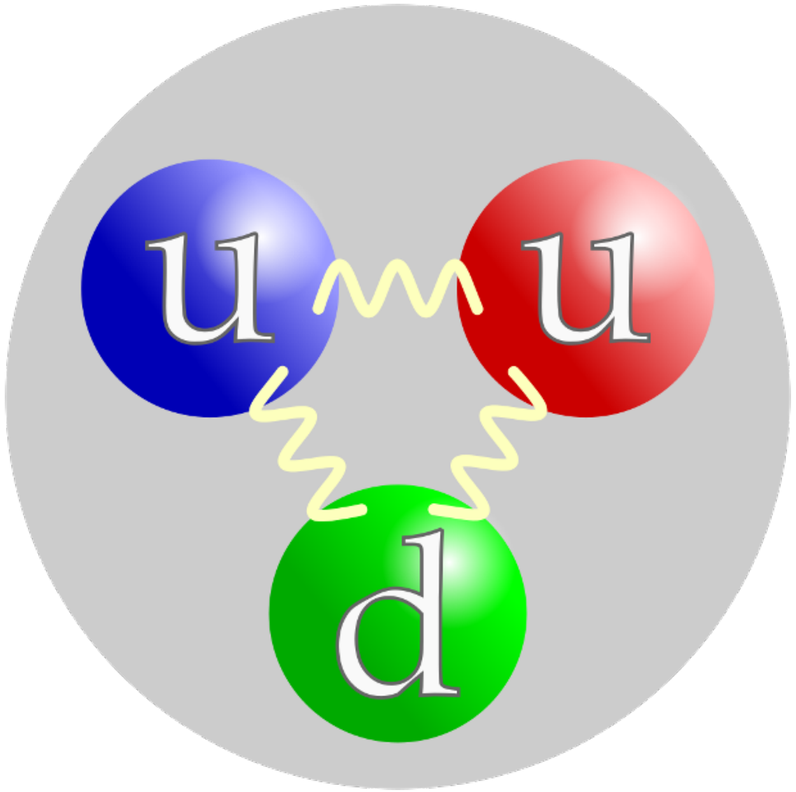
Peering into the proton: Insights from Theory

Alessio Carmelo Alvaro, XXXIX Cicle
End of year seminars, University of Pavia
Supervisors: Barbara Pasquini, Simone Rodini

A long story short

Quark model (1964)



A long story short

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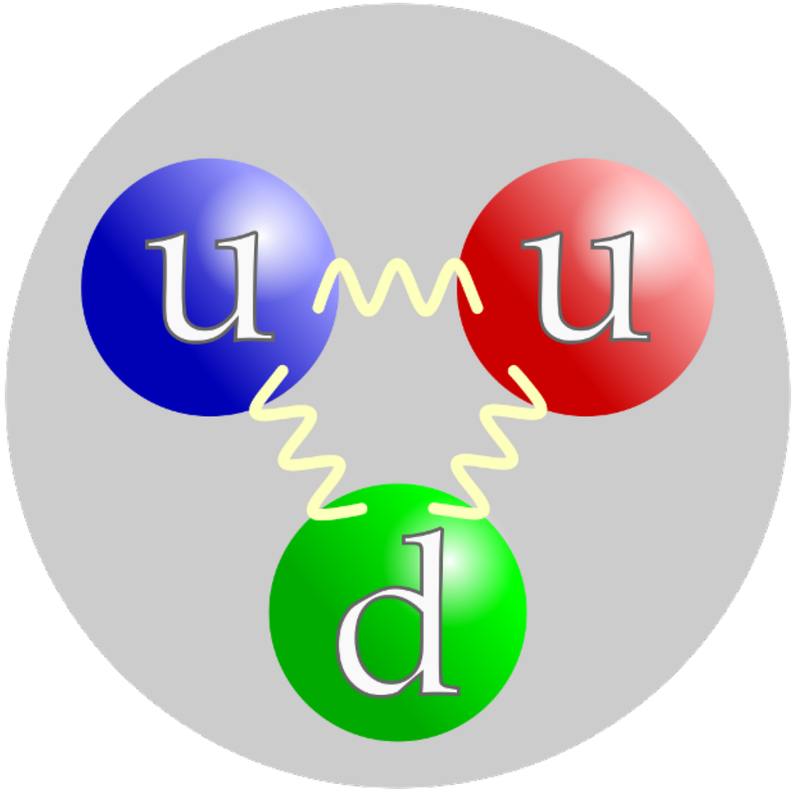
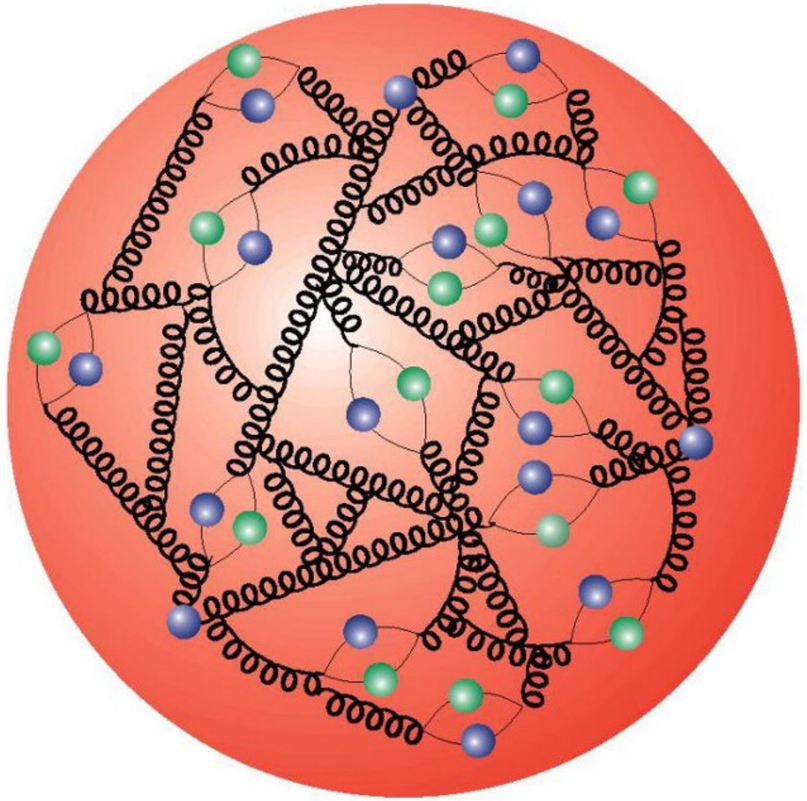


Today



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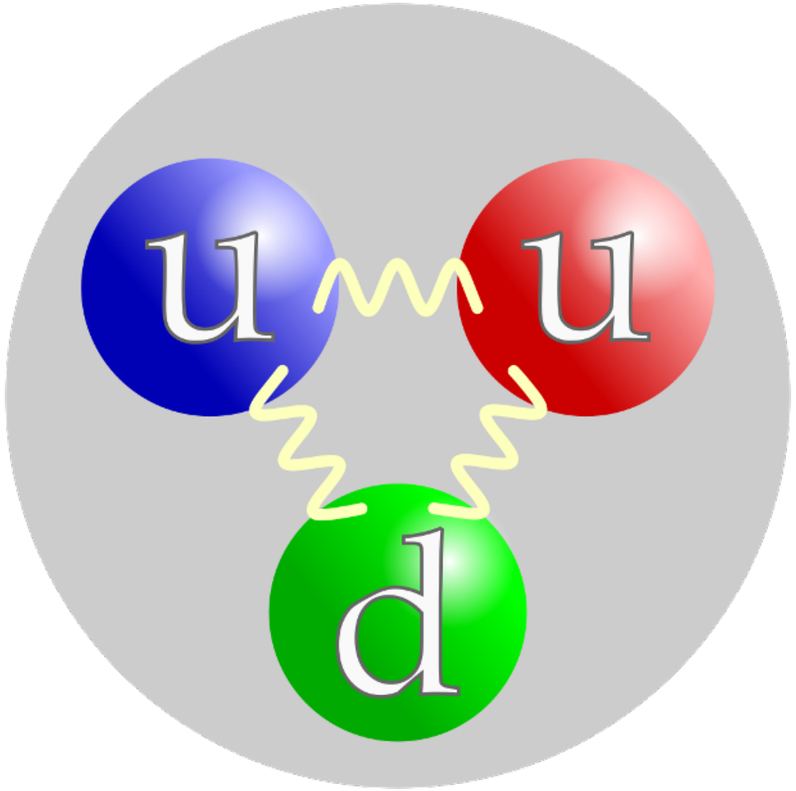

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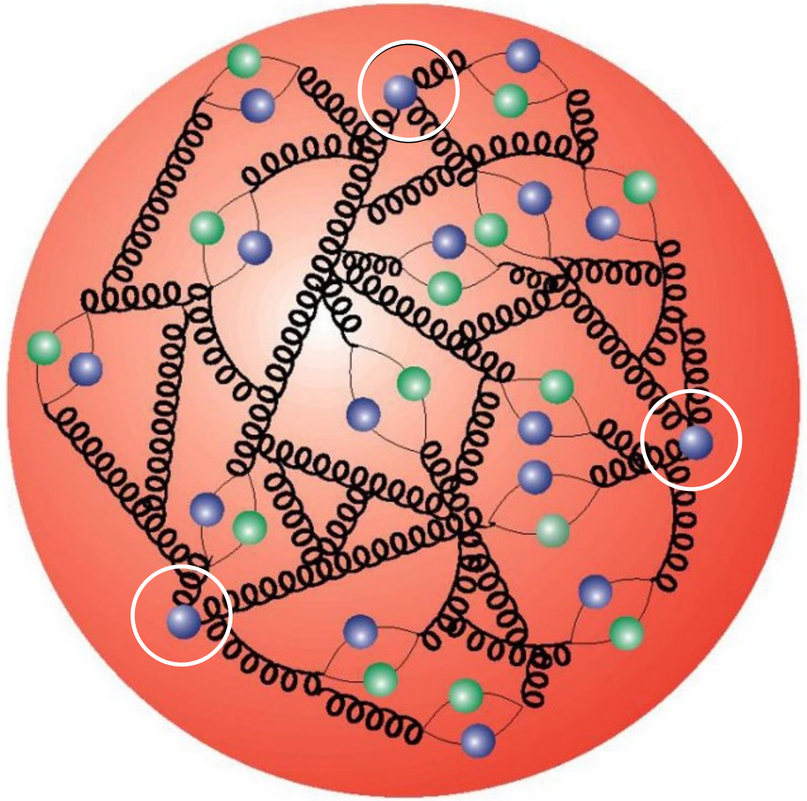

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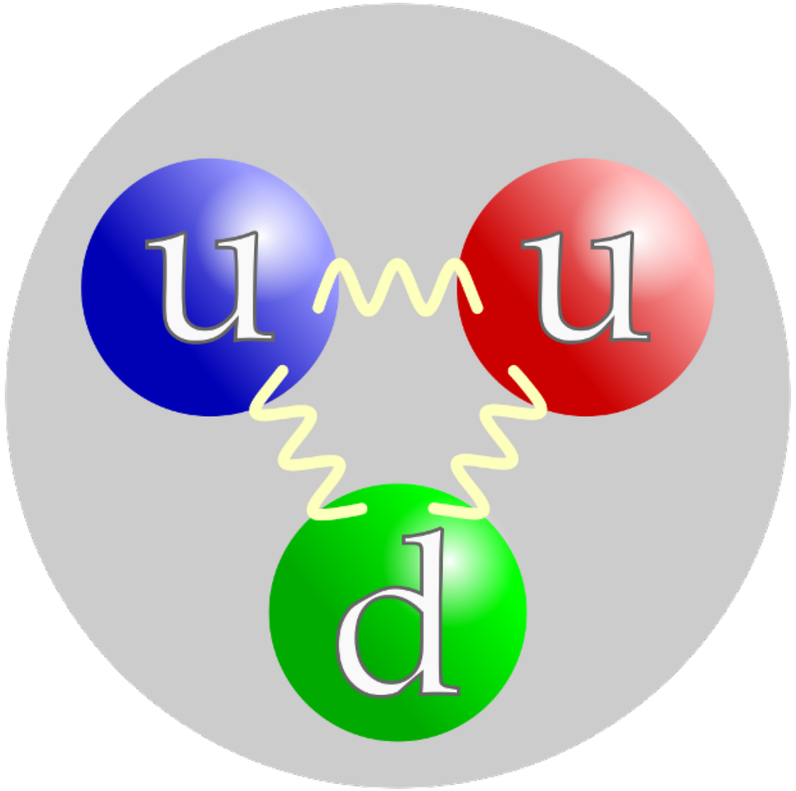

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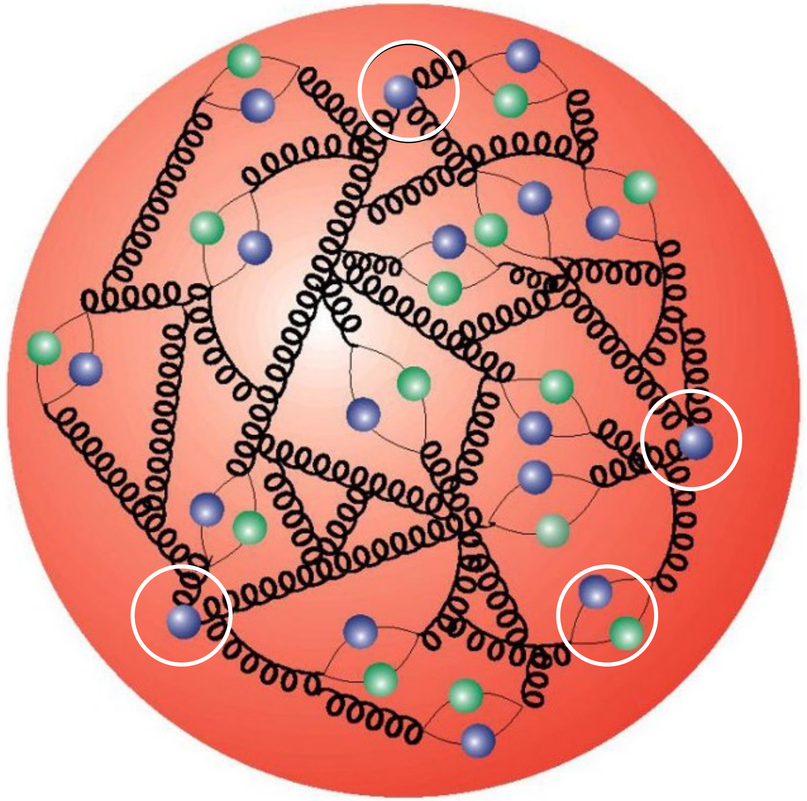

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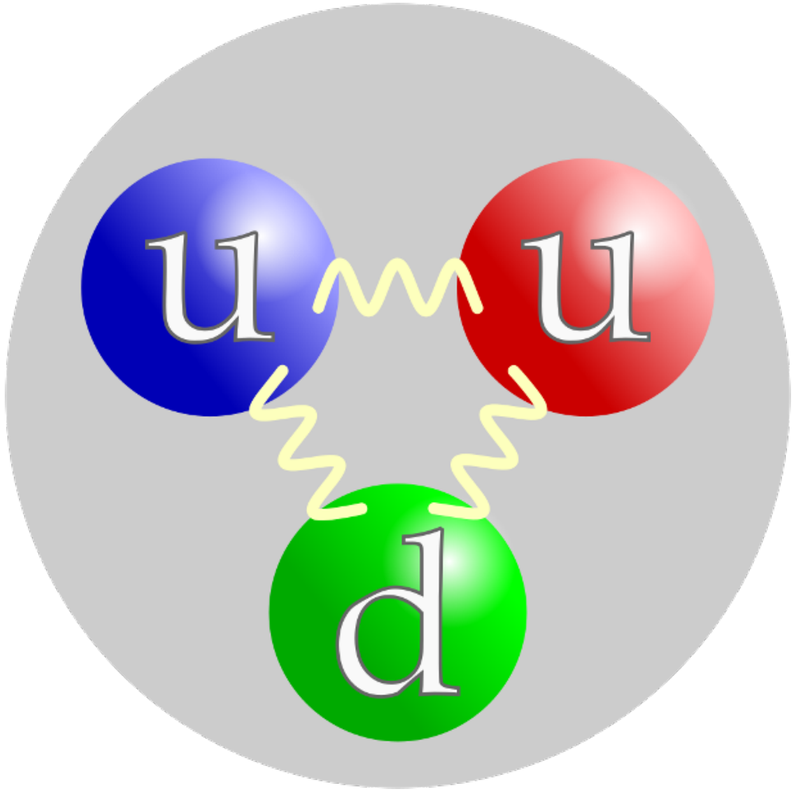

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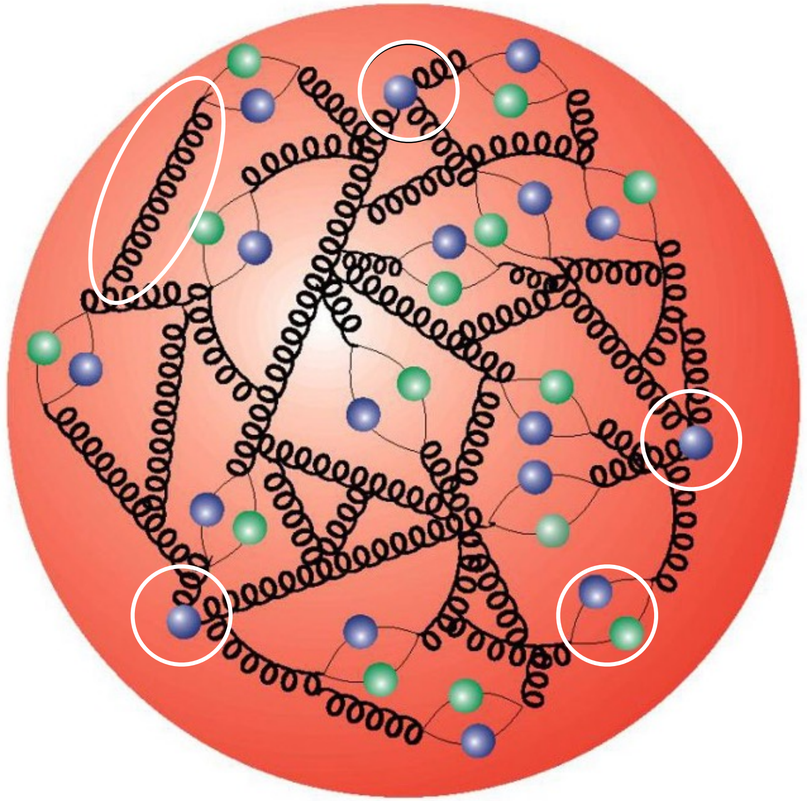
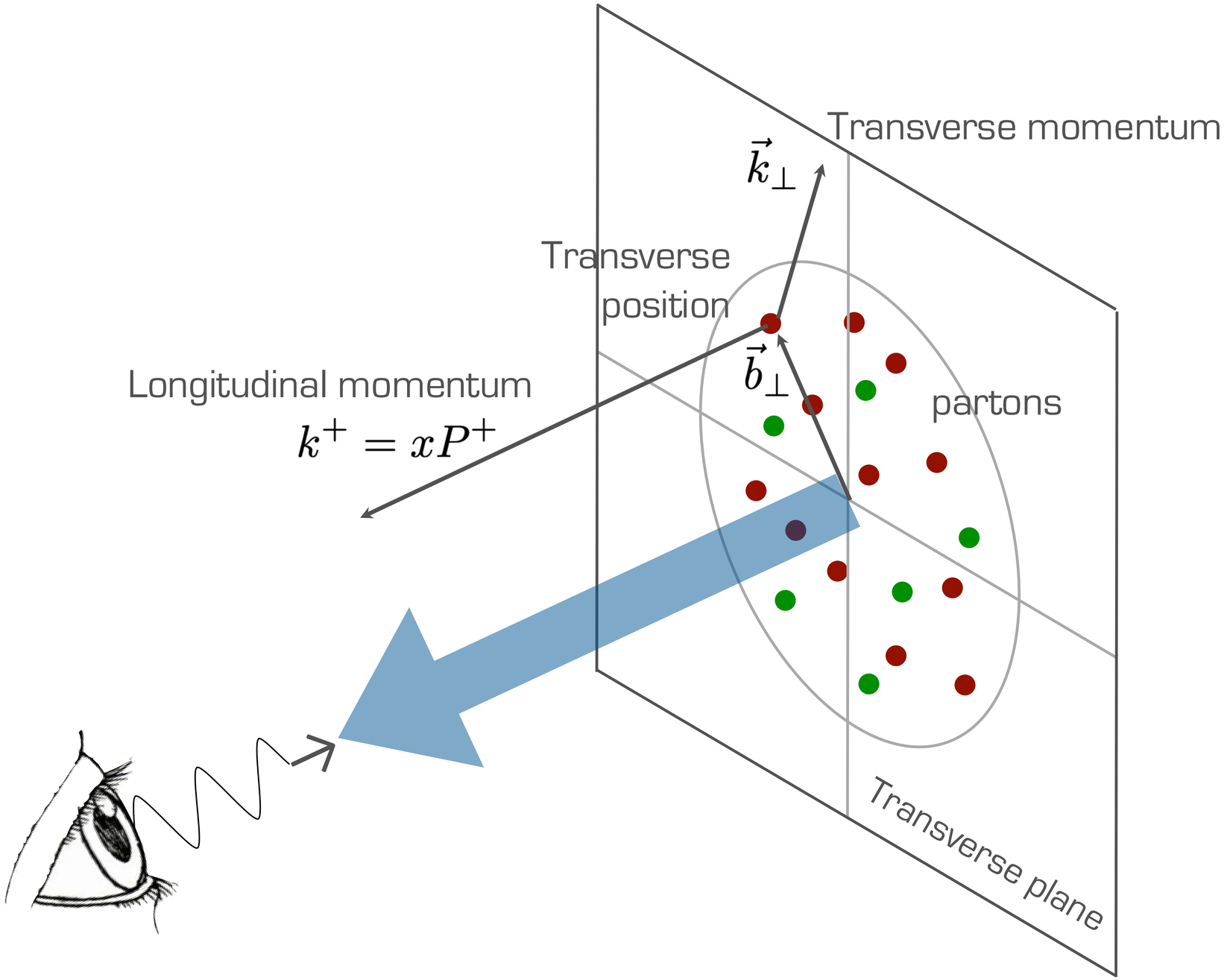


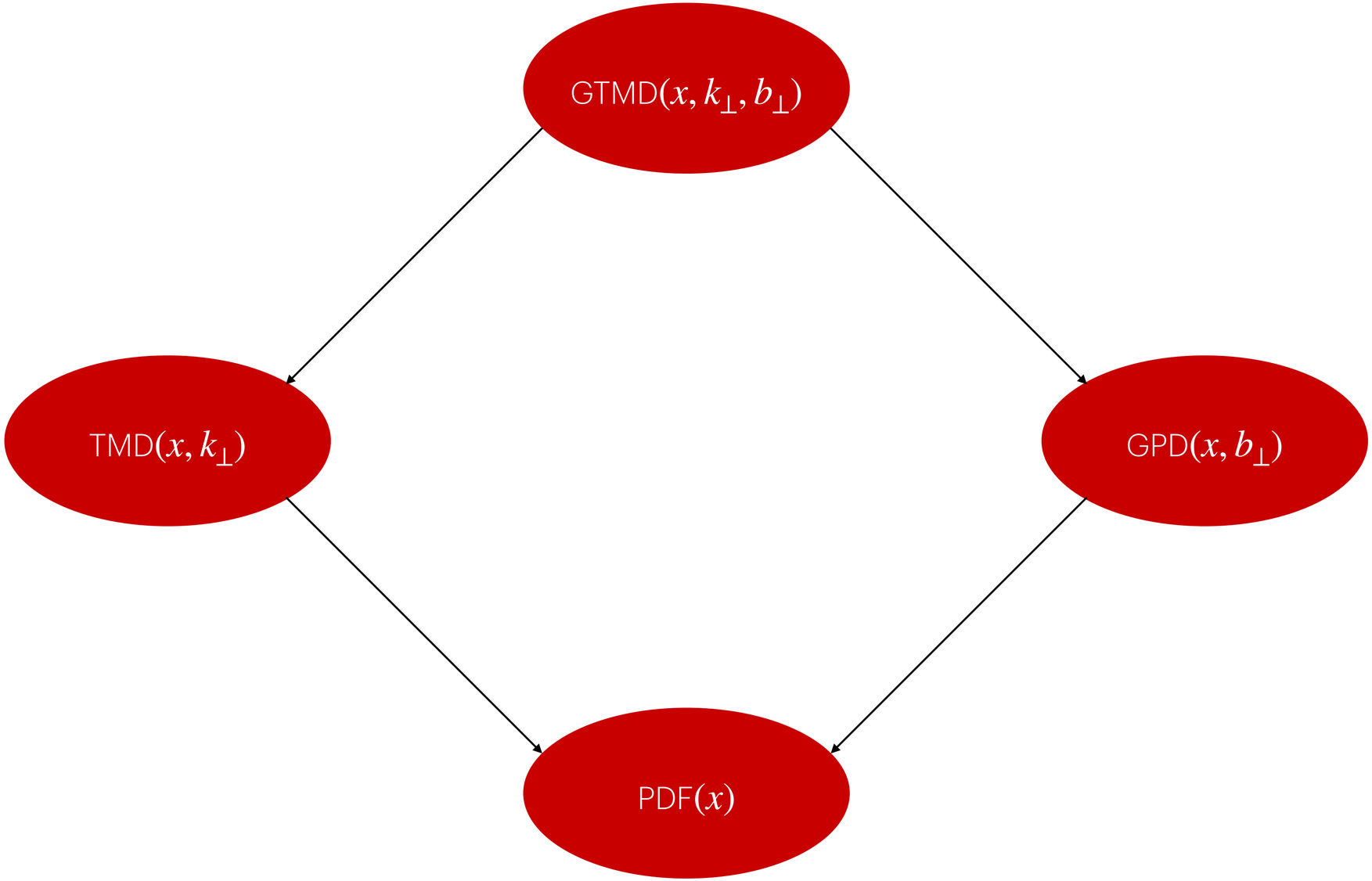
Open problems

Can we build maps of the internal structure of the proton?

How do the global properties (mass, spin) of the proton emerge from its elementary constituents?


Transverse momentum
$\vec{k}_\perp$
Transverse
position
$\vec{b}_\perp$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

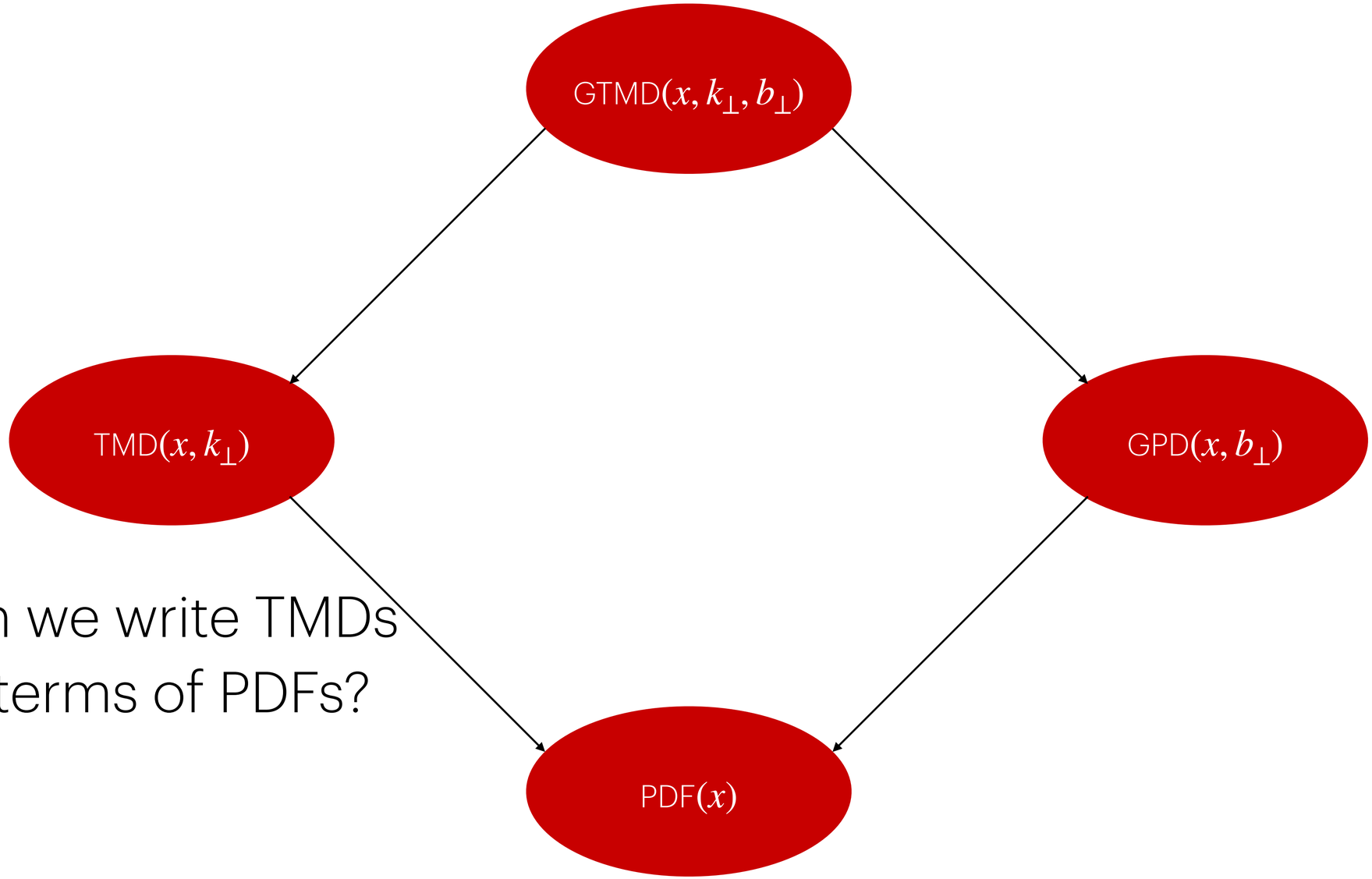
Different tools to unveil proton structure



A mixed problem

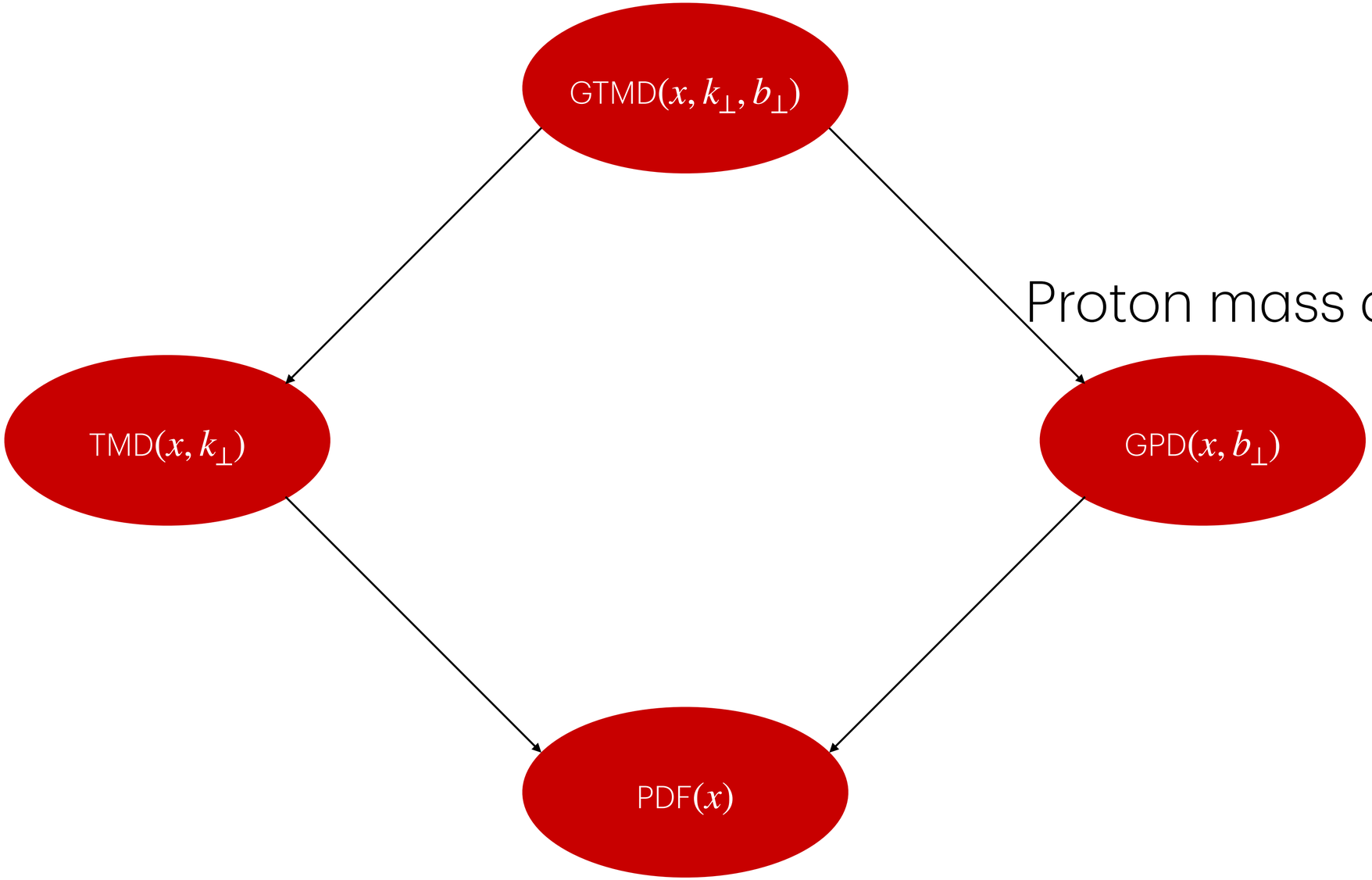
We cannot (completely) predict these maps from theory!

My projects



Can we write TMDs in terms of PDFs?

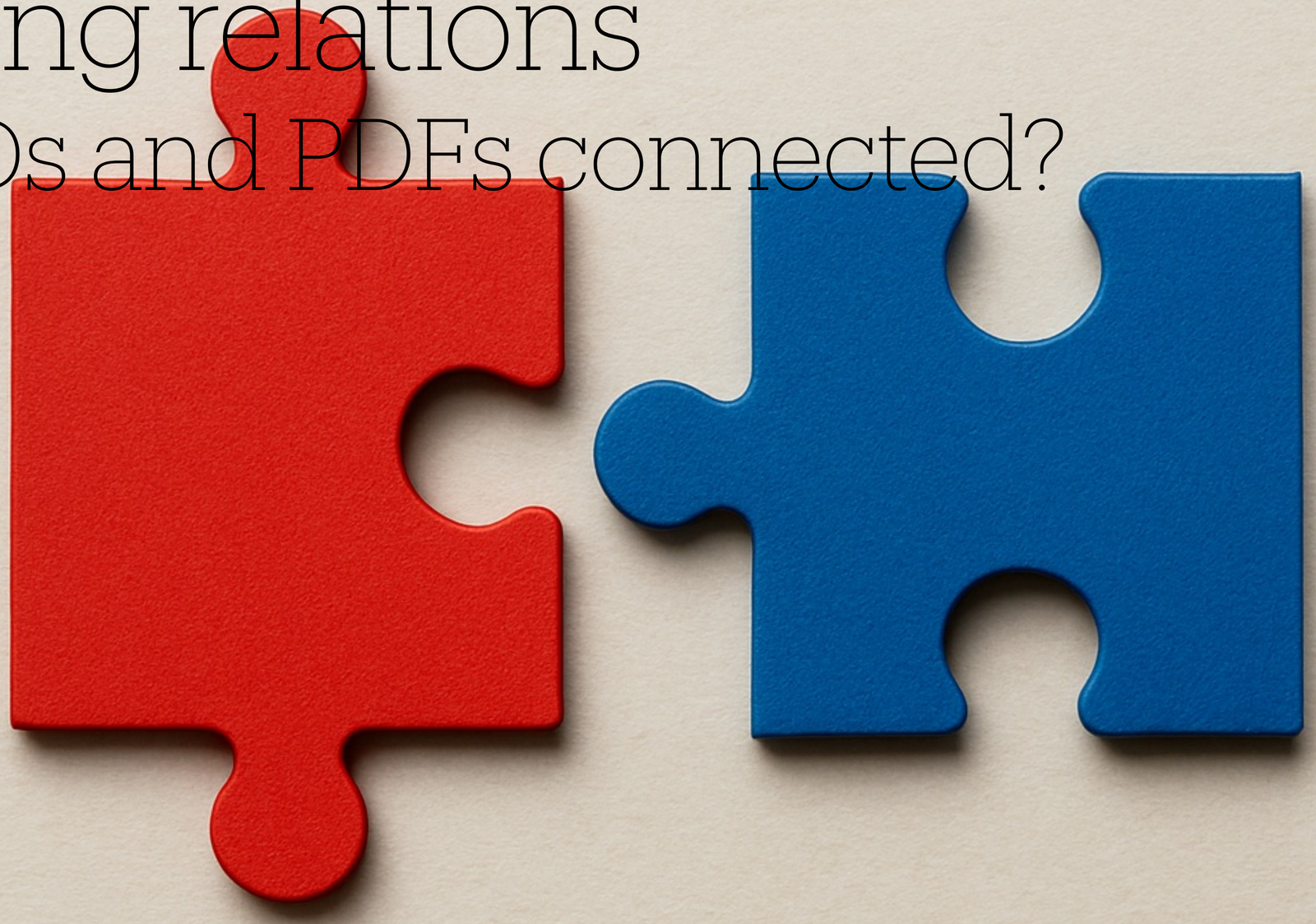
My projects



Proton mass and GPDs

Matching relations

Are TMDs and PDFs connected?



Physical intuition

Collinear kinematic $\longleftrightarrow$ Integration over $k_\perp$ transverse kinematic?

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"Yes" $\Longrightarrow$ Relation between TMDs and PDFs

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Collinear kinematic $\longleftrightarrow$ Integration over $k_\perp$ transverse kinematic?

"Yes" $\Longrightarrow$ Relation between TMDs and PDFs

Simple ansatz: $\int dk_\perp \mathrm{TMD}(x, k_\perp) = \mathrm{PDF}(x)$

Too simple

A bit of rigor...

$$\mathrm{TMD}_i(x, k_\perp) = \sum_j \mathrm{C}_{ij}(x, k_\perp) \otimes \mathrm{PDF}_j(x) + O\left(\frac{M^2}{k_\perp^2}\right)$$

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Why? Some PDFs are very well known

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Prediction of physical observables

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Why? Some PDFs are very well known

Prediction of physical observables

Constraints to TMDs expression (toward TMD fit)

Matching relations: current status

Distribution	Tw2	Tw3	Accuracy
f_1^g	f_g	-	$\mathrm{N^3LO}$
$h_1^{\perp g}$	f_g	-	$\mathrm{N^3LO}$
g_{1L}^g	Δf_g		$\mathrm{N^3LO}$
g_{1T}^g			
$f_{1T}^{\perp g}$	-		
h_{1T}^g	-		
$h_{1L}^{\perp g}$			
$h_{1T}^{\perp g}$			

Dash: no matching

Blank space: unknown

Matching relations: my results

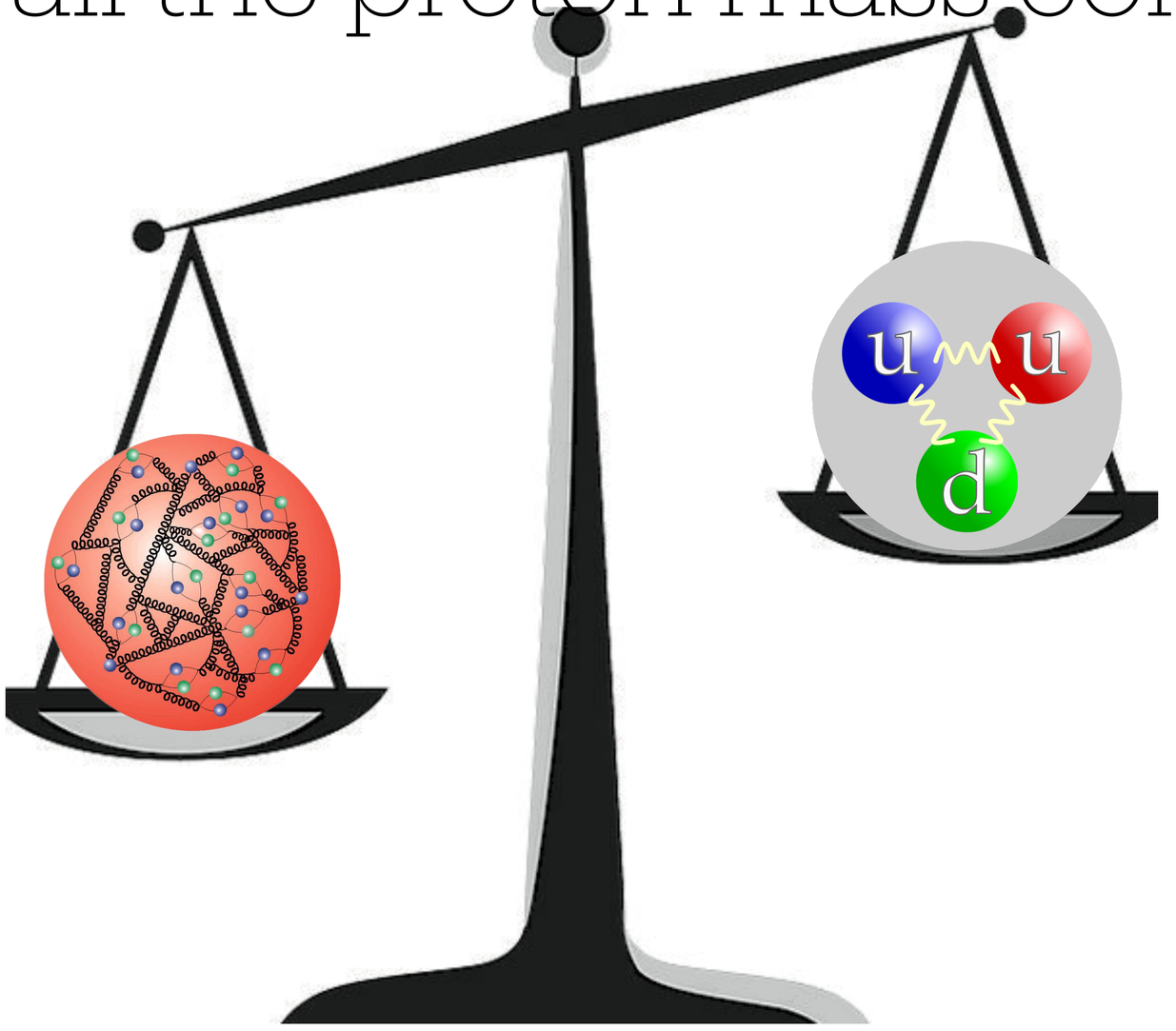
Distribution	Tw2	Tw3	Accuracy
f_1^g	f_g	-	$\mathrm{N^3LO}$
$h_1^{\perp g}$	f_g	-	$\mathrm{N^3LO}$
g_{1L}^g	Δf_g	$\surd$	$\mathrm{N^3LO}$
g_{1T}^g	$\surd$	$\surd$	LO
$f_{1T}^{\perp g}$	-	$\surd$	LO
h_{1T}^g	-	$\surd$	LO
$h_{1L}^{\perp g}$	$\surd$	$\surd$	LO
$h_{1T}^{\perp g}$	$\surd$	$\surd$	LO

Tree level: filled all the gaps
+ all mass corrections

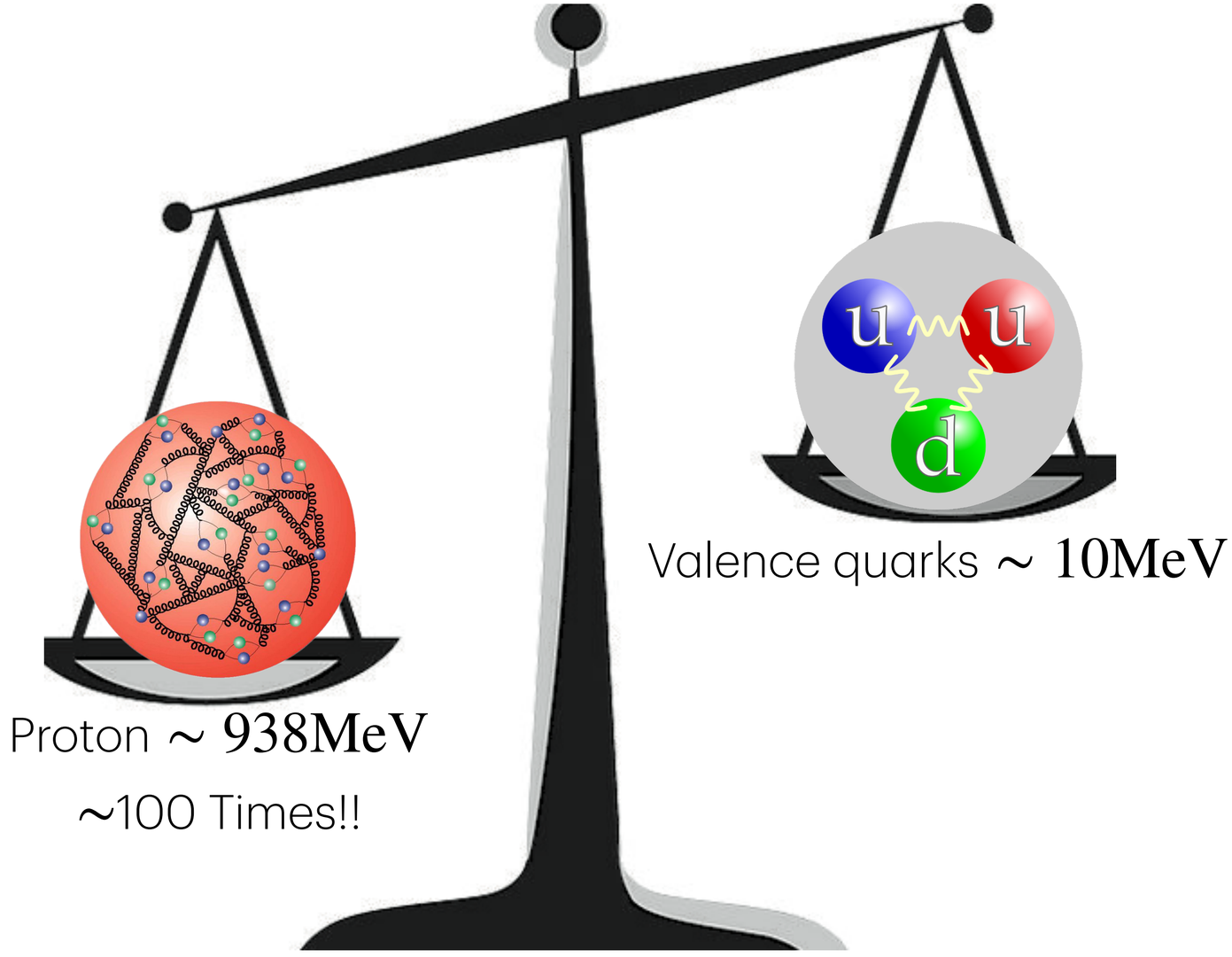
Plan: $\boldsymbol{g_{1T}^g}$ at Tw2-NLO
+ phenomenology

The trace anomaly and the mass

Where does all the proton mass come from?



The mass problem

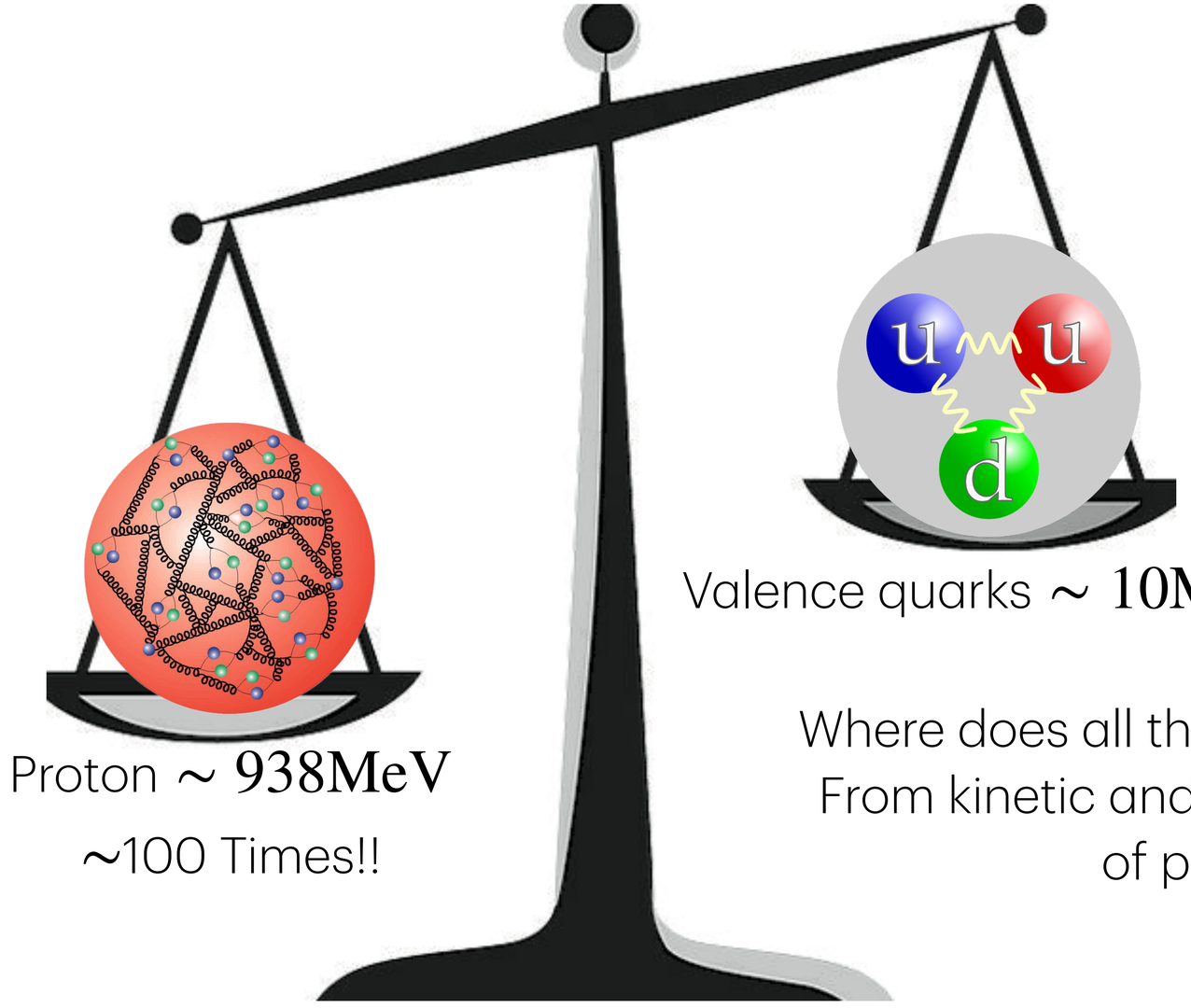


Valence quarks ~ 10MeV

Proton ~ 938MeV

~100 Times!!

The mass problem



Valence quarks $\sim$ **10MeV**

Proton $\sim$ **938MeV**

~100 Times!!

Where does all the mass come from?
From kinetic and potential energies
of partons

The Trace Anomaly

Mechanical properties of a system: $T^{\mu\nu}$

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'Classical': $T^{\mu}_{\mu} = m\bar{\psi}\psi$

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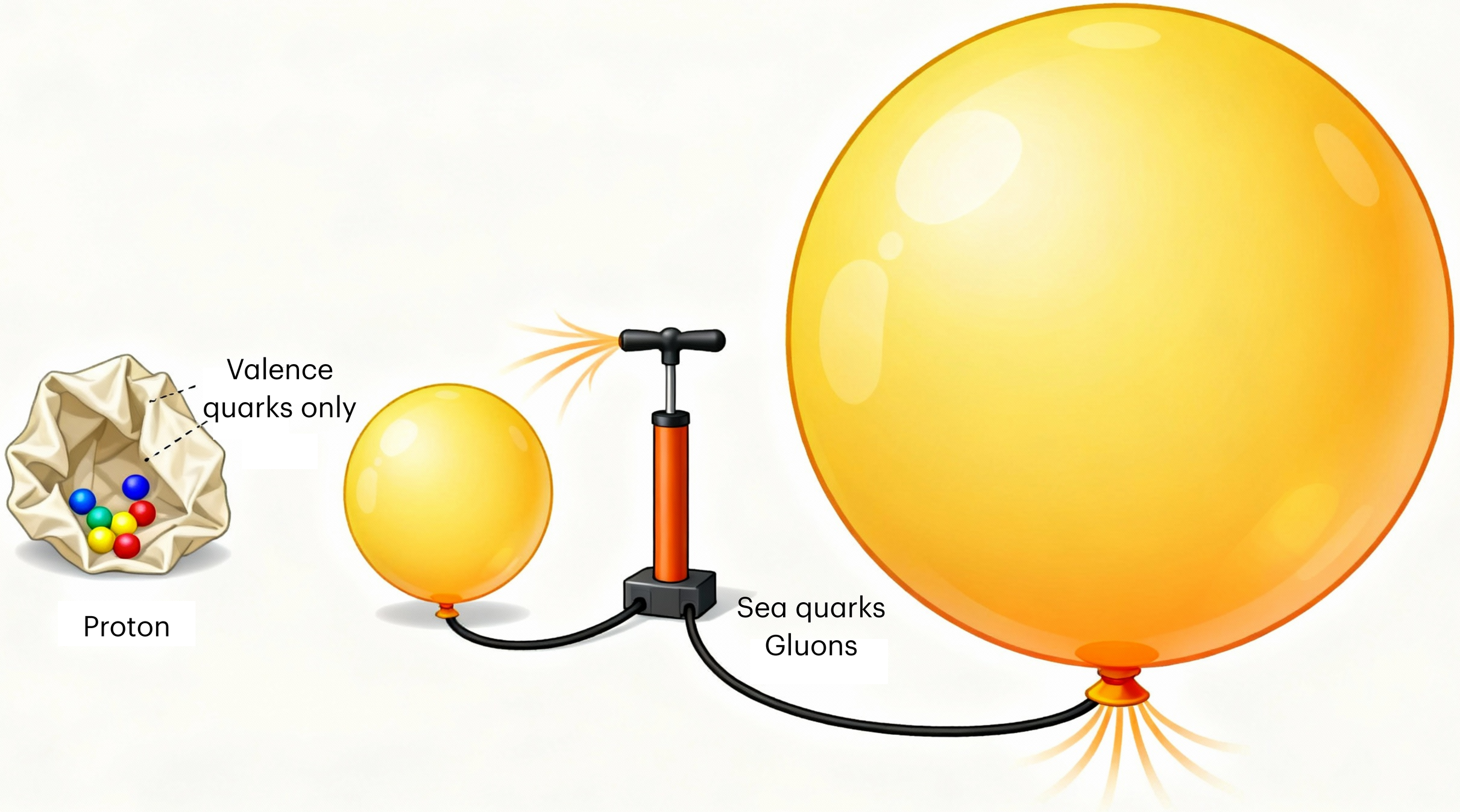
'Classical': $T^{\mu}_{\mu} = m\bar{\psi}\psi$

After renormalization:

$$T^{\mu}_{\mu} = m\bar{\psi}\psi + \gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F_{\alpha\beta}F^{\alpha\beta}$$

'Classical' term

Anomaly term


Valence
quarks only
Proton
Sea quarks
Gluons

Deep Virtual Compton Scattering

EMT $T^{\mu\nu} \Longleftrightarrow$ GPDs$(x, b_\perp)$

GPDs are measured in $lp \rightarrow l'p'\gamma$

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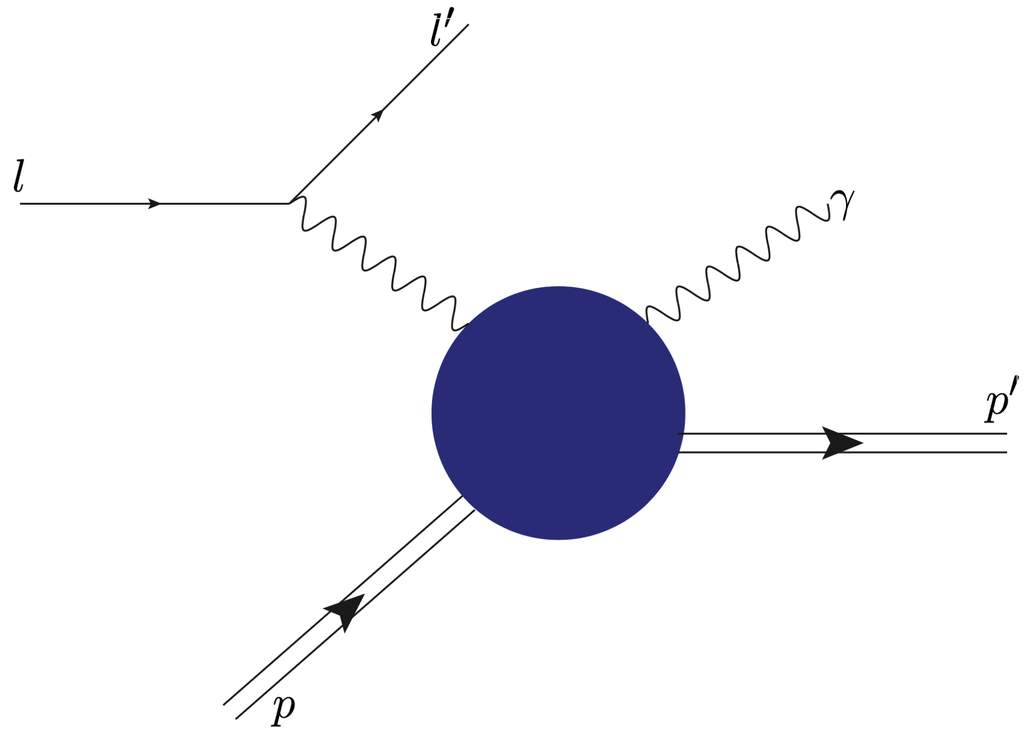

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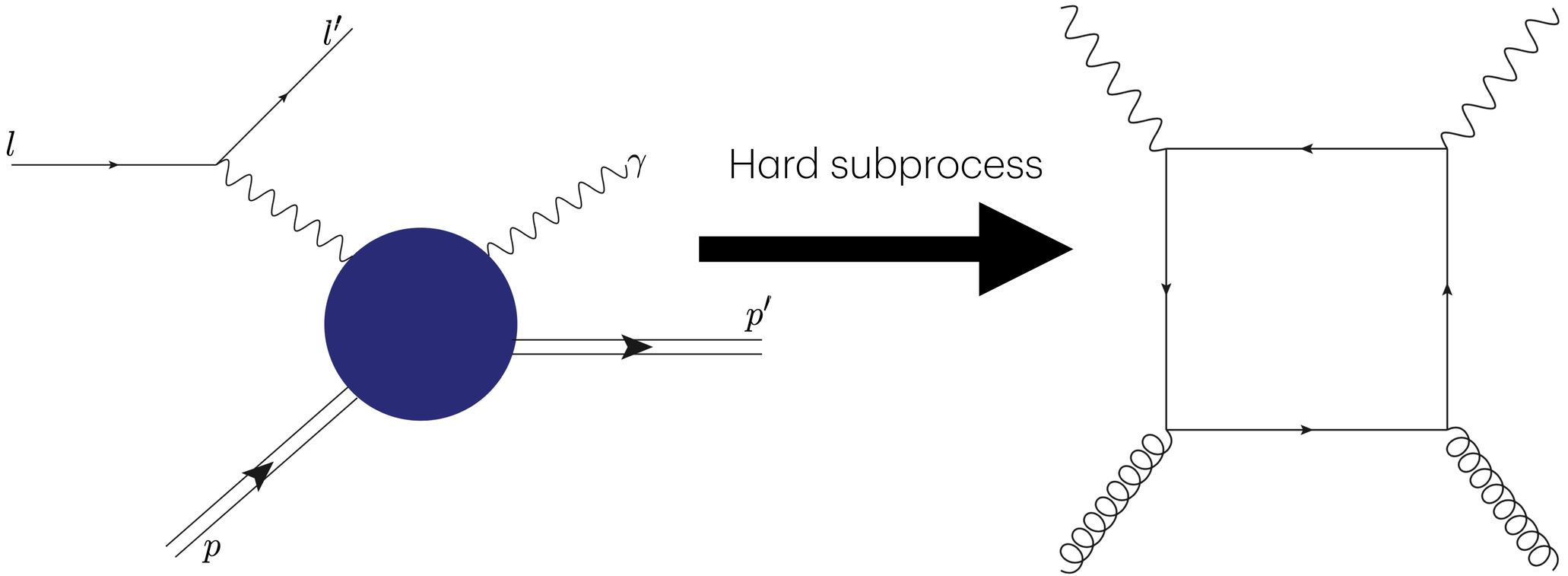
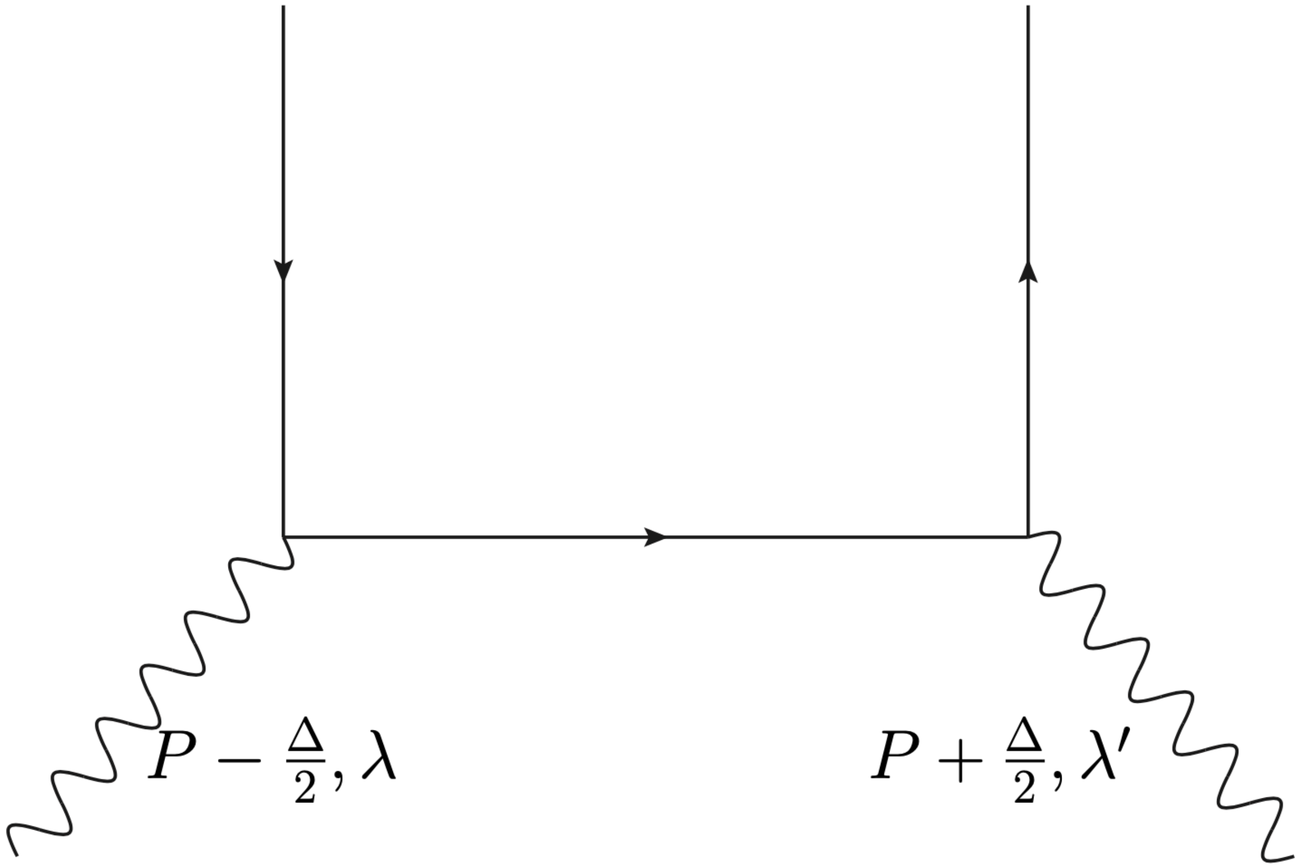


My work

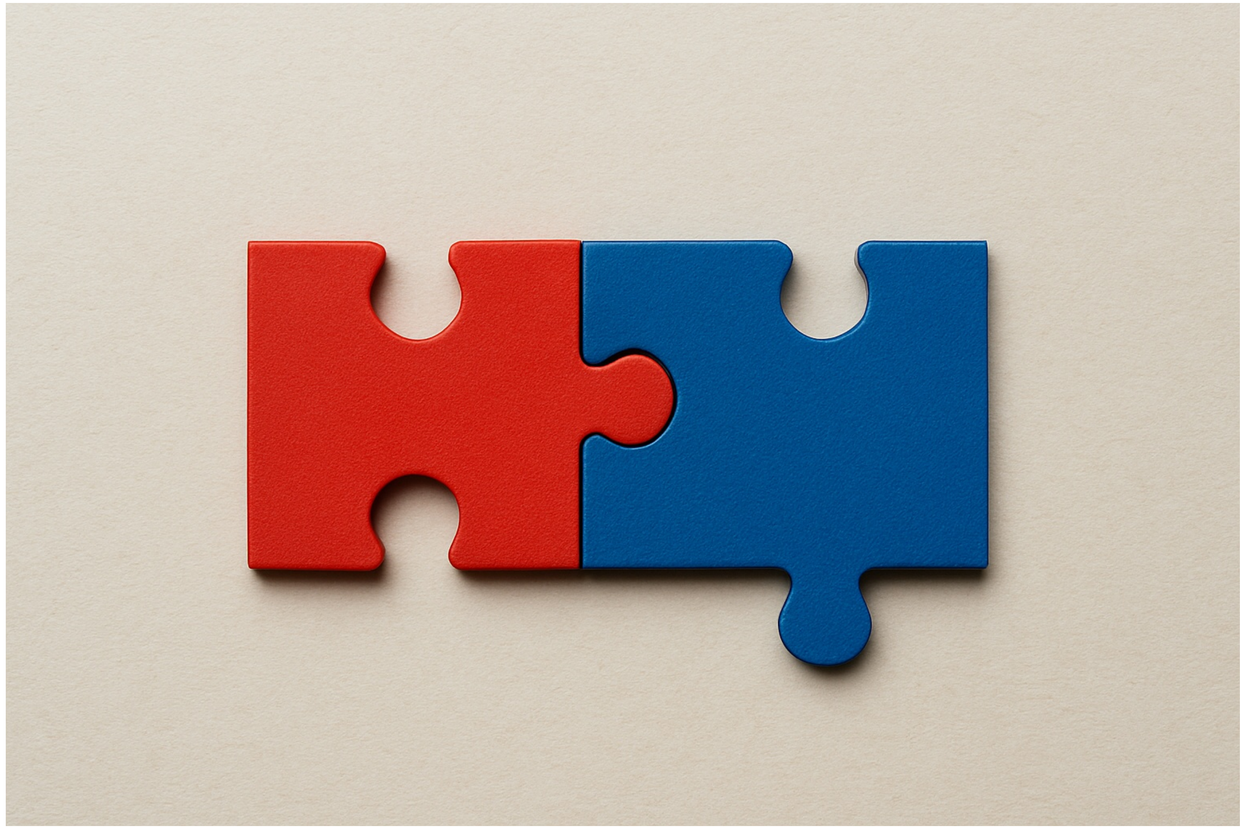


Perturbative computation of the GPDs

Trace anomaly in terms of GPDs

Plan: different regulators + gluon GPDs

Conclusions



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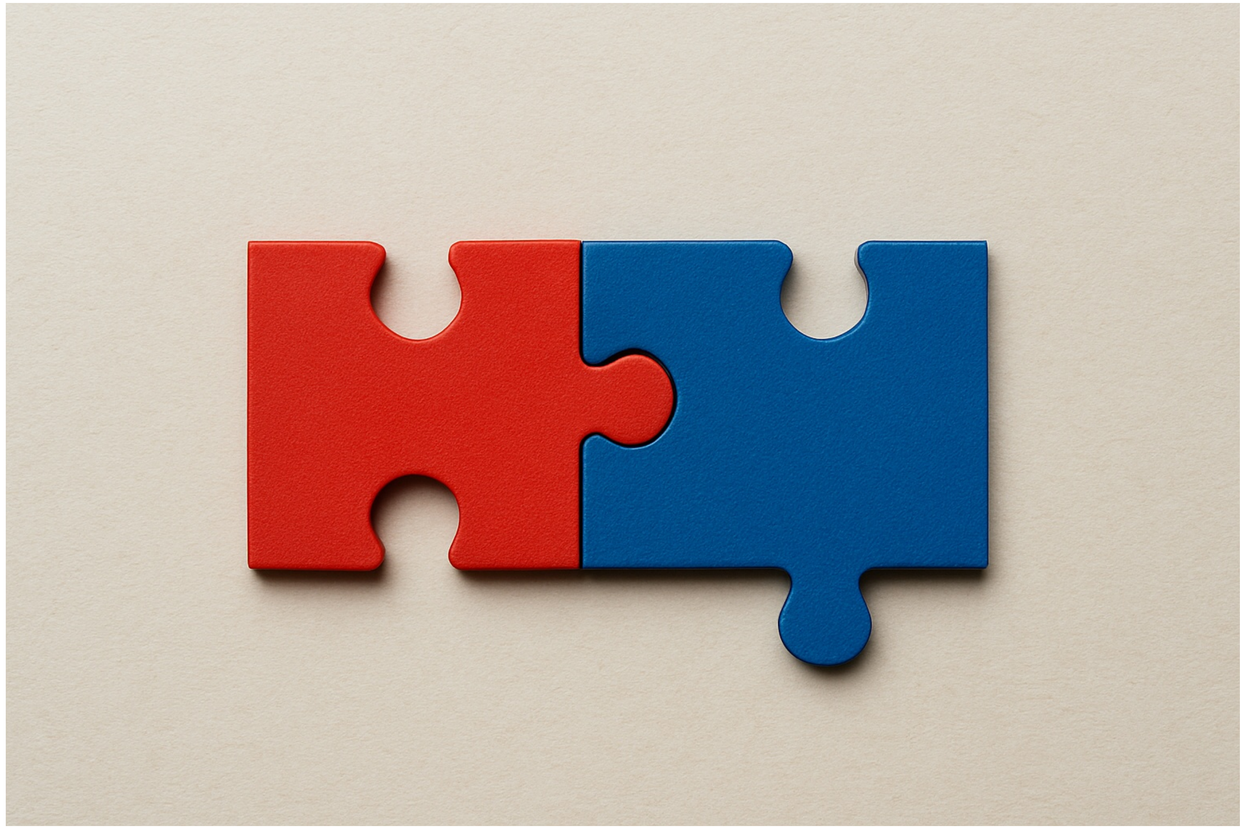
Conclusions



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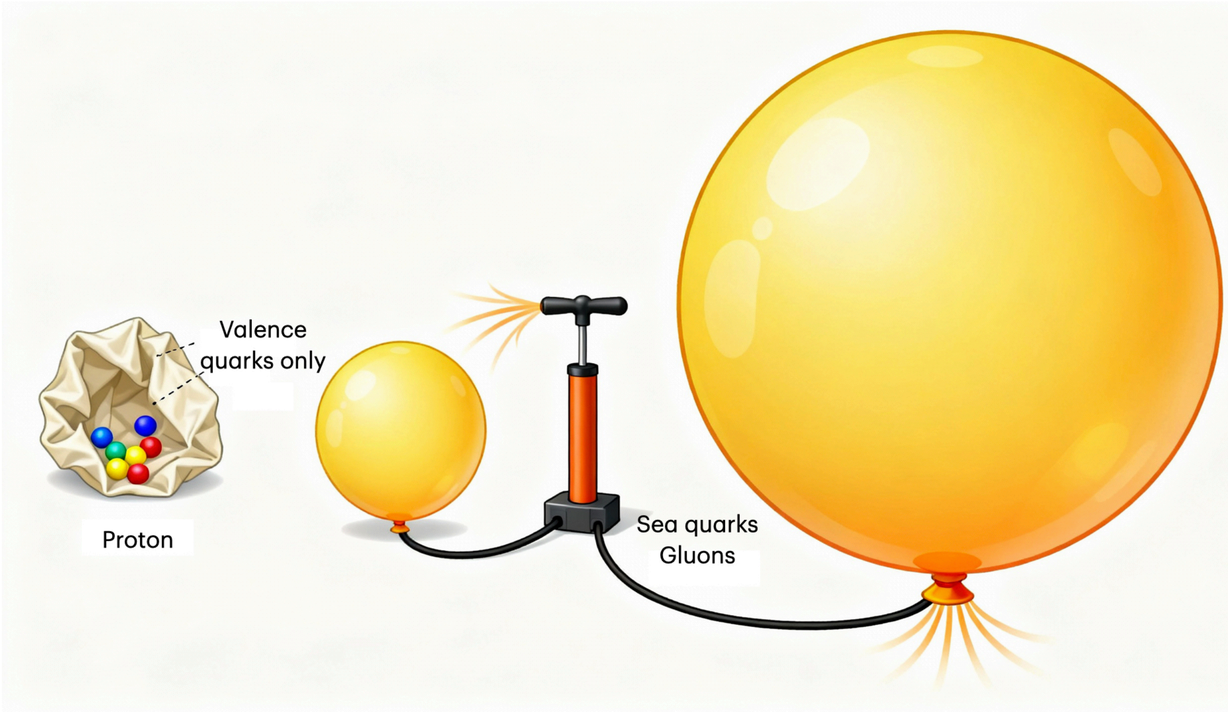

Proton mass and trace anomaly