A Quantum Cellular Automata framework for interacting quantum fields

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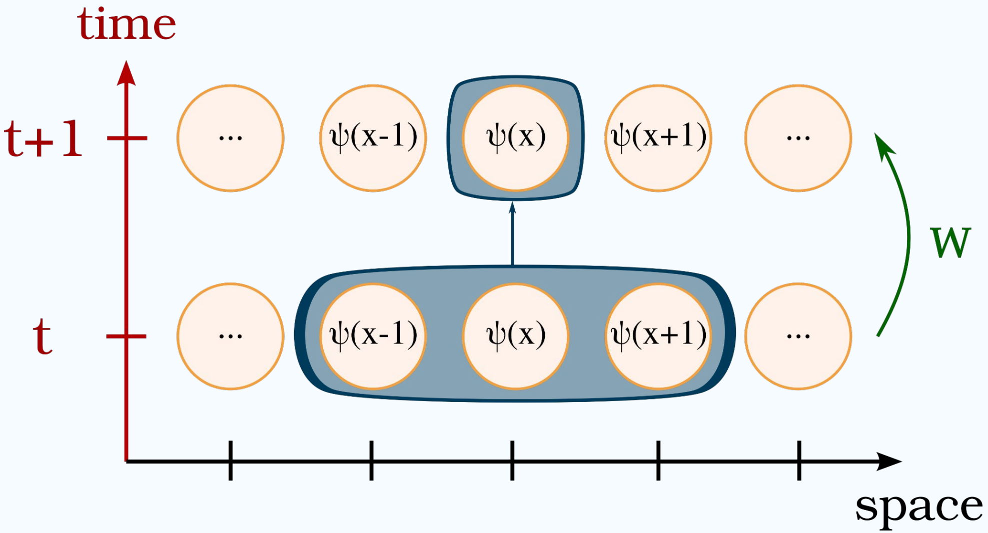
What is a Quantum Cellular Automaton?

QCA: most general
- unitary
- local
- time-discrete

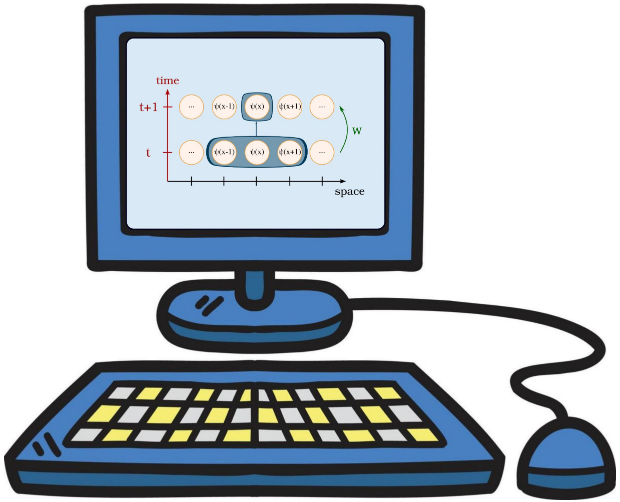
dynamics of a lattice of quantum systems

$$W \in \mathrm{Lin}\left[\mathbb{C}^d \otimes l^2(\mathbb{Z})\right]$$



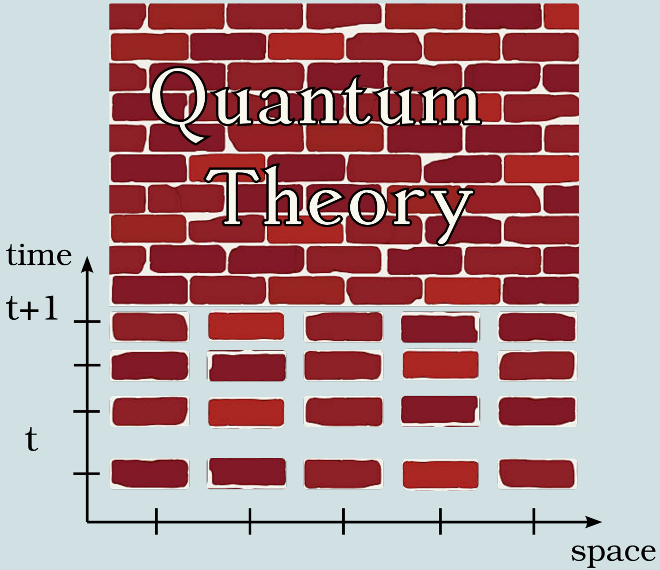
Why study QCA?

Simulation of Quantum Physics



- Algorithms for quantum computing
- Universal computation
- Simulation of Quantum Field Theory

Quantum physics from information processing



Physical systems: memory cells
Physical laws: algorithms

What makes an algorithm *physical*?

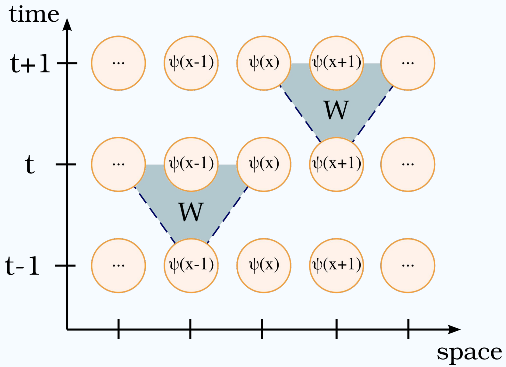
Cell Minimal internal dimension:  $\simeq \mathbb{C}^2$ $\quad \psi(x,t) = \begin{pmatrix} \psi_R(x,t) \\ \psi_L(x,t) \end{pmatrix}$

Update rule

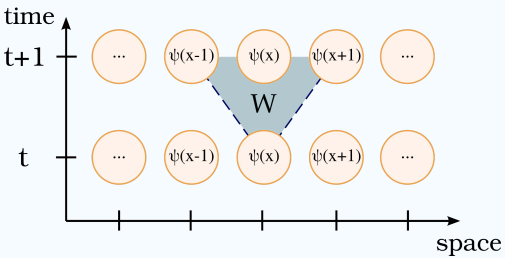
- **Unitarity**: reversibility of the evolution

$$WW^\dagger = W^\dagger W = \mathbb{I}$$

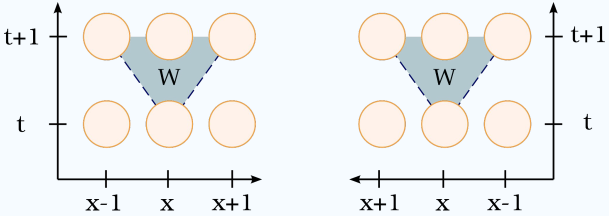
➕ **Homogeneity**: no preferred cell



- **Locality**: bounded propagation speed

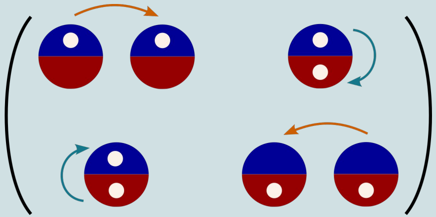


➕ **Isotropy**: no preferred direction



The Dirac QCA: a free theory

Rule: either flip or shift

W= 

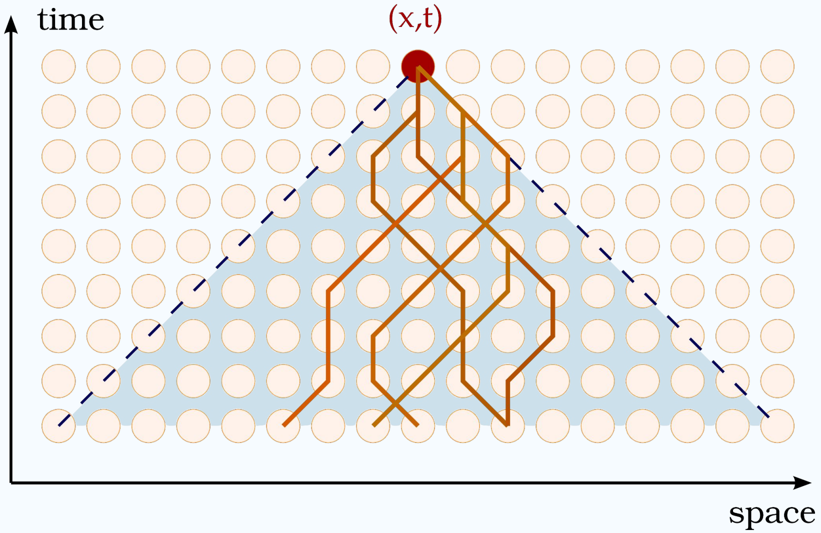
$$= \begin{pmatrix} nT & im \\ im & nT^\dagger \end{pmatrix} \xrightarrow{\text{FT}} \begin{pmatrix} ne^{-ikx} & im \\ im & ne^{ikx} \end{pmatrix}$$

In the low mass and momentum limit it recovers the Dirac dynamics

Path-sum solution

$$W = \begin{pmatrix} nT & im \\ im & nT^\dagger \end{pmatrix} = W_R \otimes T + W_L \otimes T^\dagger + W_F \otimes \mathbb{I}$$

= **action on the state** ⊗ **motion on the lattice**



path $s = s_1 \cdots s_t \in \{R, L, F\}^t$

transition matrix $\mathcal{W}(s) = W_{s_t} \cdots W_{s_2} W_{s_1}$

$$\psi(x,t) = \sum_{\text{starting point } y} \sum_{\text{path } s} \mathcal{W}(s)\psi(y,0)$$

↓

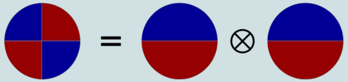
Discrete "Dirac propagator"

Introducing interactions: Thirring QCA

Why?

- QCA counterpart of Thirring model (integrable)
- most general on-site, number preserving interaction

Cell

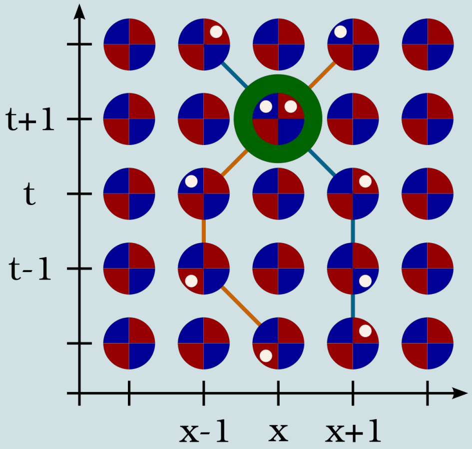


Update rule

$$U = J(\chi) W^{\otimes 2} = e^{i\chi\, \delta_{x_1,x_2}} W^{\otimes 2}$$

Same site + opposite spins = **interaction**

Evolution

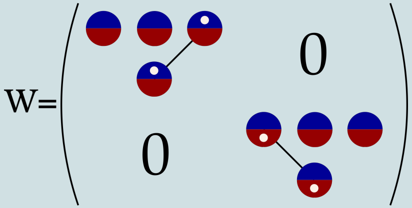


Phenomenology

- **not** a discretization of Thirring model
- broader family of scattering solutions
- bound states for every momentum value

Bethe Ansatz approach

Case study:
massless Thirring
QCA



$$= \begin{pmatrix} T & 0 \\ 0 & T^\dagger \end{pmatrix}$$

Rule: always shift

Ansatz: anti-symmetric superposition of

free eigenstates

$$W|\psi\rangle = \lambda|\psi\rangle \qquad |\psi_{\mathbf{k}}^{(N)}(\mathbf{a})\rangle = \sum_{\mathbf{x}\in\mathbb{Z}^N} e^{i\langle\mathbf{k}|\mathbf{x}\rangle}|\mathbf{a}\rangle|\mathbf{x}\rangle$$

with same-energy

$$\lambda(\mathbf{k},\mathbf{a}) = e^{-i\langle\mathbf{k}|\mathbf{a}\rangle}$$ is invariant

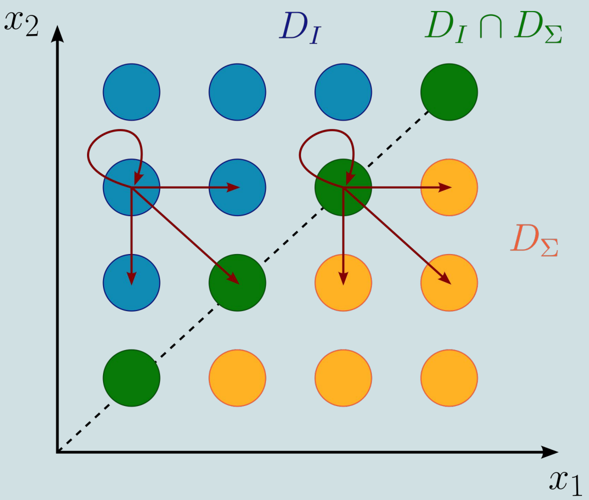
- under simultaneous permutation of spins and momenta
- mod(2π)

Aim:

- determine coefficients of the ansatz & retrieve **Yang-Baxter** equation
- prove **completeness**

Future perspectives

Generalisation to **massive** Thirring QCA

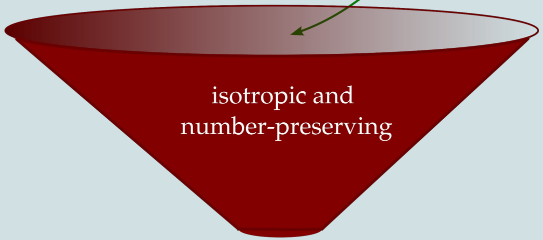


Challenges:

- free solution **not diagonal** in spin-position basis
- **no longer purely translational**

& **other interactions**

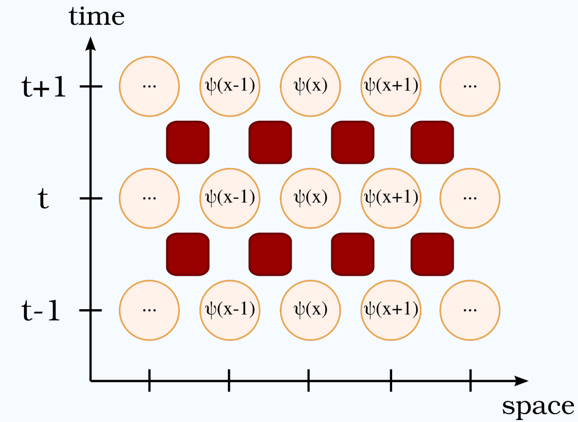
$$J_{int} = e^{H_{int}}$$



classification

- **analytical** solution?
- if not, **approximation** techniques?

QCA description of **gauge theories**?



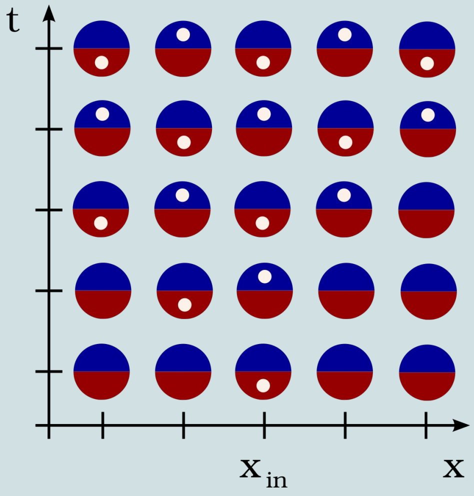
- **auxiliary cells** to restore local symmetries?

Thank you!

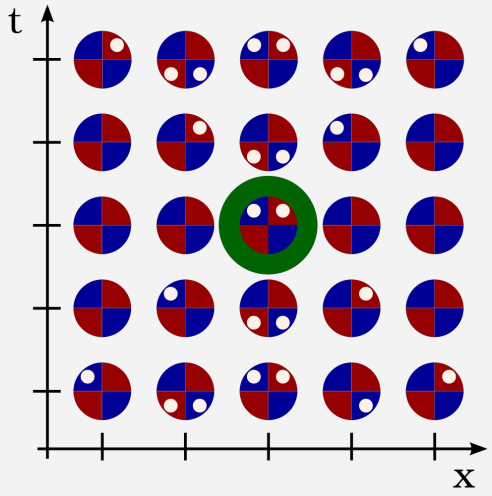
Backup slides

Preliminary results

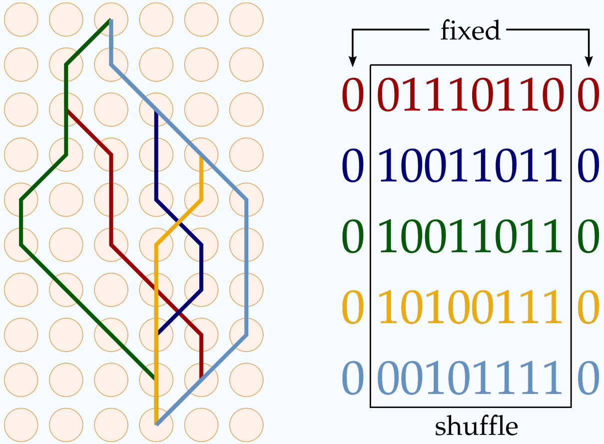
The free dynamics univocally determines the state at each site



Necessary condition for **interaction**:
- even distance, opposite spins
- odd distance, same spins



Paths connecting two sites share the same number of up and down spins



Bethe Ansatz approach

Ansatz: superposition of **same-energy** free eigenstates

$$|\psi_{\mathbf{k}}^{(N)}(\mathbf{a})\rangle = \sum_{\mathbf{x}\in\mathbb{Z}^N} e^{i\langle\mathbf{k}|\mathbf{x}\rangle}|\mathbf{a}\rangle|\mathbf{x}\rangle$$

$$D_\tau = \left\{\mathbf{x}\in\mathbb{Z}^N \;:\; x_{\tau(1)} < x_{\tau(2)} < \cdots < x_{\tau(N)}\right\}$$

$$|\psi_{\mathbf{k}}(\mathbf{a})\rangle = \sum_{\tau\in S_N}\sum_{\mathbf{x}\in D_\tau}\sum_{\sigma\in S_N} f_{\sigma,\tau}(\mathbf{x})e^{i\langle\sigma\mathbf{k}|\mathbf{x}\rangle}|\sigma\mathbf{a}\rangle|\mathbf{x}\rangle$$

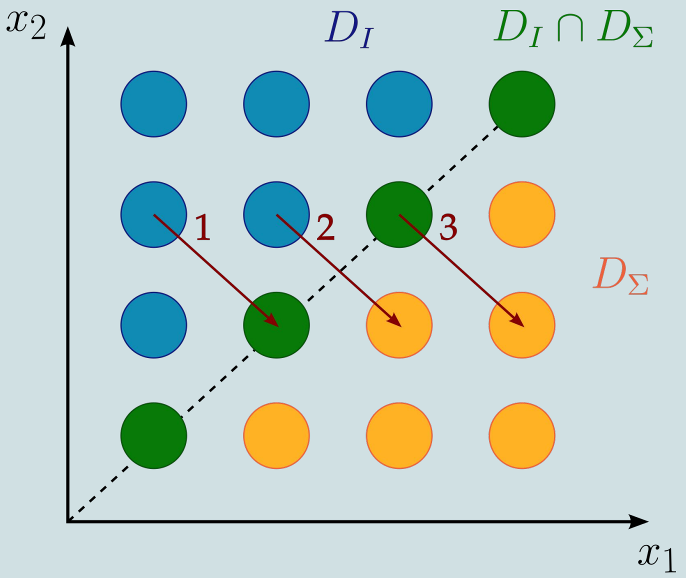
$\lambda(\mathbf{k},\mathbf{a}) = e^{-i\langle\mathbf{k}|\mathbf{a}\rangle}$ is invariant

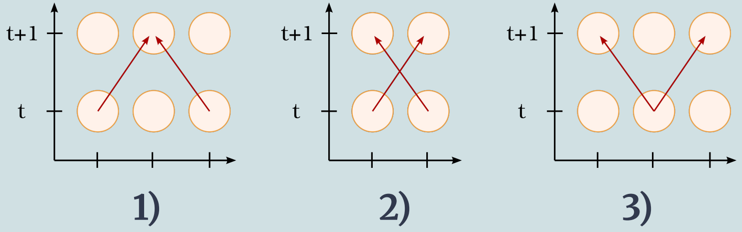
- under simultaneous permutation of spins and momenta
- mod(2π)

Impose **anti-symmetry**:

$$f_{\sigma\rho^{-1},\tau}(\rho^{-1}\mathbf{x}) = \mathrm{sign}(\rho)f_{\sigma,\rho^{-1}\tau}(\mathbf{x})$$

Bethe Ansatz approach: 2-particle sector





$$|\psi_{\mathbf{k}}(+-)\rangle = |\psi^I_{\mathbf{k}}(+-)\rangle + |\psi^\Sigma_{\mathbf{k}}(+-)\rangle + |\psi^\cap_{\mathbf{k}}(+-)\rangle$$

1) $f_{II}(x-1, x+1) = Ce^{i\chi}$

2) $f_{II}(x-1, x) = f_{I\Sigma}(x, x-1)$

3) $C = f_{I\Sigma}(x+1, x-1)$

Solution

$$f_{I\Sigma}(x_1, x_2) = \Omega(\chi, x_1, x_2) f_{II}(x_1, x_2)$$

$$\Omega(\chi, x_1, x_2) = (e^{-i\chi}+1)+(-1)^{x_1+x_2}(e^{-i\chi}-1)$$