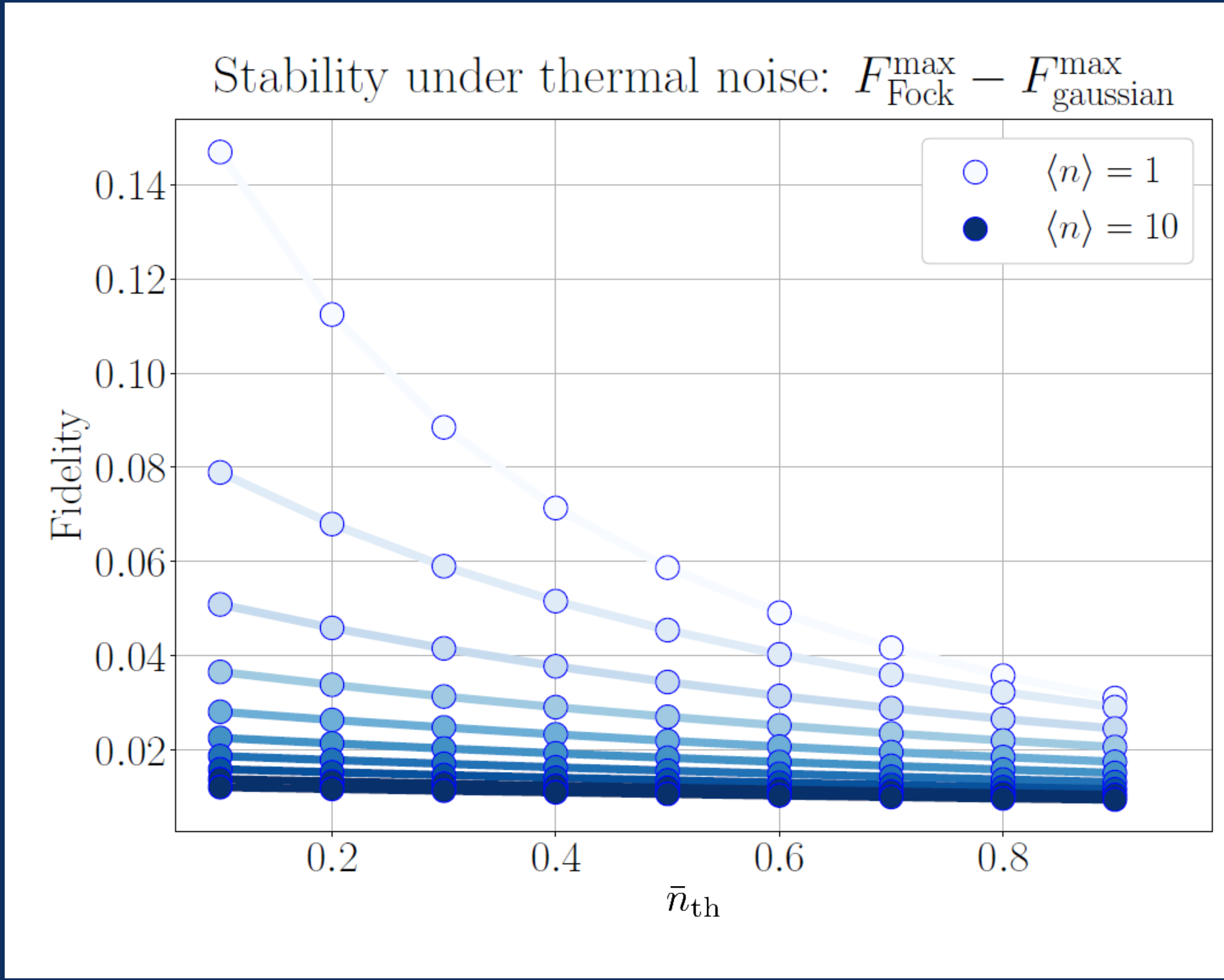
Noise



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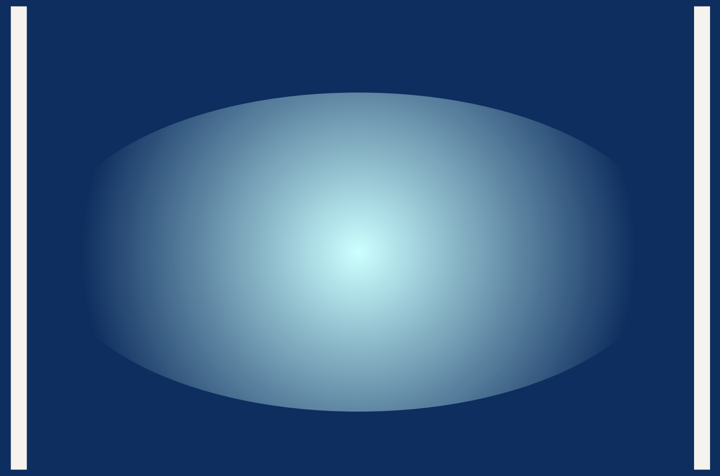
Sub-optimal conditions

Non-perfect resonance between qubit and cavity



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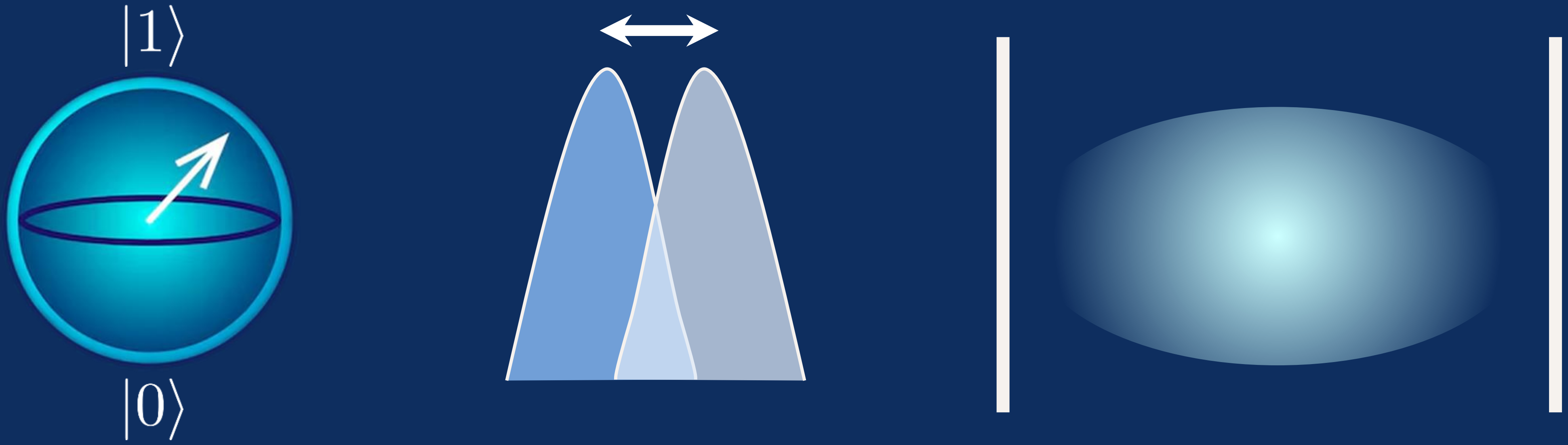
Sub-optimal conditions

Non-perfect resonance between qubit and cavity

$$\Delta\omega = \omega_{\mathrm{qub}} - \omega_{\mathrm{cav}}$$



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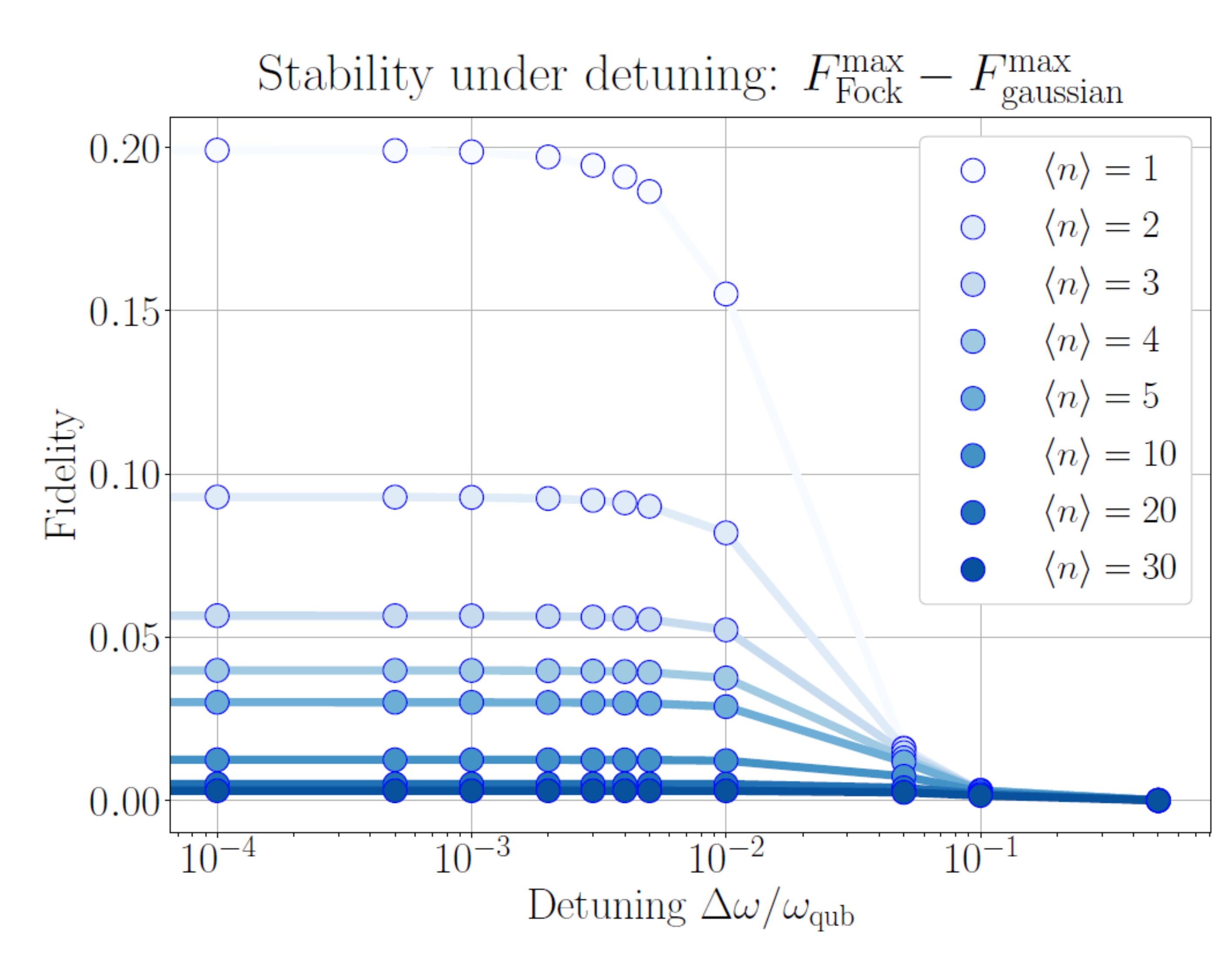

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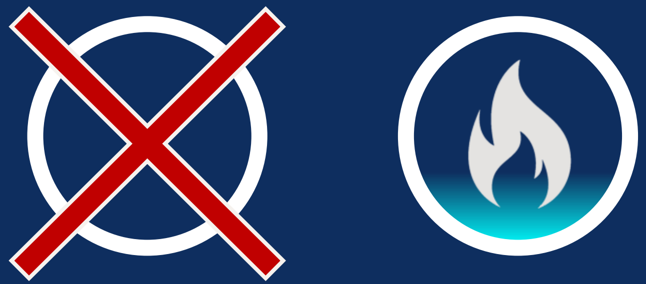
Sub-optimal conditions

Qubit in a thermal state

$$\hat{\rho}_{\mathrm{qub}}(0) = \begin{pmatrix} q & \\ & 1-q \end{pmatrix}$$



$$|e\rangle\langle e| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad |g\rangle\langle g| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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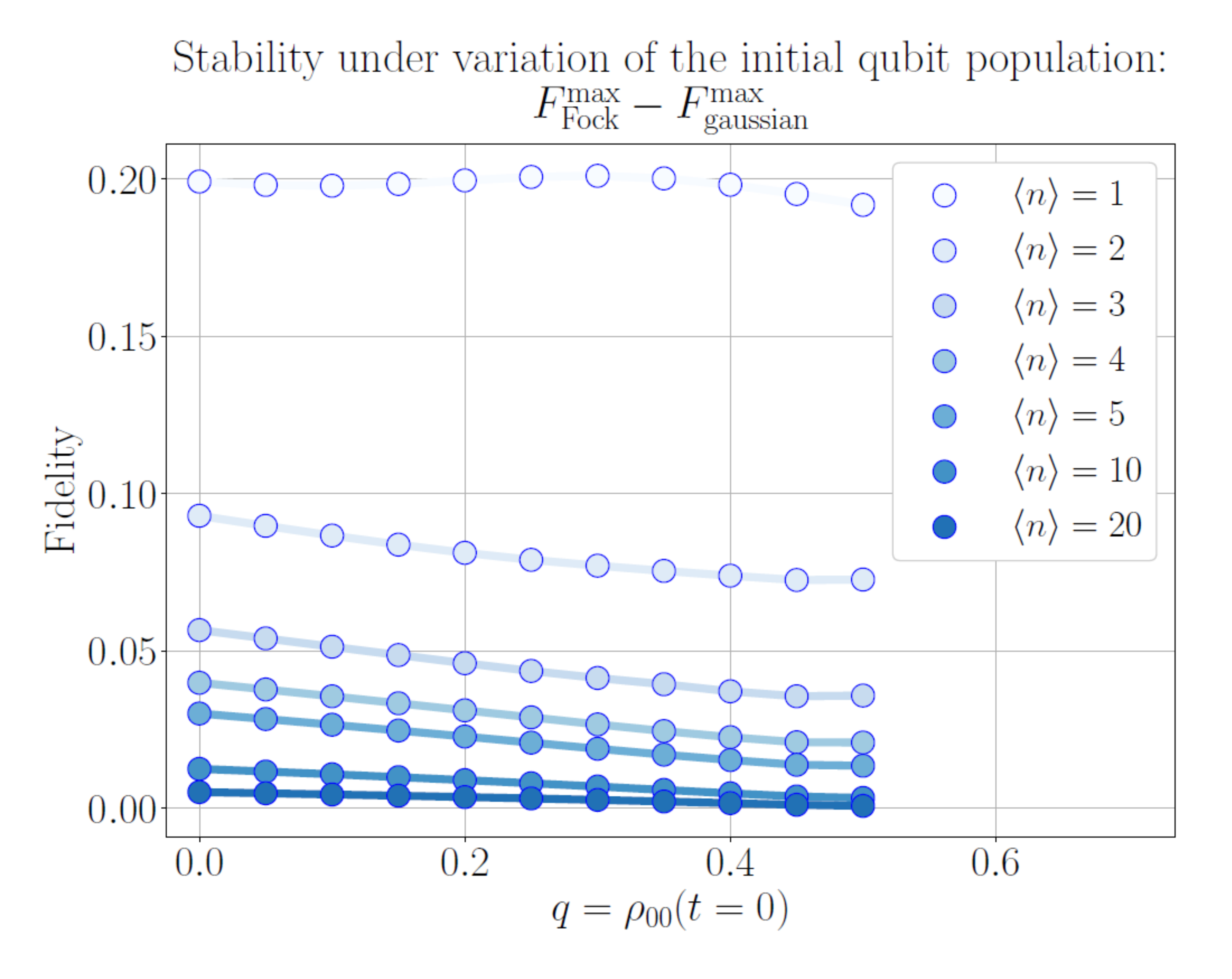
Sub-optimal conditions



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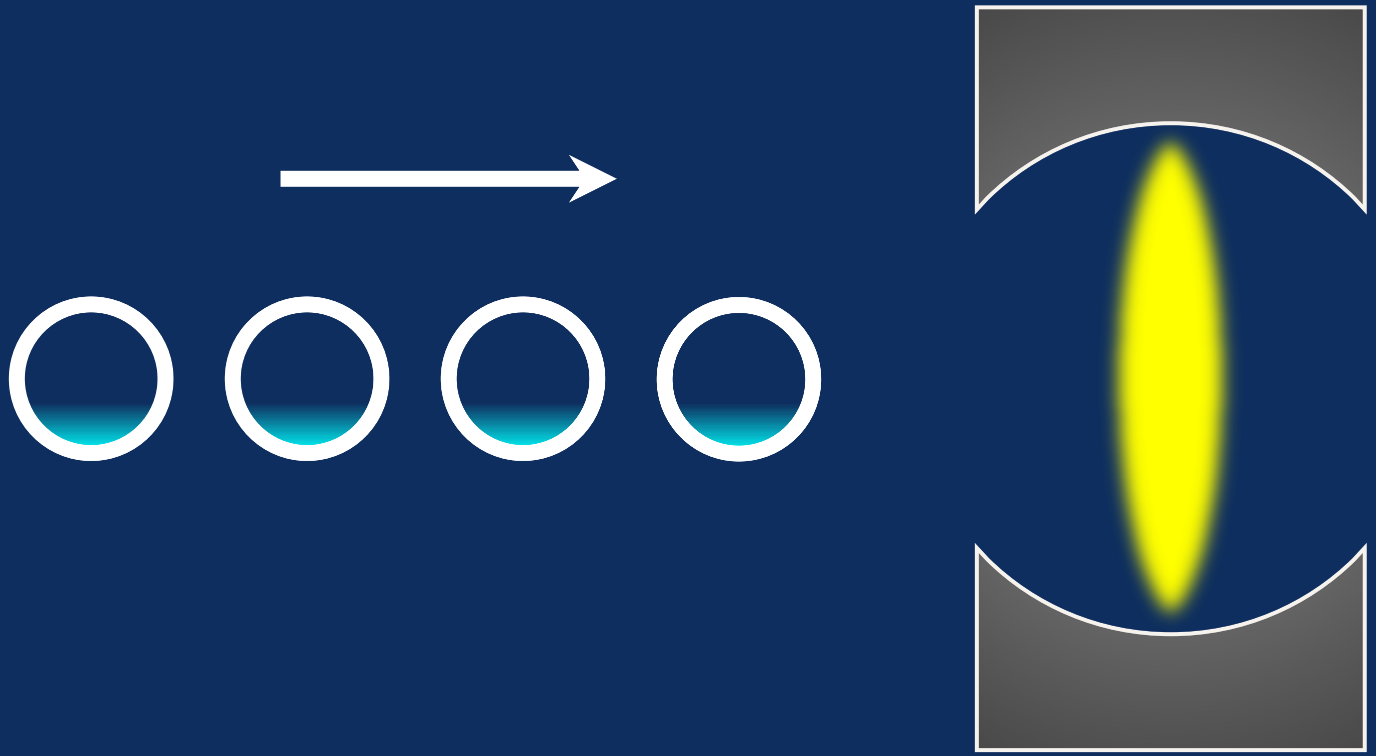
Result 3:

The advantage can be more appreciated if **multiple qubits** are charged

Multi-qubit charge

Sequential charge



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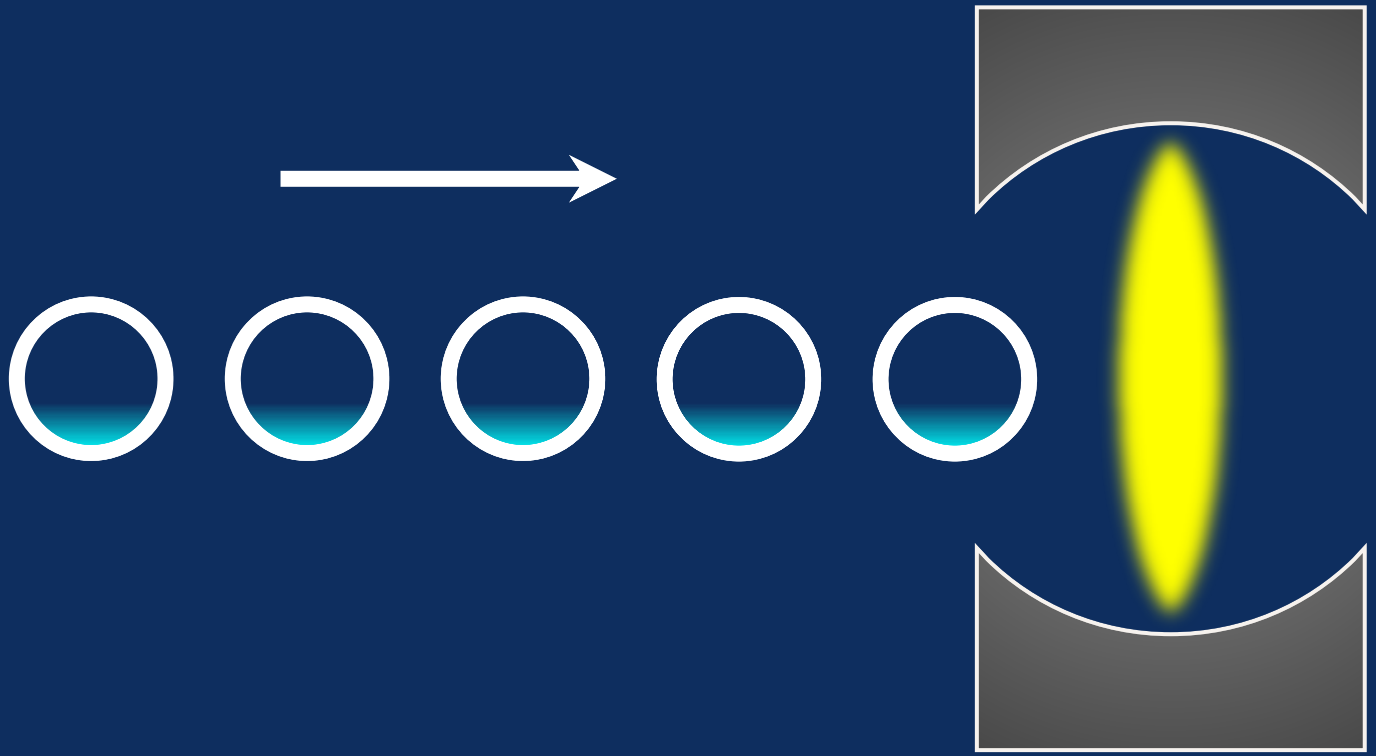
Multi-qubit charge

Sequential charge



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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Multi-qubit charge

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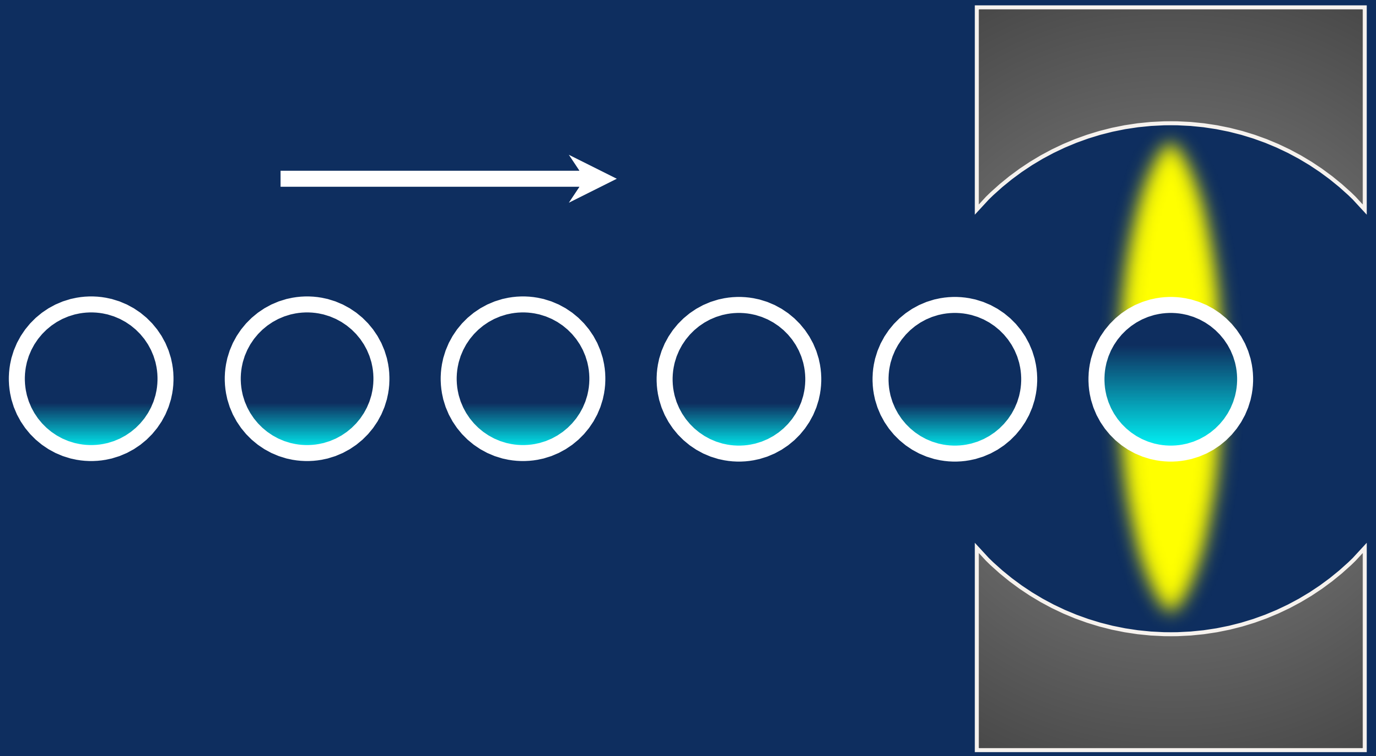

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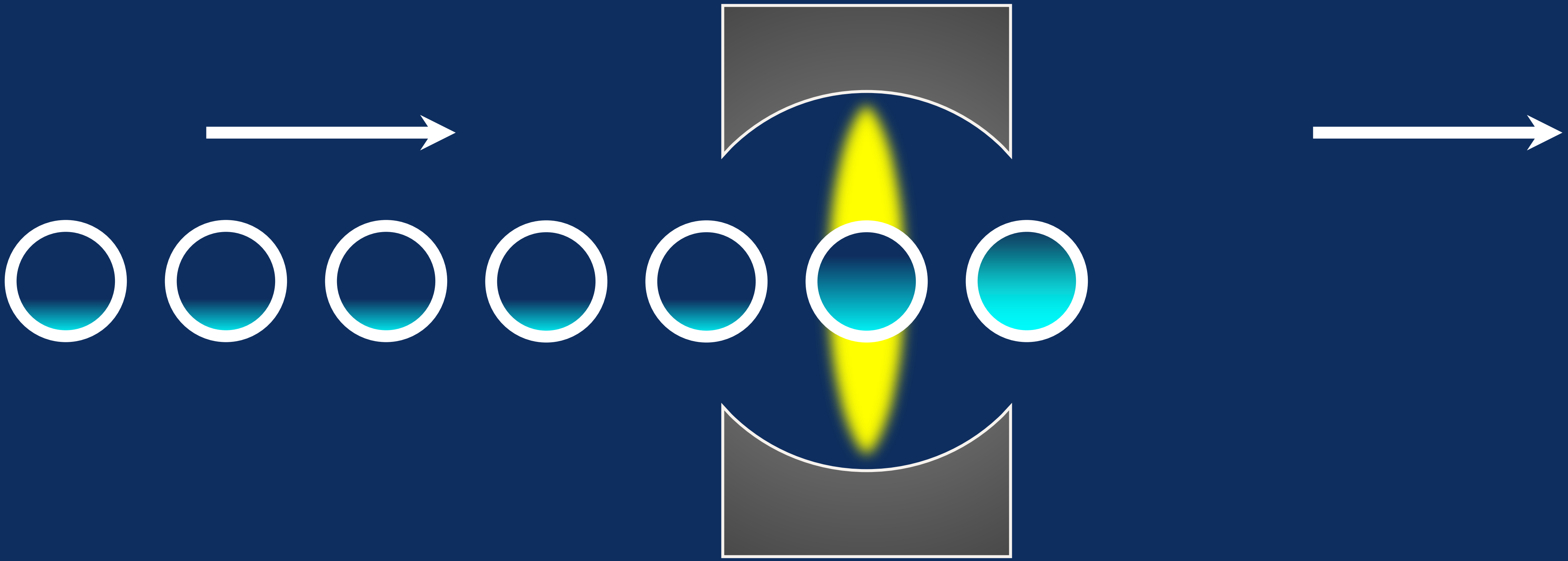

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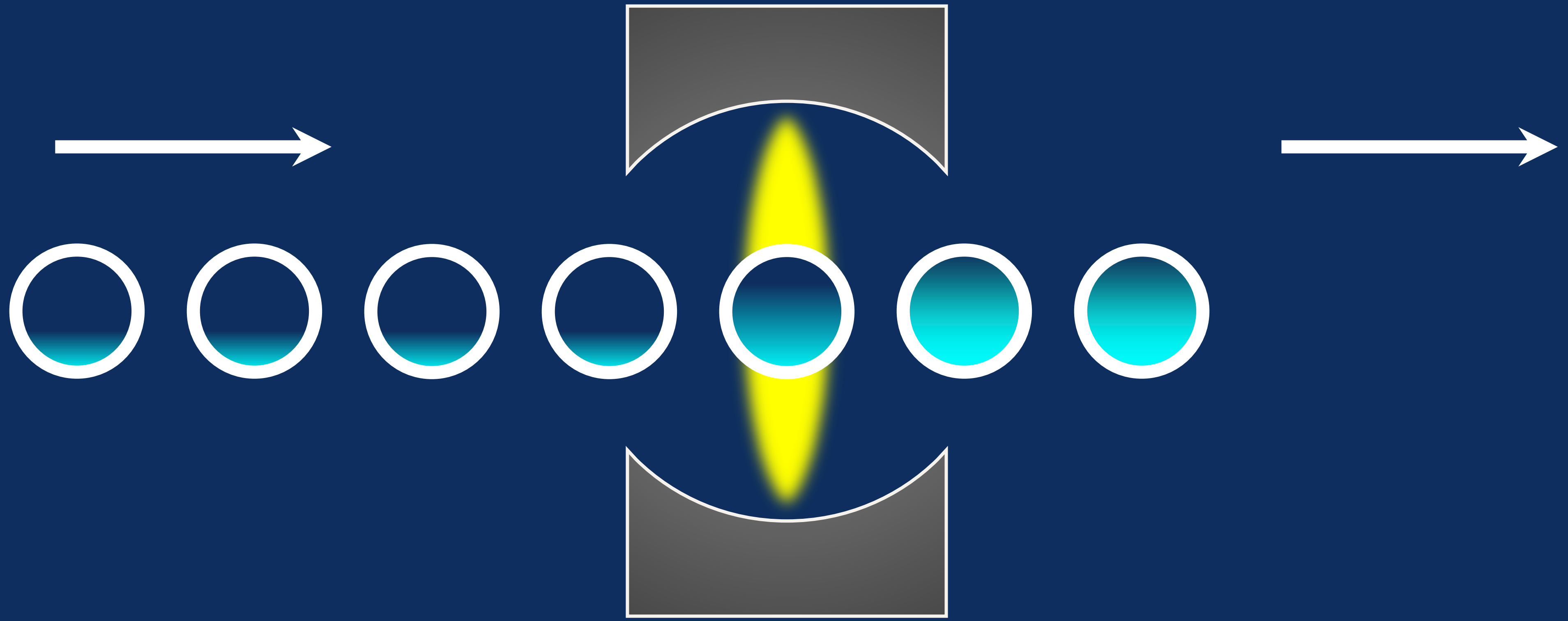

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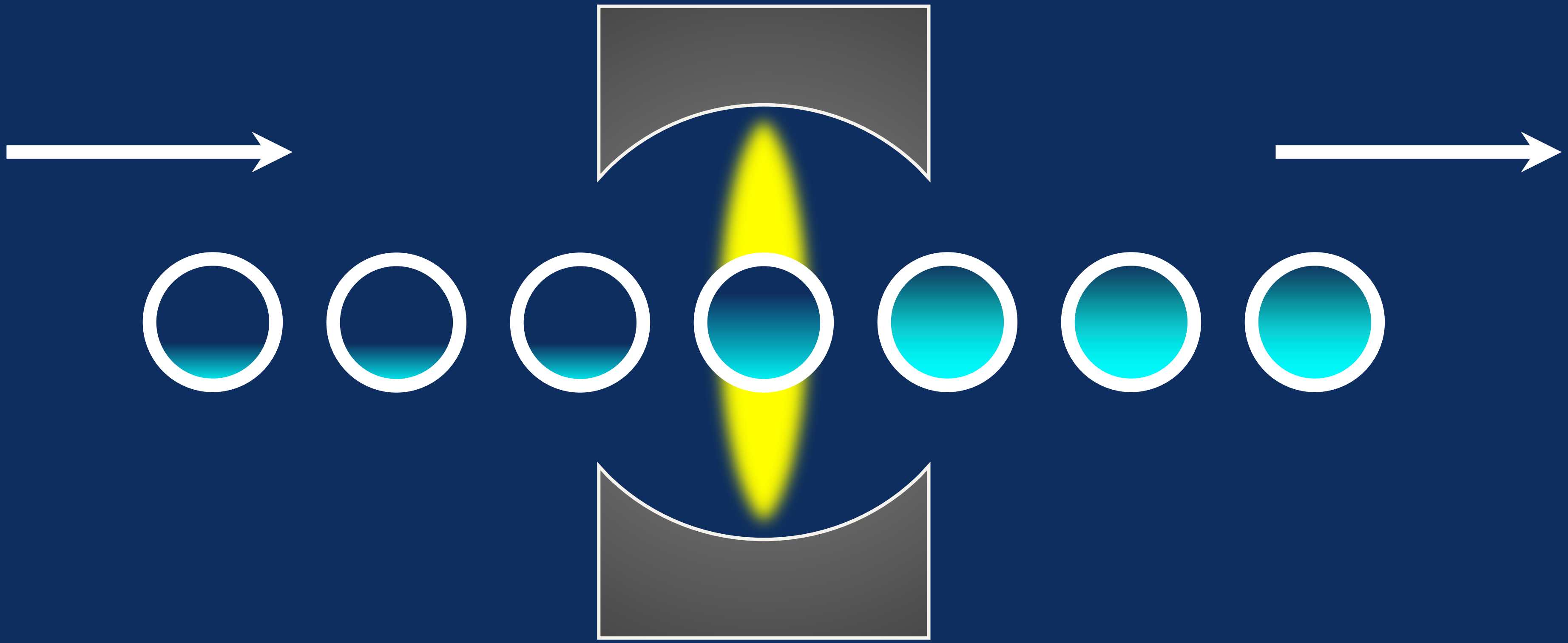

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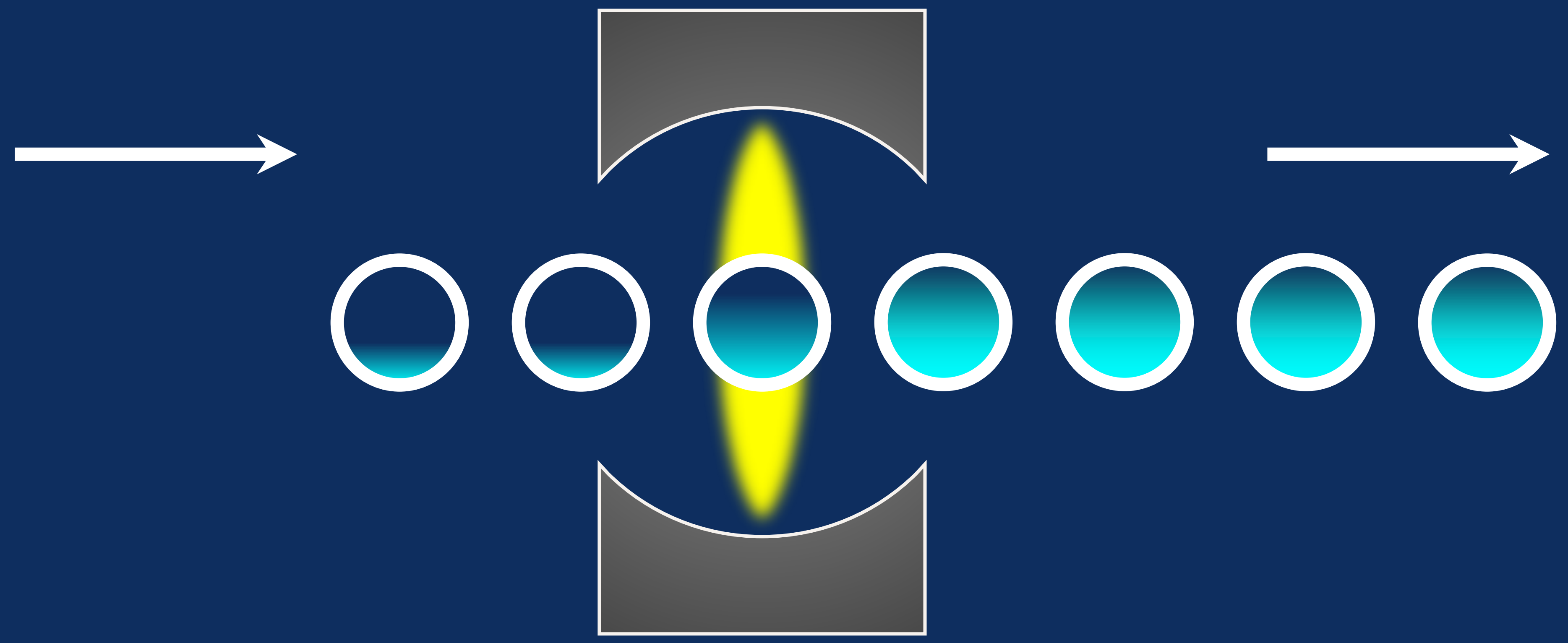

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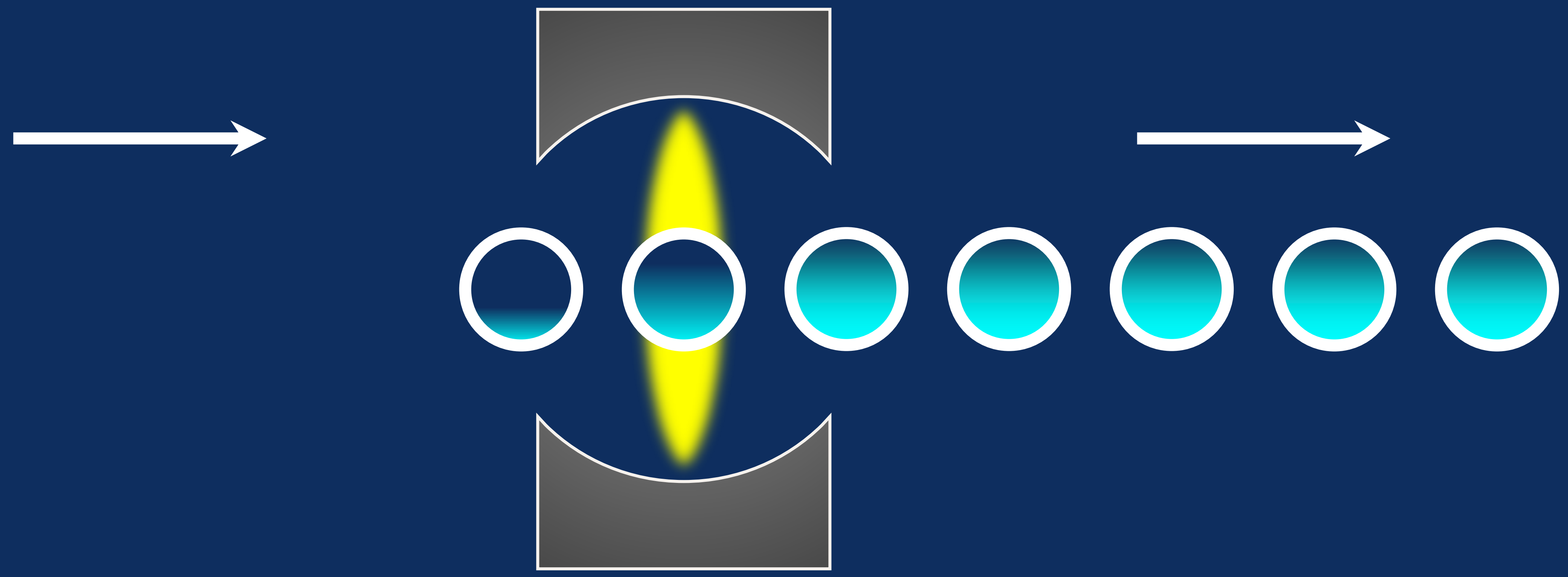

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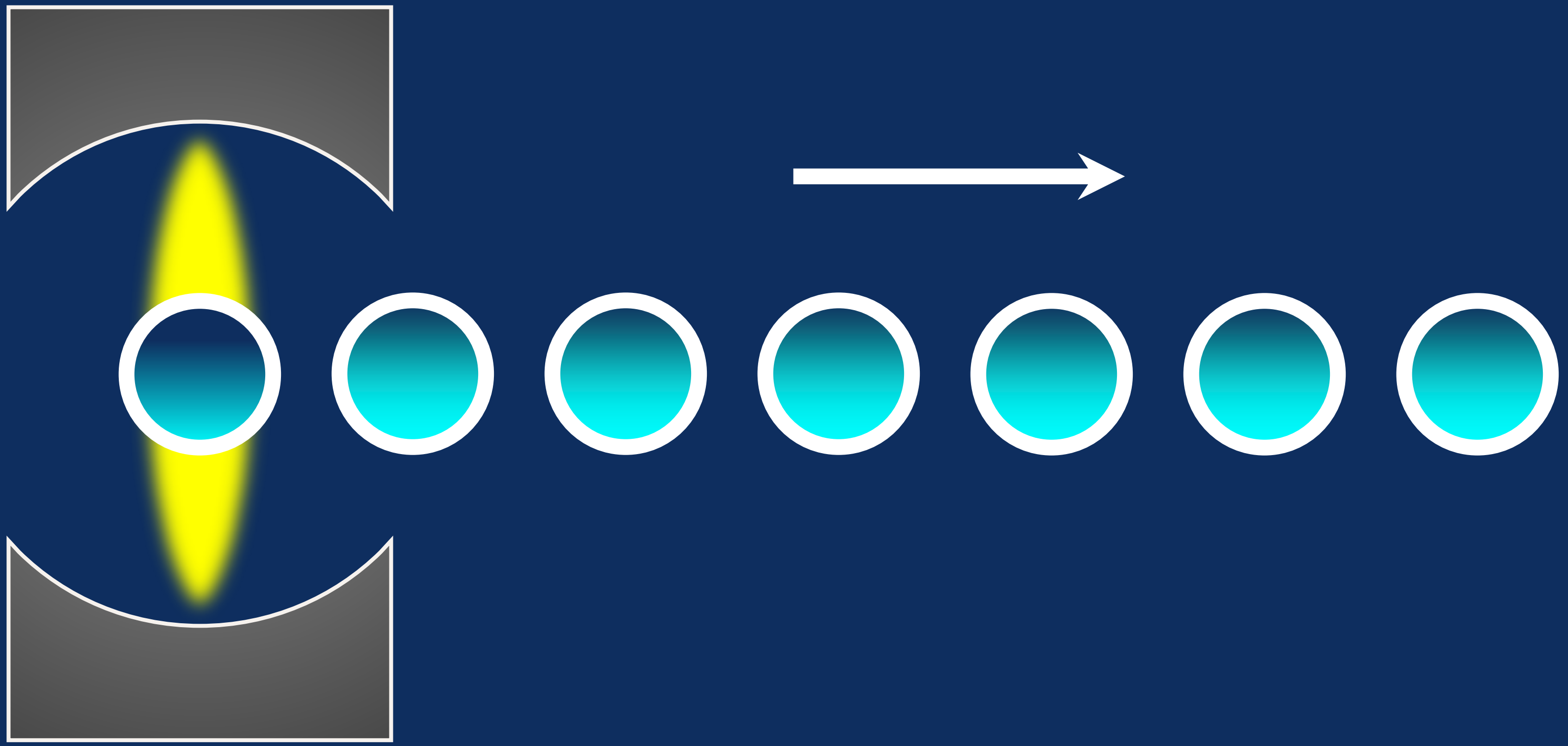

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Multi-qubit charge

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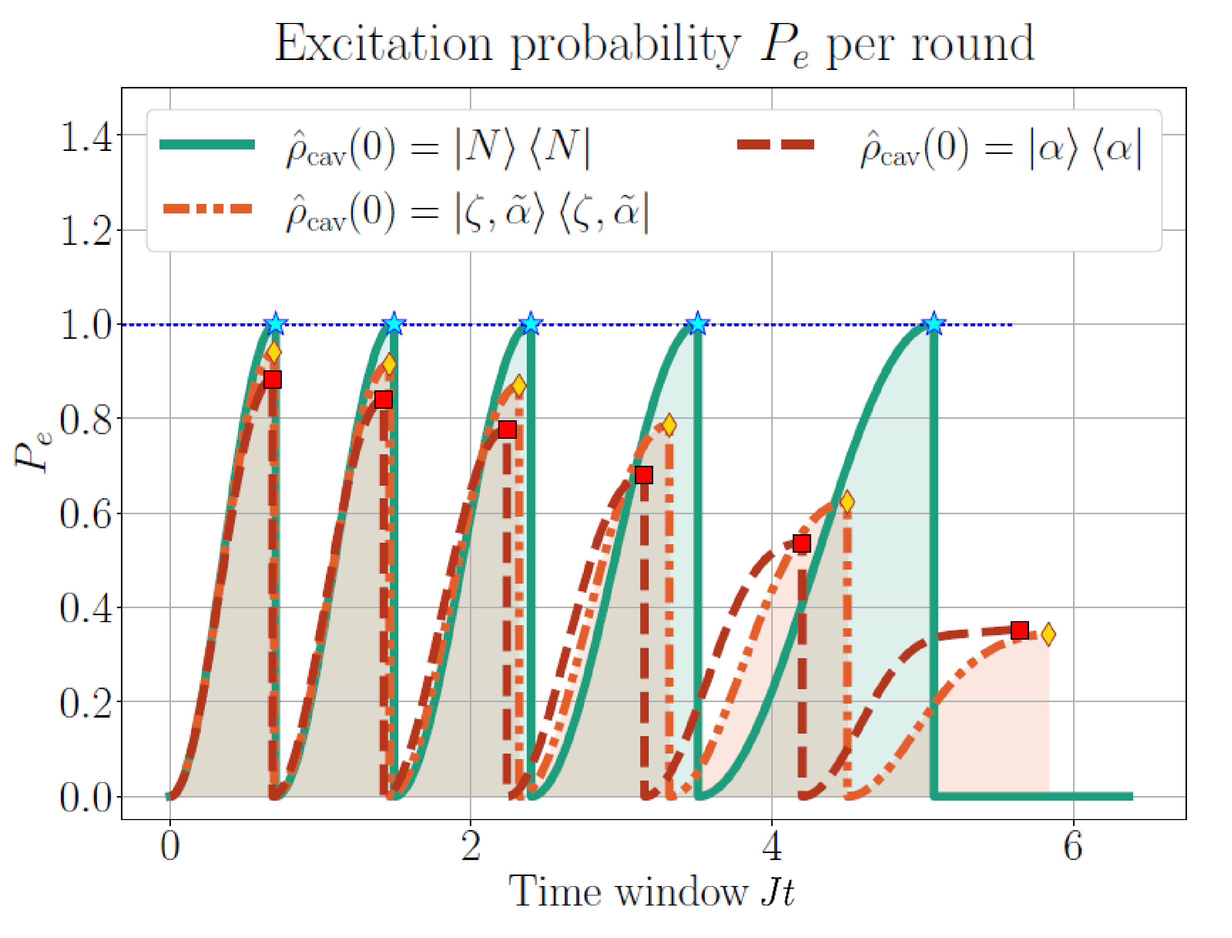

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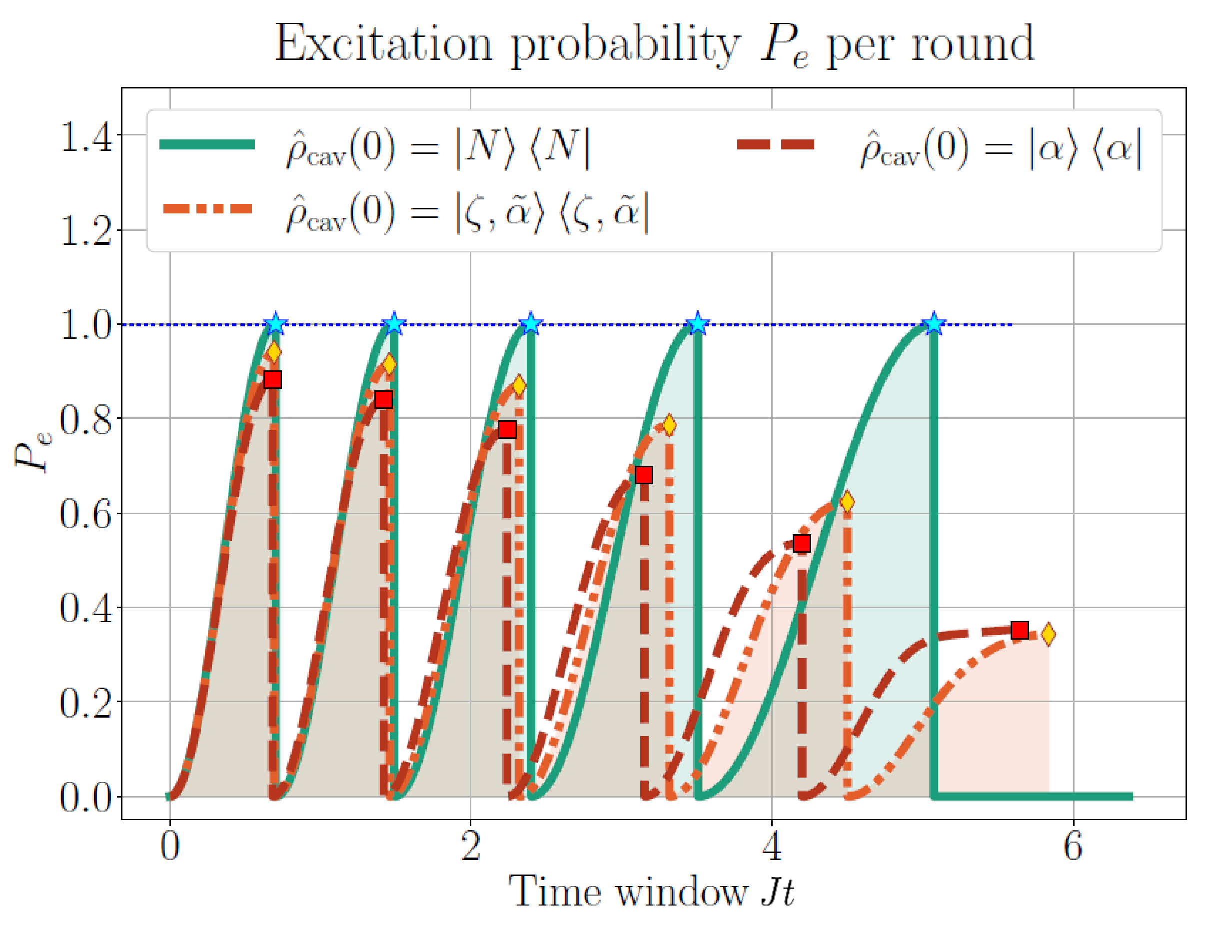
D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Multi-qubit charge

Sequential charge

→ The cavity is coupled to a qubit per time



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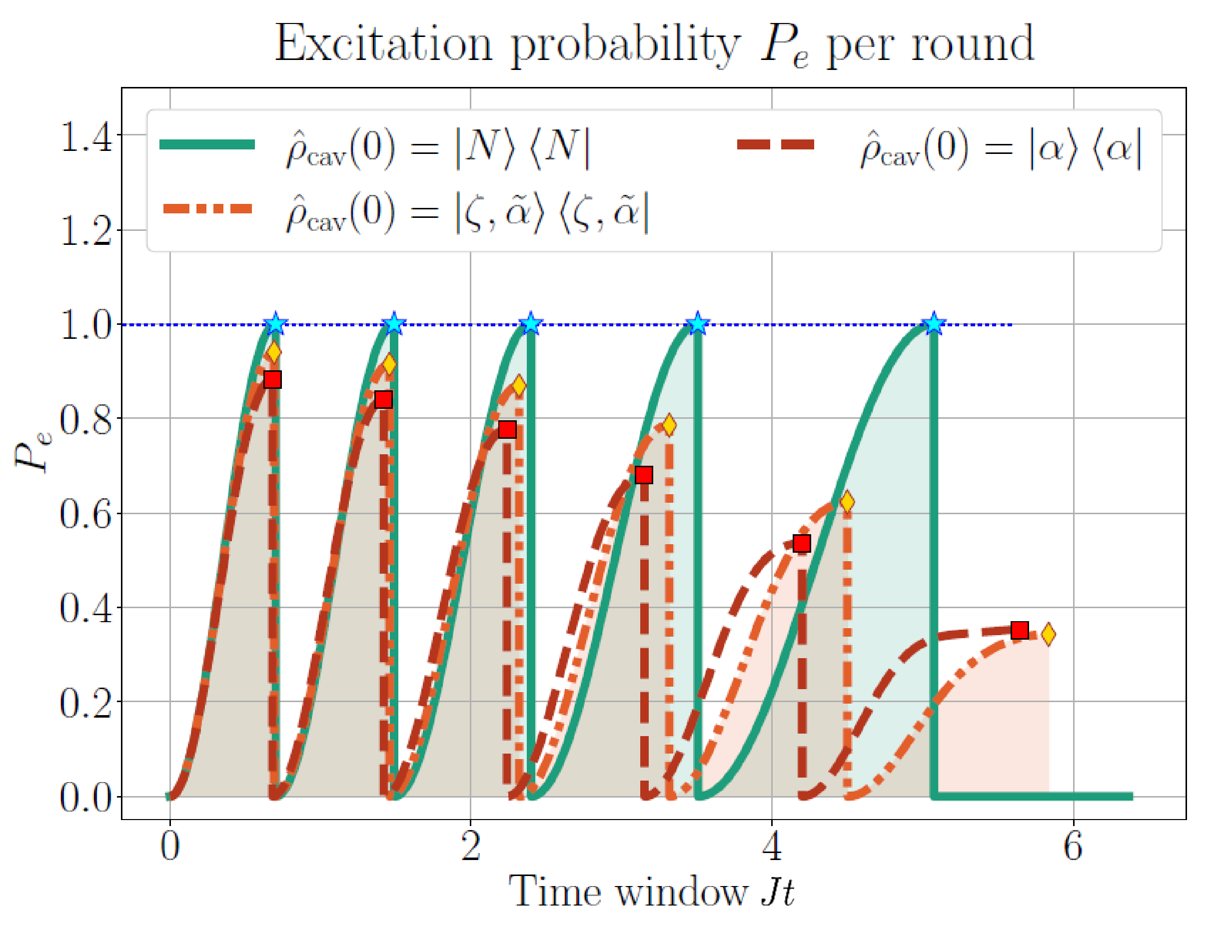
D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Multi-qubit charge

Sequential charge

- The cavity is coupled to a qubit per time
- Perfect charge of N qubits with a N-photon cavity Fock state



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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

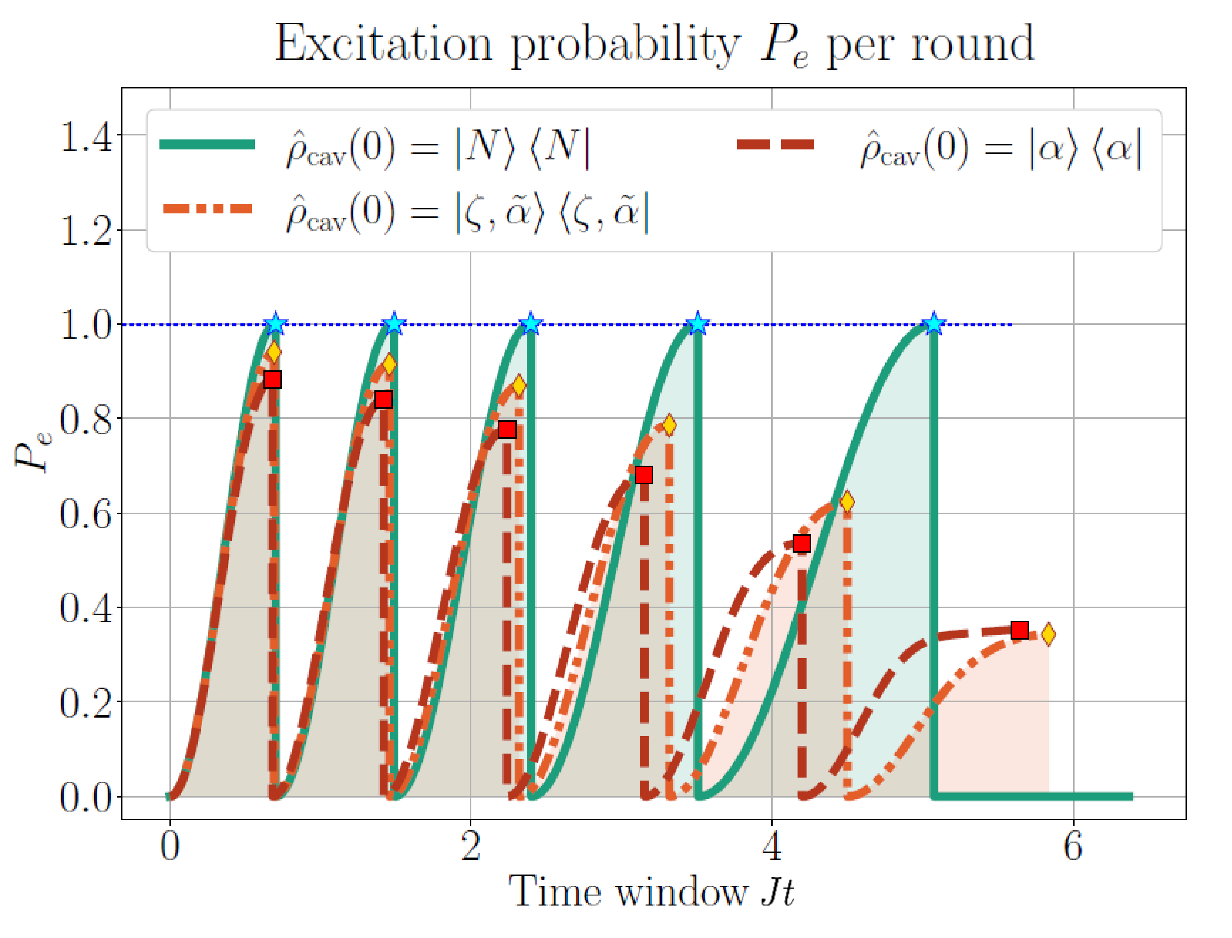
Multi-qubit charge

Sequential charge

→ The cavity is coupled to a qubit per time

→ Perfect charge of *N* qubits with a *N*-photon cavity Fock state

...with no noise!



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Result 4:

The advantage in charging **multiple qubits** is robust in presence of noise

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Multi-qubit charge and noise

Sequential charge and thermal noise

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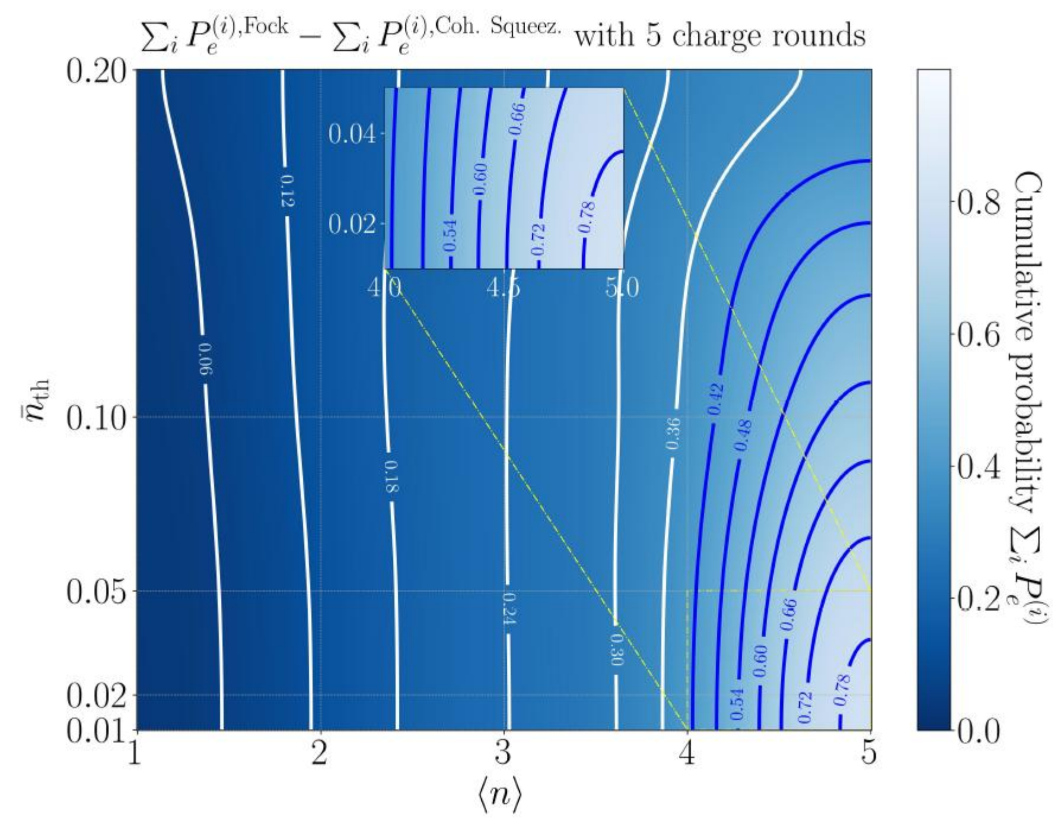
Multi-qubit charge and noise

Sequential charge and thermal noise



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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

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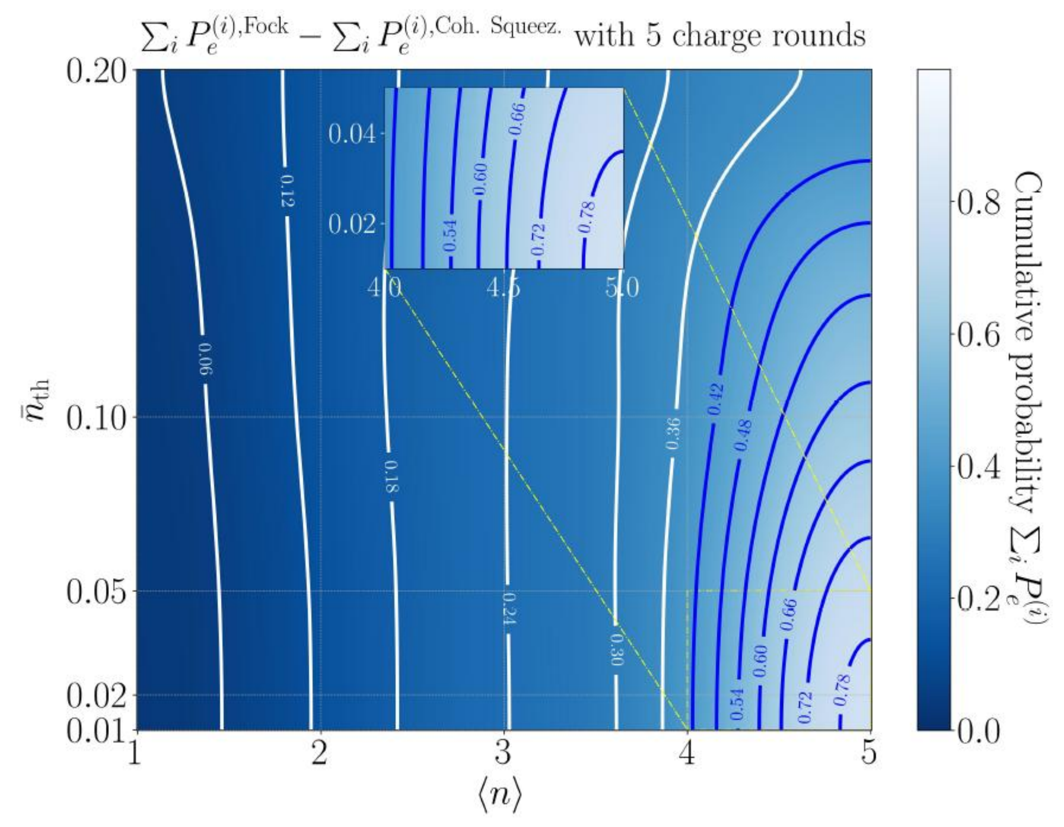
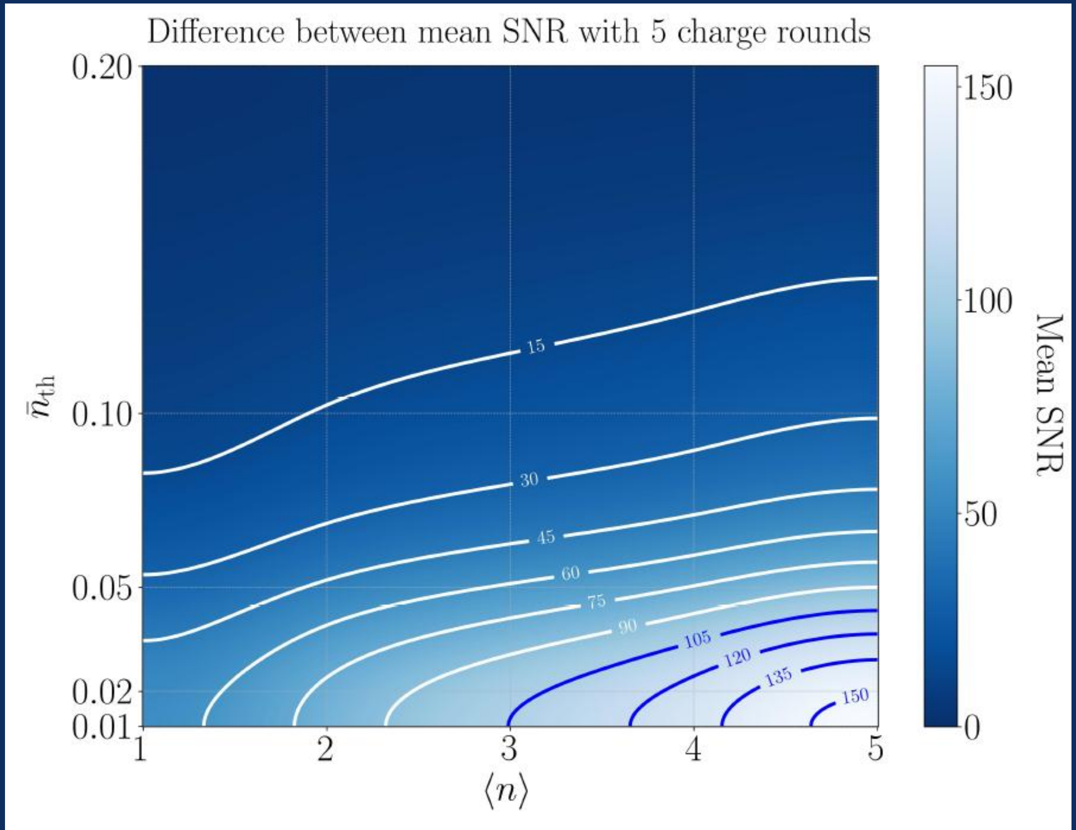
Sequential charge and thermal noise





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Multi-qubit charge and noise

Sequential charge and non-perfect state preparation

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Multi-qubit charge and noise

Sequential charge and non-perfect state preparation

$$
\begin{aligned}
|\psi\rangle\langle\psi| \simeq\ & 0.95\,|5\rangle\langle 5| \\
& + 0.025\,|6\rangle\langle 6| \\
& + 0.025\,|4\rangle\langle 4|
\end{aligned}
$$

Estimation from L. Podhora,
Experimental quantum non-Gaussian mechanical states of a trapped ion,
Ph.D. thesis

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Multi-qubit charge and noise

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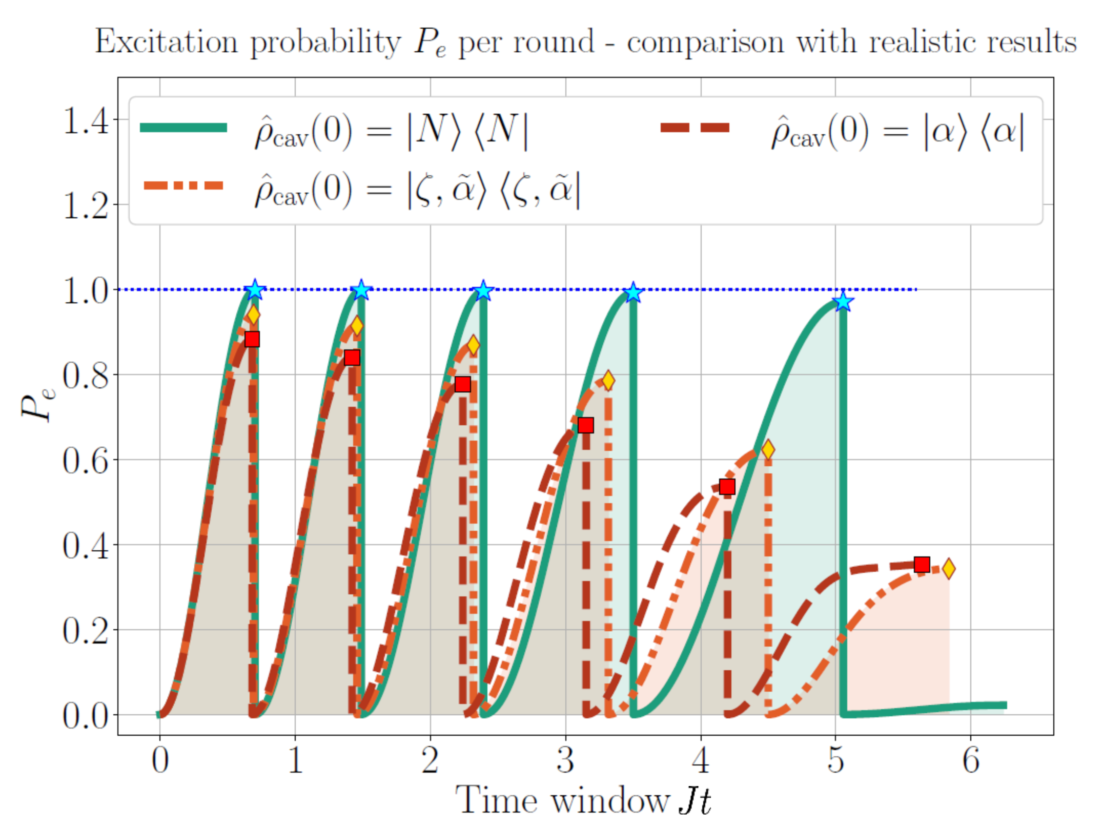
$$|\psi\rangle\langle\psi| \simeq 0.95\,|5\rangle\langle 5| + 0.025\,|6\rangle\langle 6| + 0.025\,|4\rangle\langle 4|$$



Estimation from L. Podhora, *Experimental quantum non-Gaussian mechanical states of a trapped ion*, Ph.D. thesis

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

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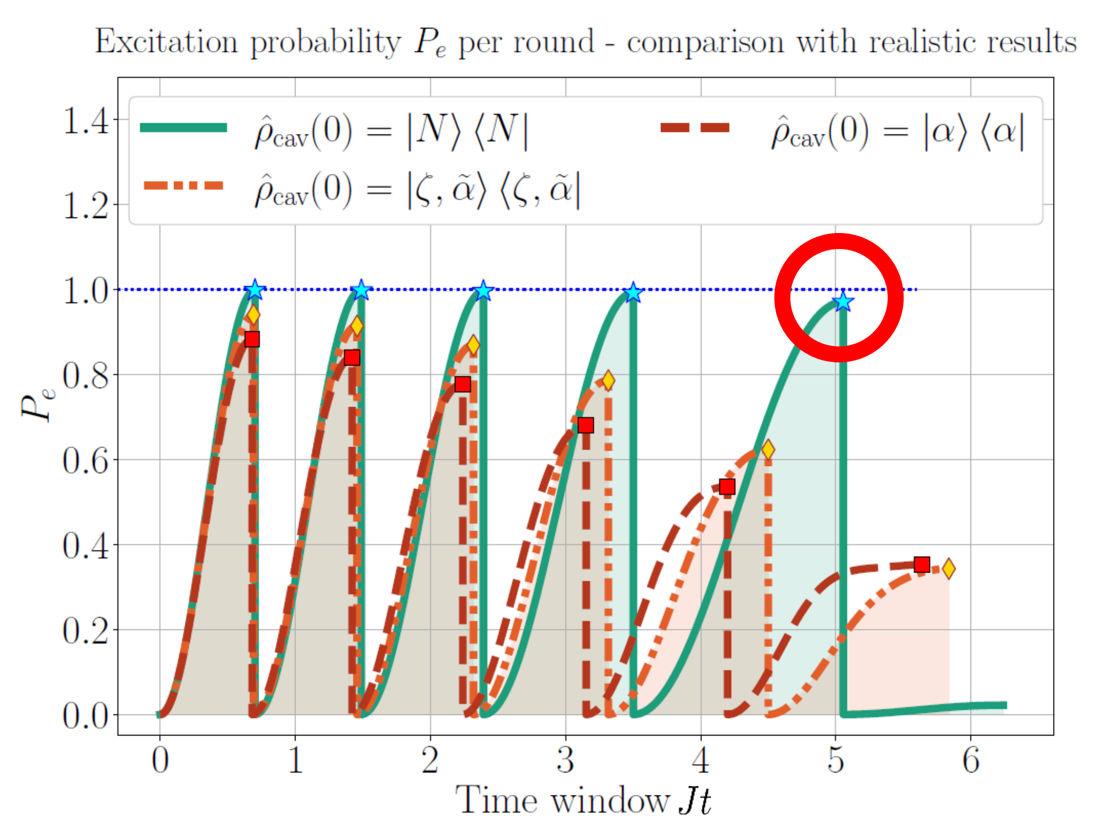
Estimation from L. Podhora, *Experimental quantum non-Gaussian mechanical states of a trapped ion*, Ph.D. thesis



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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

Wrap-up

Quantum batteries* can:

*not included in the package

Wrap-up

Quantum batteries* can:

→ **test the thermodynamics** of highly-controlled quantum systems

*not included in the package

Wrap-up

Quantum batteries* can:

→ **test the thermodynamics** of highly-controlled quantum systems

→ find out **the most efficient** energy exchange protocol

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Wrap-up

Quantum batteries* can:

→ **test the thermodynamics** of highly-controlled quantum systems

→ find out **the most efficient** energy exchange protocol

→ be experimentally implemented as **proof-of-principle** devices

*not included in the package

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Wrap-up

Quantum batteries* can:

→ **test the thermodynamics** of highly-controlled quantum systems

→ find out **the most efficient** energy exchange protocol

→ be experimentally implemented as **proof-of-principle** devices

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Wrap-up

Why *not included?*

Wrap-up

Why *not included?*

Alessandro Volta

Courtesy of Nonciclopedia

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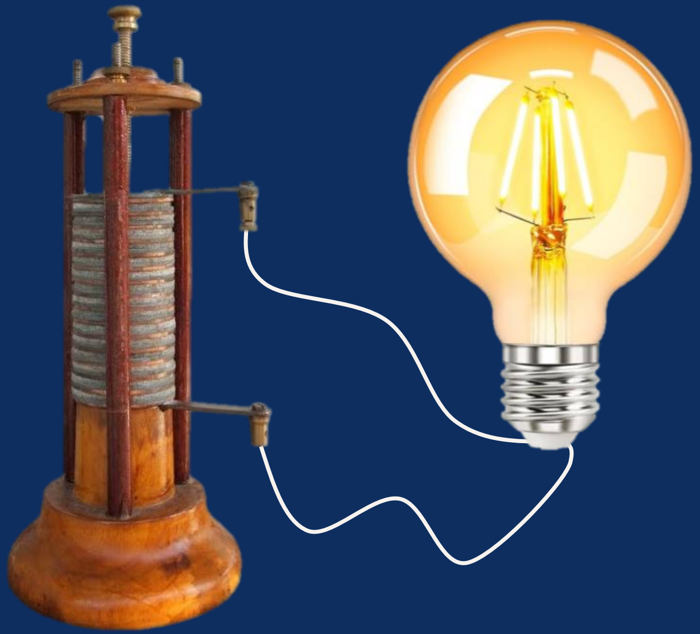


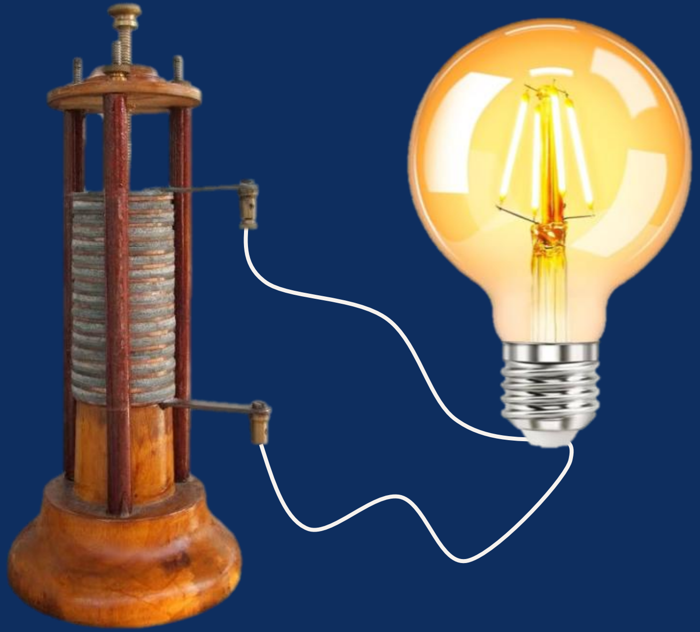
Normal battery

Wrap-up

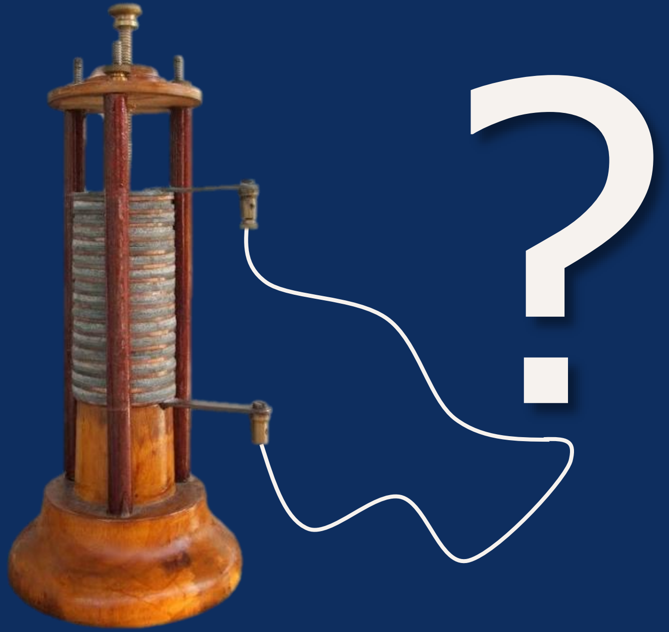
Why *not included?*

Alessandro Volta

Courtesy of Nonciclopedia



Normal battery



Quantum battery

Conclusions

Conclusions

Quantum batteries*

Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

   → **Energy fluctuations**

Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

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2. **Robust advantage** even in presence of:

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Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

   → **Energy fluctuations**

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   → Noise

Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

   ⟶ **Energy fluctuations**

2. **Robust advantage** even in presence of:

   ⟶ Noise

   ⟶ Multiple qubits

Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

   ⟶ **Energy fluctuations**

2. **Robust advantage** even in presence of:

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   ⟶ Multiple qubits

3. No realistic usage yet:

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Conclusions

Quantum batteries*

1. Key: **precision & efficiency**

   → **Energy fluctuations**

2. **Robust advantage** even in presence of:

   → Noise

   → Multiple qubits

3. No realistic usage yet:

   → ***to be included** in the package!
   …further project on qubit reset

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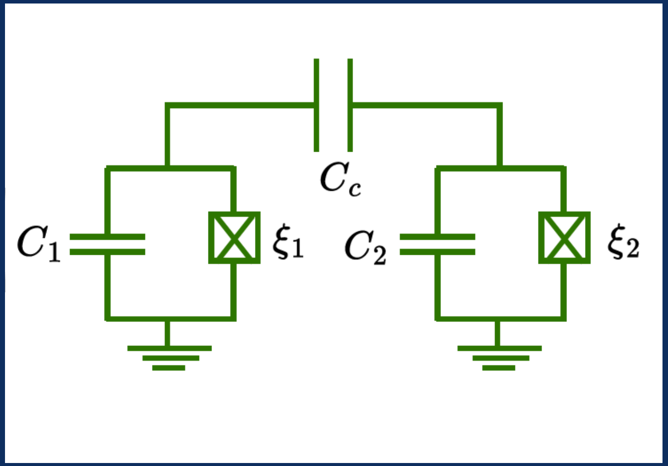
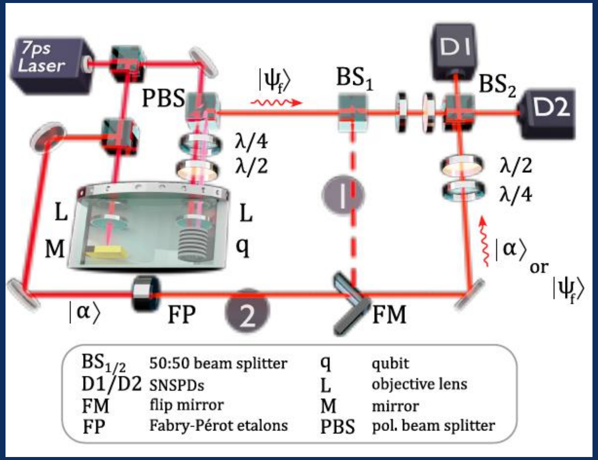
Supplementary material

Proof-of-principles



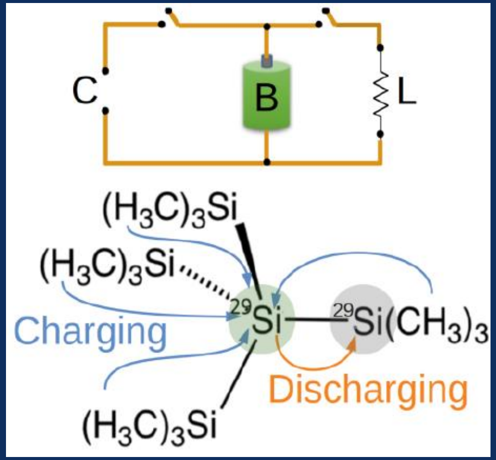
Superconducting circuits

S. Elghaayda *et al.*, Advanced Quantum Technologies, 2025, 2400651.



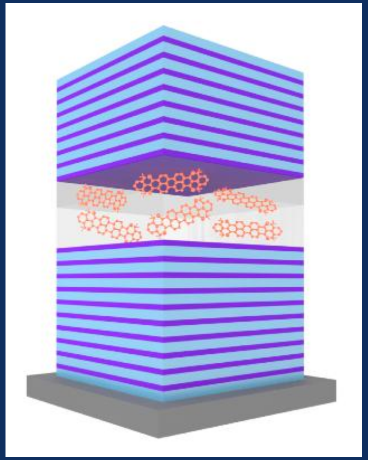
Quantum dots

Wenniger *et al.*, 2023, Phys. Rev. Lett. 131, 260401



Nuclear spin systems

Joshi, J., and T. S. Mahesh, 2022, Phys. Rev. A 106, 042601



Organic microcavities

Quach *et al.*, 2022, Sci. Adv. 8, 3160.

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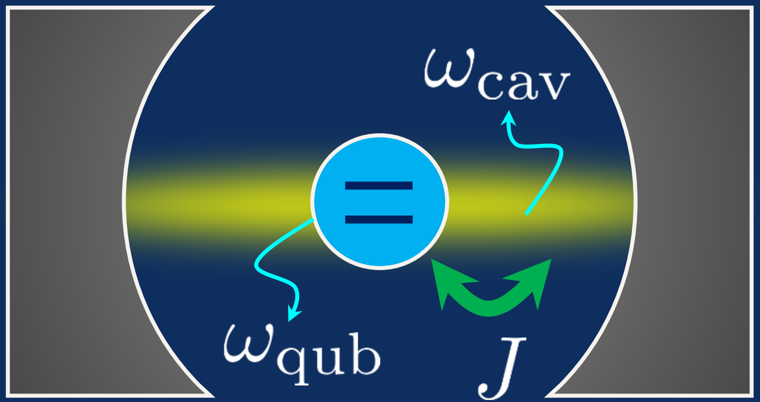
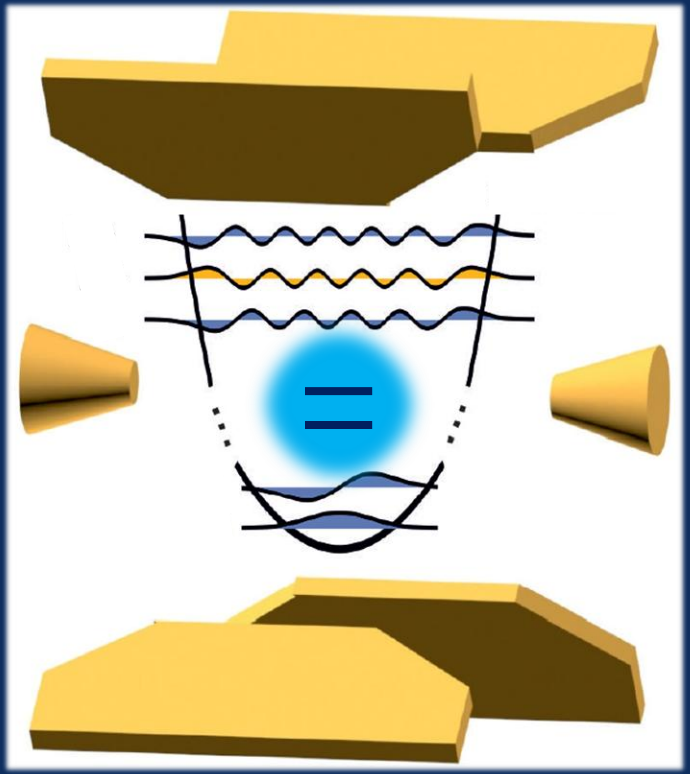
The Jaynes-Cummings model

Implementation: trapped ions





L. Podhora *et al.*, Physical Review Letters, 2022, 129.1: 013602.

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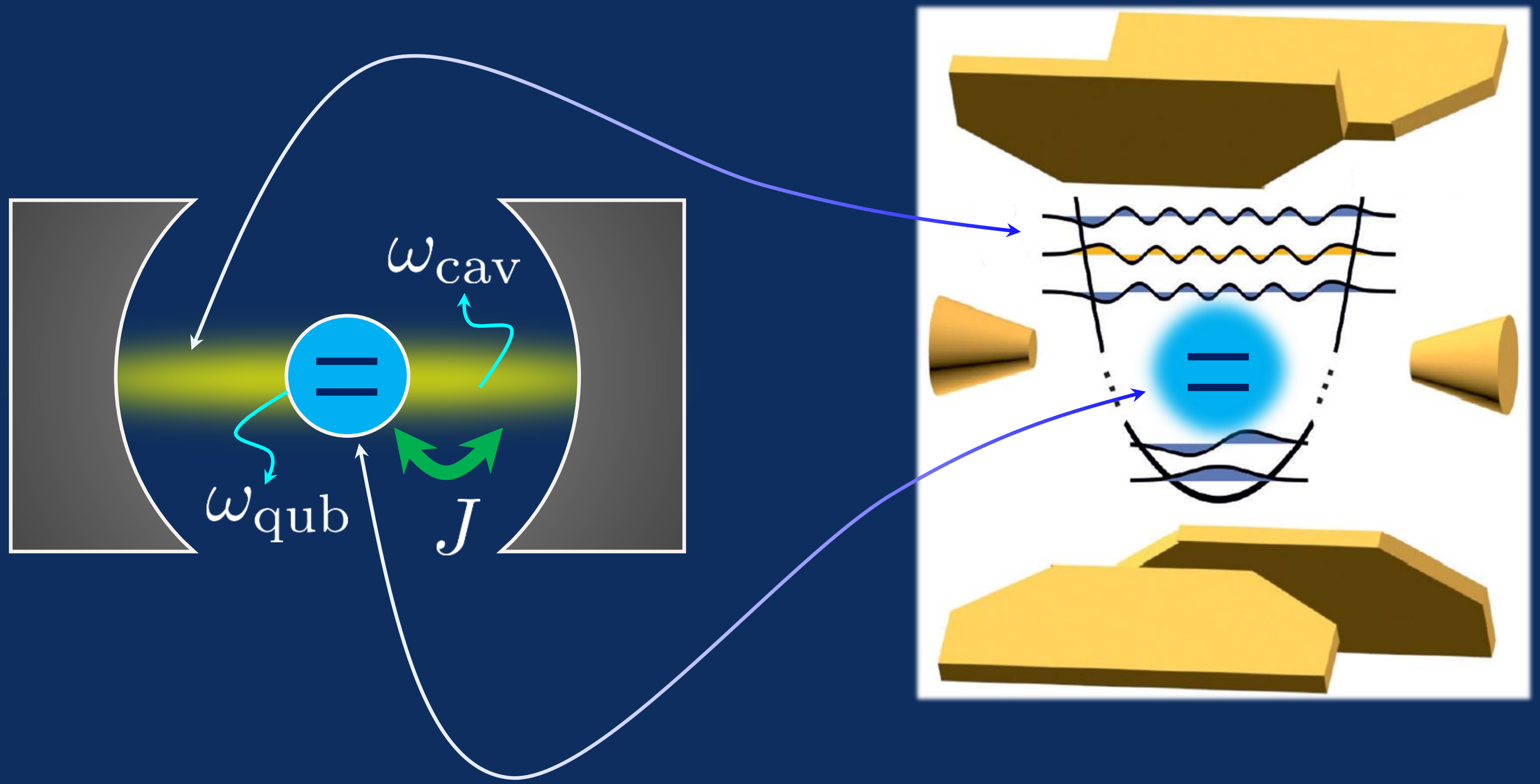

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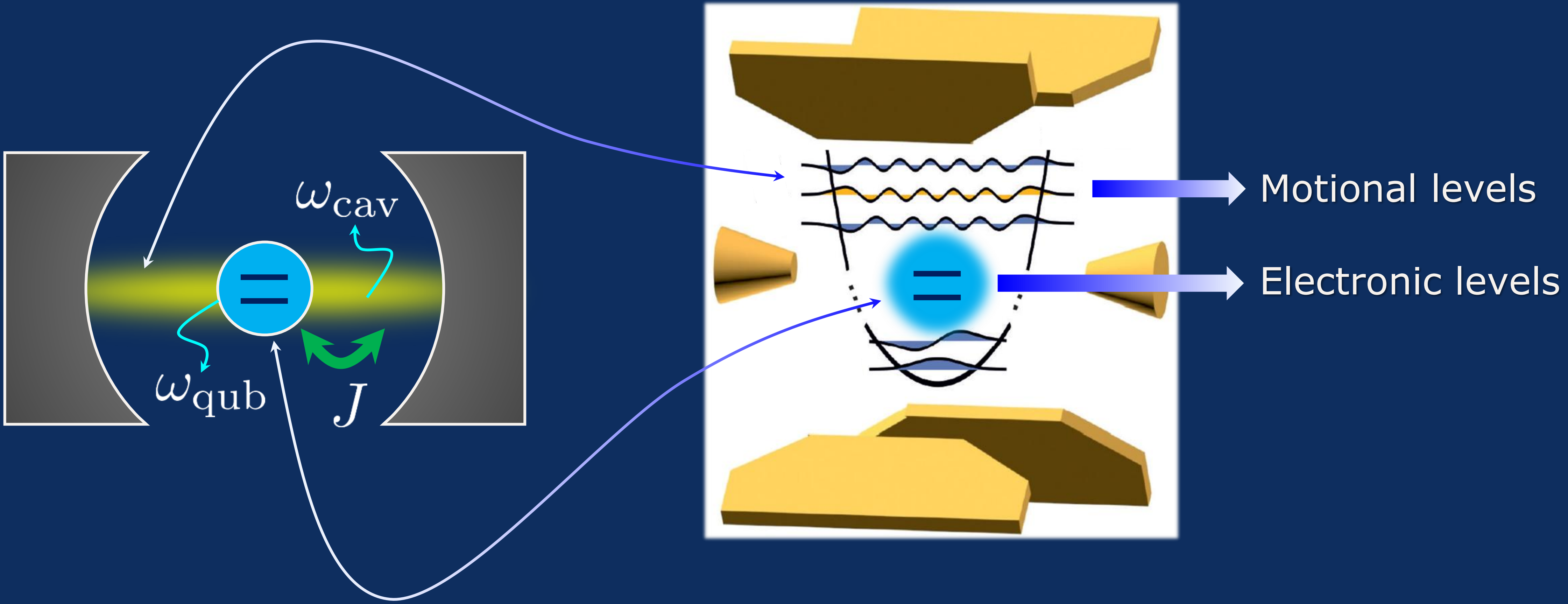

L. Podhora *et al.*, Physical Review Letters, 2022, 129.1: 013602.

Results

Start from the JC evolution:

$$\hat{U}_\tau = \begin{pmatrix} \hat{U}_{00} & \hat{U}_{01} \\ \hat{U}_{10} & \hat{U}_{11} \end{pmatrix}$$

Analytically known

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S. Stenholm, Physics Reports 6, 1 (1973).

Results

FCS for the JC model: statistics of the qubit energy

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$\hat{U}_{\tau,\chi} \equiv e^{+i\frac{\chi}{2}\hat{H}_{\tau}}\hat{U}_{\tau}e^{-i\frac{\chi}{2}\hat{H}_0}$$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

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$$\hat{U}_{\tau,\chi} = \begin{pmatrix} \hat{U}_{00} & \hat{U}_{01}e^{+i\frac{\chi}{2}\hbar\omega_{\mathrm{qub}}} \\ \hat{U}_{10}e^{-i\frac{\chi}{2}\hbar\omega_{\mathrm{qub}}} & \hat{U}_{11} \end{pmatrix}$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

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FCS for the JC model: statistics of the qubit energy

$$\hat{\rho}_{\tau,\chi} \equiv \hat{U}_{\tau,+\chi}\hat{\rho}_0\hat{U}^{\dagger}_{\tau,-\chi}$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

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$$\hat{\rho}_{\tau,\chi} \equiv \hat{U}_{\tau,+\chi}\hat{\rho}_0\hat{U}^{\dagger}_{\tau,-\chi}$$

$$G(\chi) = \mathrm{Tr}\left\{\hat{\rho}_{\tau,\chi}\right\}$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$\hat{\rho}_0 = \hat{\rho}_0^{\mathrm{qub}} \otimes \hat{\rho}_0^{\mathrm{cav}}$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$\hat{\rho}_0 = \hat{\rho}_0^{\mathrm{qub}} \otimes \hat{\rho}_0^{\mathrm{cav}}$$

$$\downarrow \qquad \downarrow$$

Example: $|g\rangle\langle g| \quad |N\rangle\langle N|$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$G(\chi) = \mathrm{Tr}\{\hat{\rho}_{\tau,\chi}\}$$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$G(\chi) = \mathrm{Tr}\{\hat{\rho}_{\tau,\chi}\}$$

$$= J^2 N e^{i\chi\hbar\omega_{\mathrm{qub}}} \mathcal{S}^2(\tau) + \mathcal{C}^2(\tau) + \left(\frac{\Delta\omega}{2}\right)^2 \mathcal{S}^2(\tau)$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

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FCS for the JC model: statistics of the qubit energy

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$$= J^2 N e^{i\chi\hbar\omega_{\mathrm{qub}}} \mathcal{S}^2(\tau) + \mathcal{C}^2(\tau) + \left(\frac{\Delta\omega}{2}\right)^2 \mathcal{S}^2(\tau)$$

$$\mathcal{C}(\tau) = \cos\left(\tau\sqrt{J^2N + (\Delta\omega/2)^2}\right) \qquad \mathcal{S}(\tau) = \frac{\sin\left(\tau\sqrt{J^2N + (\Delta\omega/2)^2}\right)}{\sqrt{J^2N + (\Delta\omega/2)^2}}$$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Results

FCS for the JC model: statistics of the qubit energy

$$\langle \Delta E_\tau \rangle = -i \frac{d}{d\chi} G(\chi) \Big|_{\chi=0} = J^2 N \hbar \omega_{\mathrm{qub}} \mathcal{S}^2(\tau)$$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

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FCS for the JC model: statistics of the qubit energy

$$\langle \Delta E_\tau \rangle = -i \frac{d}{d\chi} G(\chi) \bigg|_{\chi=0} = J^2 N \hbar \omega_{\mathrm{qub}} \mathcal{S}^2(\tau)$$

$$\begin{aligned} \mathrm{var}(\Delta E_\tau) &= \langle \Delta E_\tau^2 \rangle - \langle \Delta E_\tau \rangle^2 \\ &= J^2 N (\hbar \omega_{\mathrm{qub}})^2 \mathcal{S}^2(\tau) - [J^2 N \hbar \omega_{\mathrm{qub}} \mathcal{S}^2(\tau)]^2 \end{aligned}$$

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

Results

Charge time τ_{opt}

$$\mathrm{SNR}(\Delta E_\tau) = \frac{\langle \Delta E_\tau \rangle^2}{\mathrm{var}(\Delta E_\tau)} = \frac{J^2 N \mathcal{S}^2(\tau)}{1 - J^2 N \mathcal{S}^2(\tau)}$$

$$\hat{\rho}_0^{\text{qub}} = |g\rangle \langle g|$$
$$\hat{\rho}_0^{\text{cav}} = |N\rangle \langle N|$$

Maximize it to obtain τ_{opt}

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D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, Physical Review A, 2025, 112.1: 012205.

Results

Charge time τ_{opt}

$$\tau_{\mathrm{opt}} = \frac{\pi}{2\sqrt{g^2 N + (\Delta\omega/2)^2}} \propto \frac{1}{\sqrt{N}}$$

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Energy fluctuations

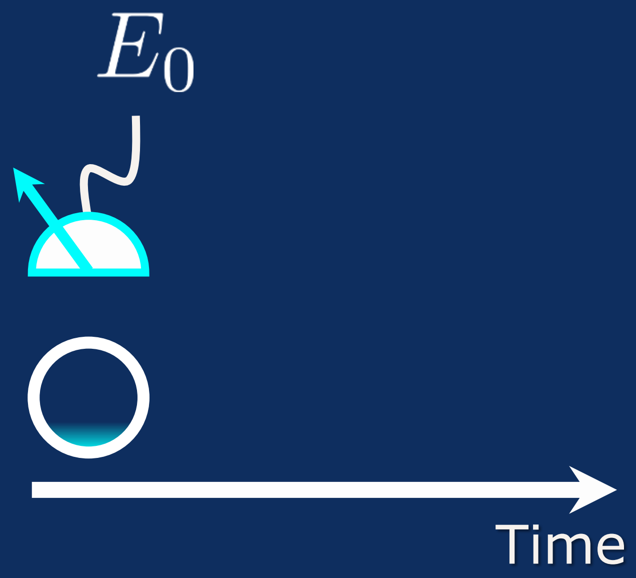
Two-point measurement (TPM) scheme

Time

M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

Energy fluctuations

Two-point measurement (TPM) scheme



M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

Energy fluctuations

Two-point measurement (TPM) scheme

Eigenvalue of $\hat{H}_0$

E_0

M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

Energy fluctuations

Two-point measurement (TPM) scheme

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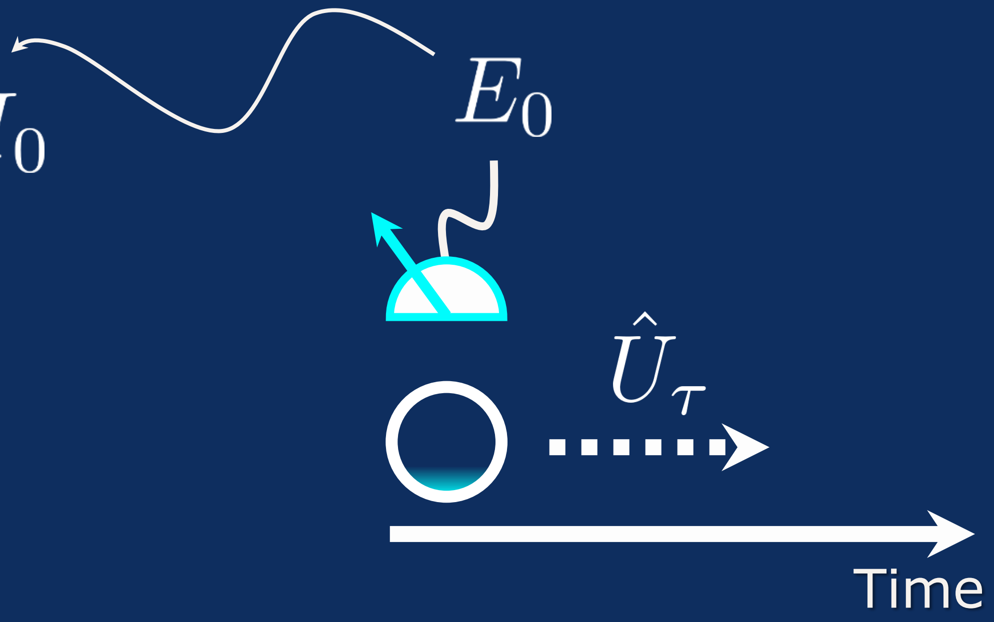

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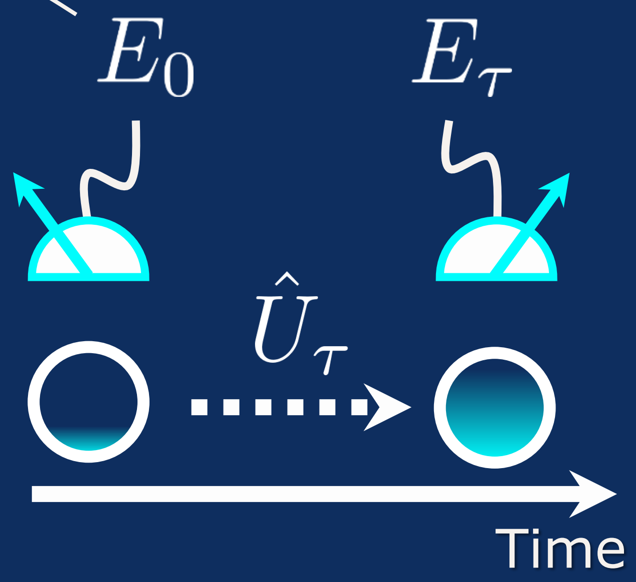

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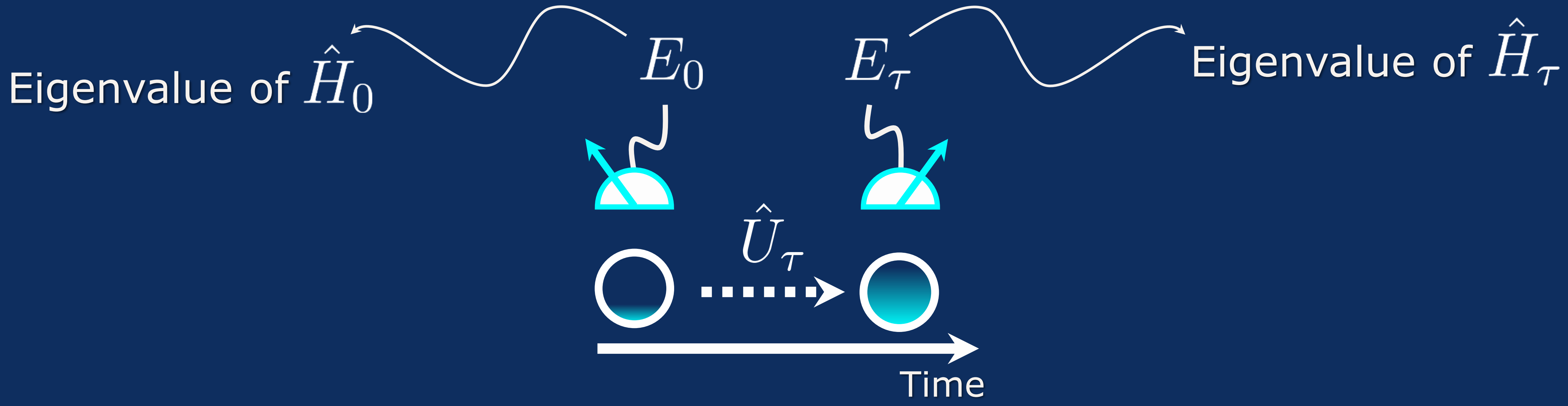

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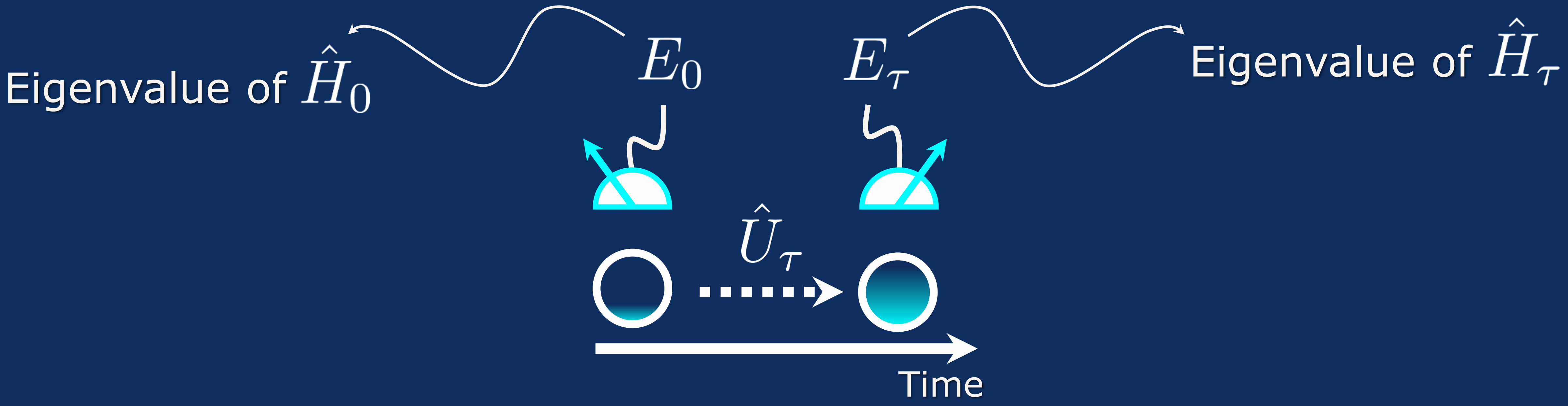
Two-point measurement (TPM) scheme



$$\Longrightarrow \quad \Delta E_\tau \equiv E_\tau - E_0$$

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M. Esposito *et al.*, Rev. of Mod. Phys., 2009, 81.4: 1665-1702.

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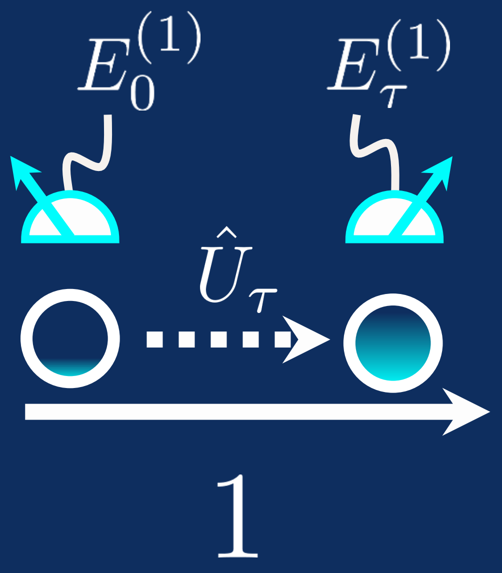

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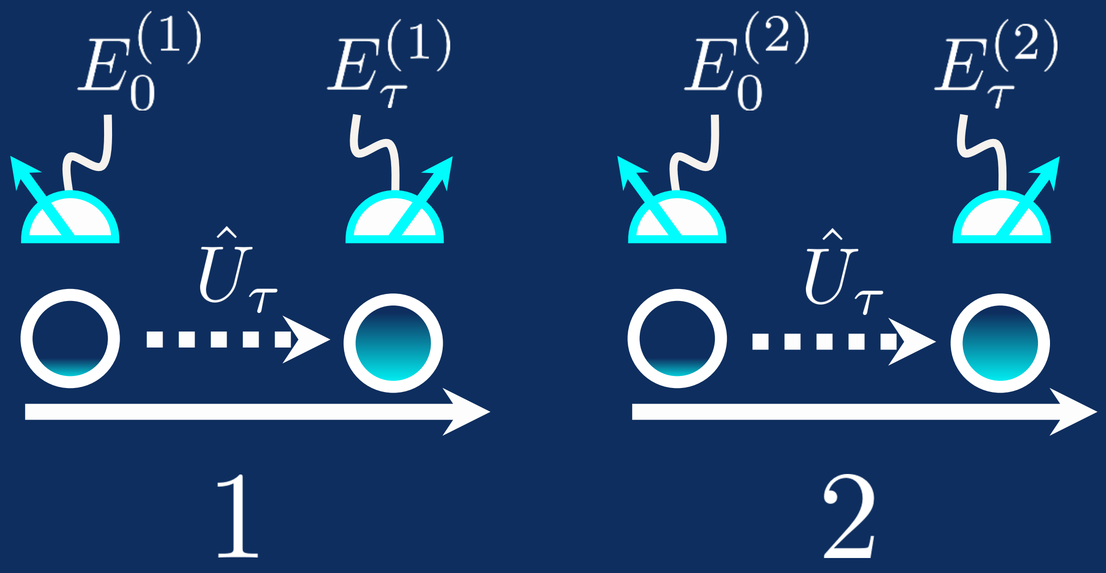

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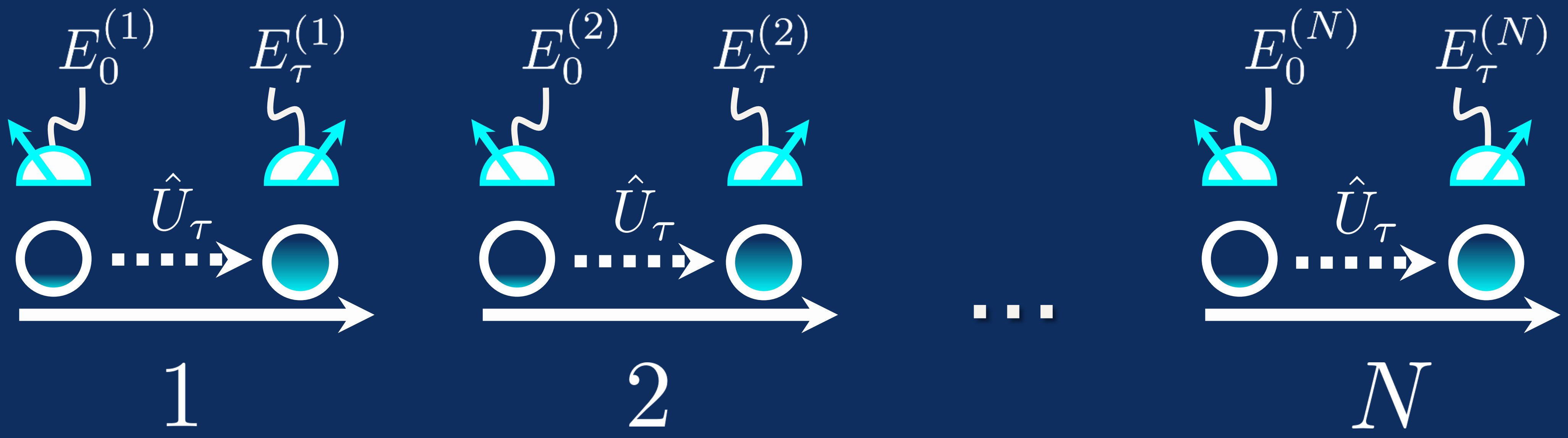

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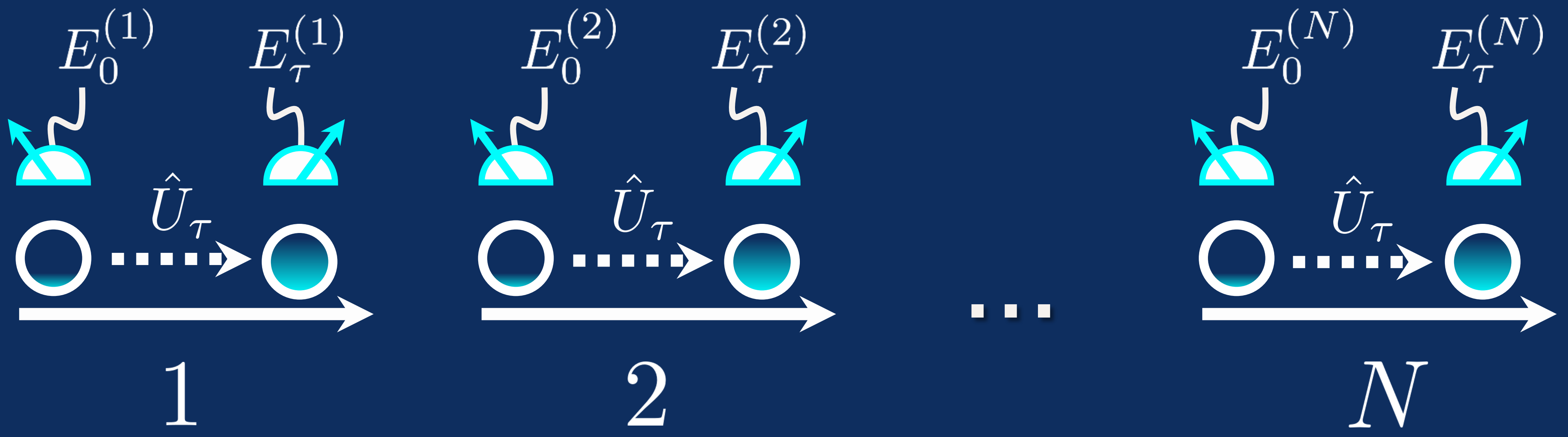

$$\rightarrow \langle \Delta E_\tau \rangle$$

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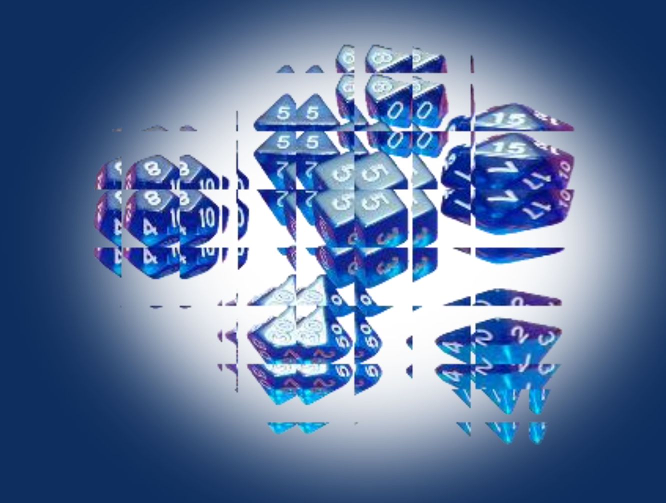
Energy fluctuations

The generating function of the statistics

$$p(\Delta E_\tau) \longleftrightarrow P[E_\tau, E_0]$$



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Energy fluctuations

The generating function of the statistics

$$G(\chi) = \int_{-\infty}^{+\infty} d(\Delta E_\tau) e^{-i\chi \Delta E_\tau} p(\Delta E_\tau)$$

Trick:

$$e^{-i\chi \Delta E_\tau} p(\Delta E_\tau) = \sum_{E_0, E_\tau} e^{-i\chi \Delta E_\tau} P[E_\tau, E_0] \delta\big(\Delta E_\tau - (E_\tau - E_0)\big)$$

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$$\Longrightarrow \; G(\chi) = \mathrm{Tr}\left\{\hat{\rho}_{\tau,\chi}\right\}$$

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Energy fluctuations

The statistics

$$\langle \Delta E_\tau^n \rangle = (-i)^n \frac{d^n}{d\chi^n} G(\chi) \bigg|_{\chi=0}$$

Once we know $G(\chi)$, we know all the momenta and cumulants associated with ΔE_τ, distributed according to $p(\Delta E_\tau)$

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Multi-qubit charge

Parallel charge

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri, in preparation

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Multi-qubit charge

Parallel charge

→ Tavis-Cummings model:

2 or more qubits coupled simultaneously to the cavity

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Multi-qubit charge

Parallel charge

→ Tavis-Cummings model:

2 or more qubits coupled simultaneously to the cavity

→ To be investigated

an analytical description is still possible

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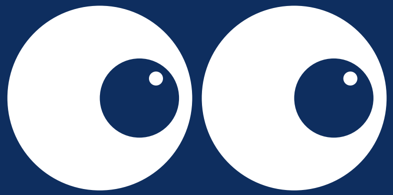
What about the advantage?



Discussion

The advantage

D. Rinaldi, R. Filip, D. Gerace, and G. Guarnieri,
Physical Review A, 2025, 112.1: 012205.

Discussion

The advantage

→ In most of the literature on quantum batteries, the word *advantage* refers to 'superlinear scaling of power':

$$\langle P \rangle \propto N_{\text{batteries}}^{\alpha}, \qquad \alpha > 1$$

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A multibody effect!

→ In our work: the advantage consists of an **efficiency improvement** with respect to other protocols

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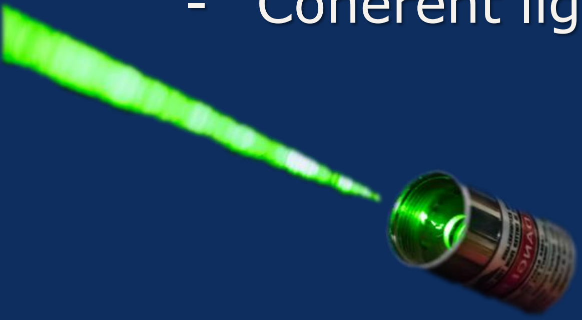
Discussion

Advantage… with respect to what?

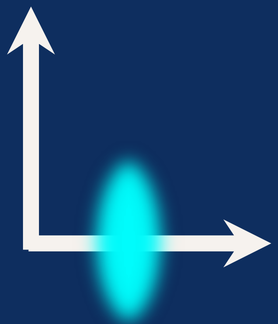
‘Classical’ protocols:

- Thermal light
- Coherent light



‘Quantum Gaussian’ protocols:

- Squeezed light



‘Quantum and non-Gaussian’ protocols:

- Genuine quantum light

Efficiency advantage

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Discussion

The Fock state is an *ideal* state

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The Fock state is an *ideal* state

$$\mathrm{SNR}(\Delta E_\tau) = \frac{J^2 N \mathcal{S}^2(\tau)}{1 - J^2 N \mathcal{S}^2(\tau)} \xrightarrow{\tau \to \tau_{\mathrm{opt}}} +\infty \quad \text{if } \Delta\omega = 0$$

for a Fock state cavity

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⟶ More realistic models can be taken into account

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→ More realistic models can be taken into account

→ Similar results involving *bosonic* quantum batteries:
B. Polo and F. Centrone, arXiv preprint arXiv:2505.24604, 2025.

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Physical Review A, 2025, 112.1: 012205.