Shake It 'till you make it

Driving precision in thermometry

UNIVERSITÀ DI PAVIA

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EoY Seminars 24/25 - Emanuele Tumbiolo

Motivation:

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I love (baking) bread

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Temperature in bread

Temperature control is key when proofing bread

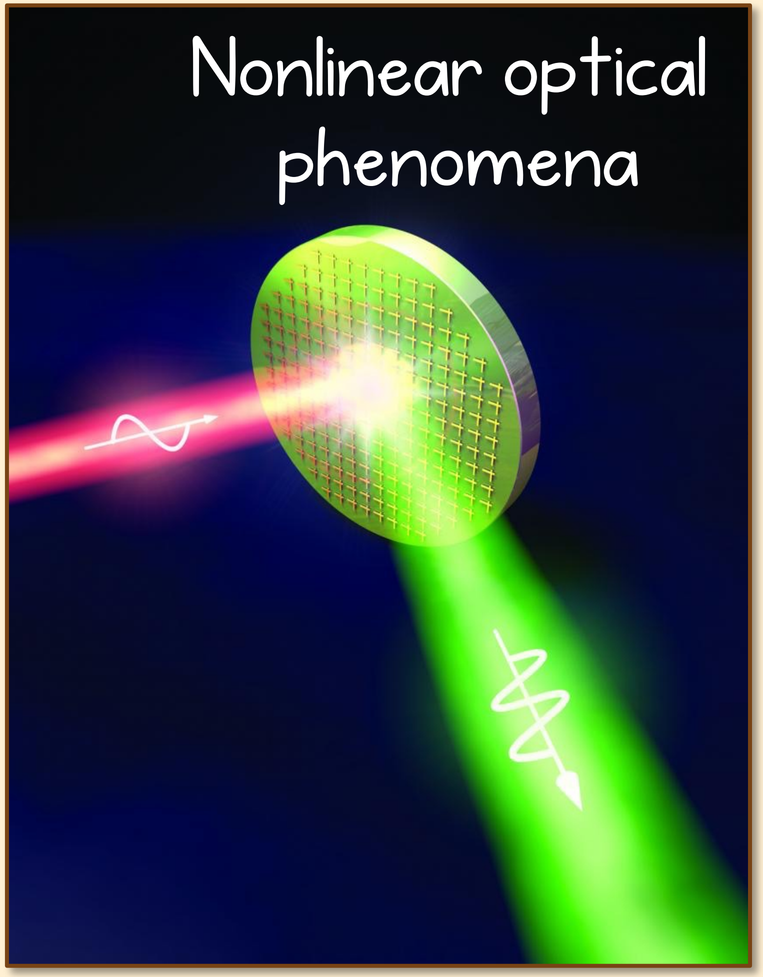
Low temp

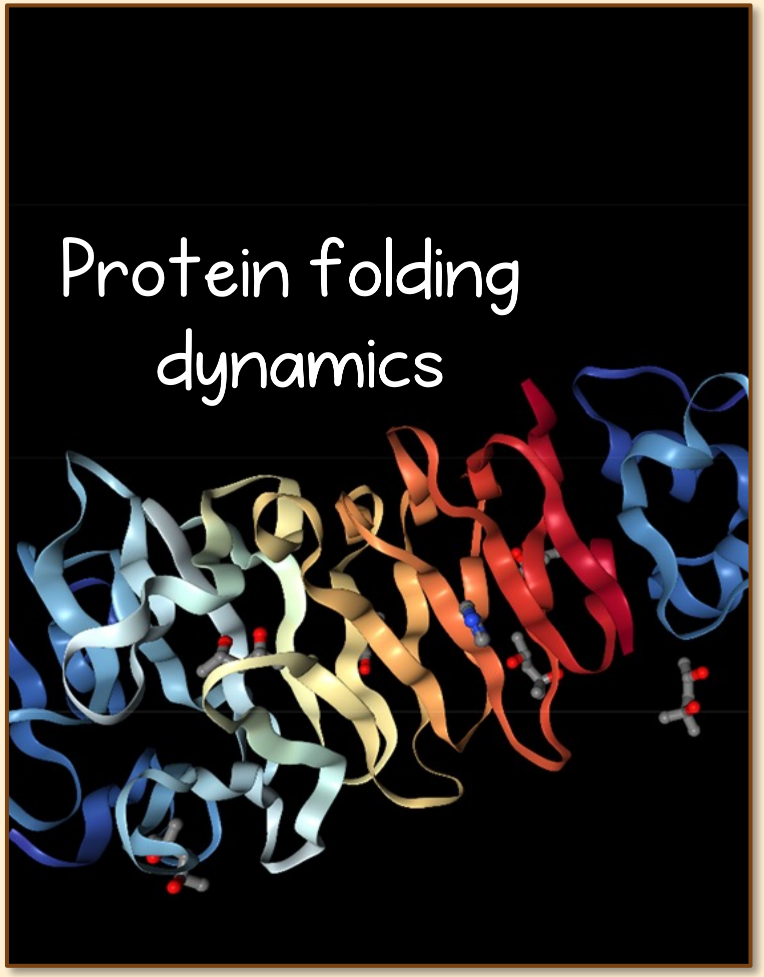
Just right

High temp

Temperature in physics

But also in all ranges of physical phenomena





Motivation (serious):

Measuring the temperature of a quantum system is non-trivial

Quantum systems are fragile

Temperature is not an observable

How do we measure it?

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"Sample" : thermodynamic bath

{State σ, H_{sample}, fixed T}

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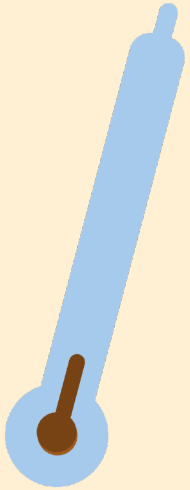
"Sample" : thermodynamic bath

"Probe"



$<<$

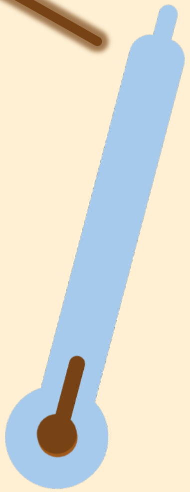
{State σ, H_{sample}, fixed T}

{State ρ, H_{probe}}

How do we measure it?

"Sample" : thermodynamic bath

"Probe"



H_{int}

{State σ, H_{sample}, fixed T}

{State ρ, H_{probe}}

Thermometric protocols at a glance

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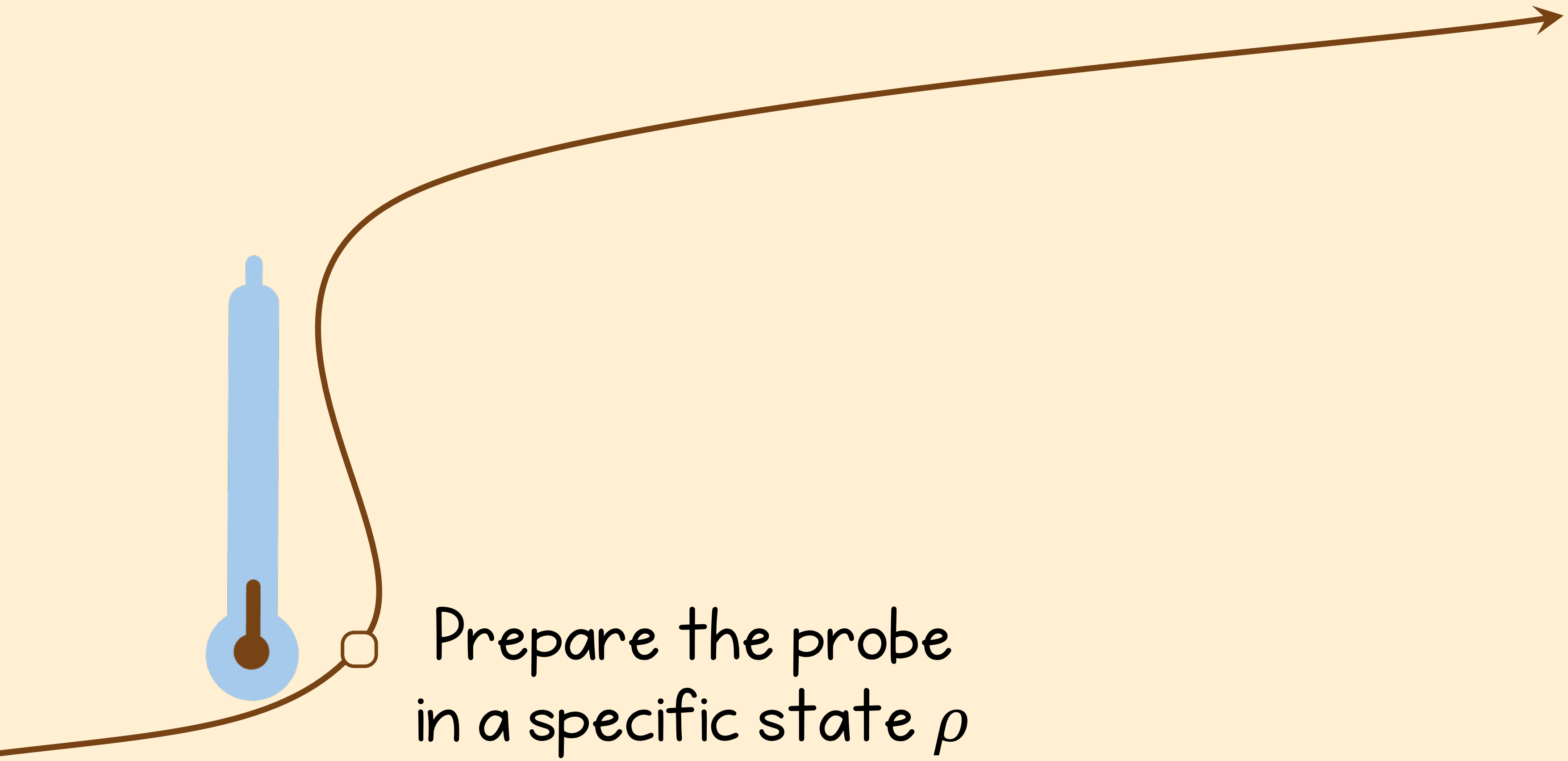


Prepare the probe
in a specific state ρ

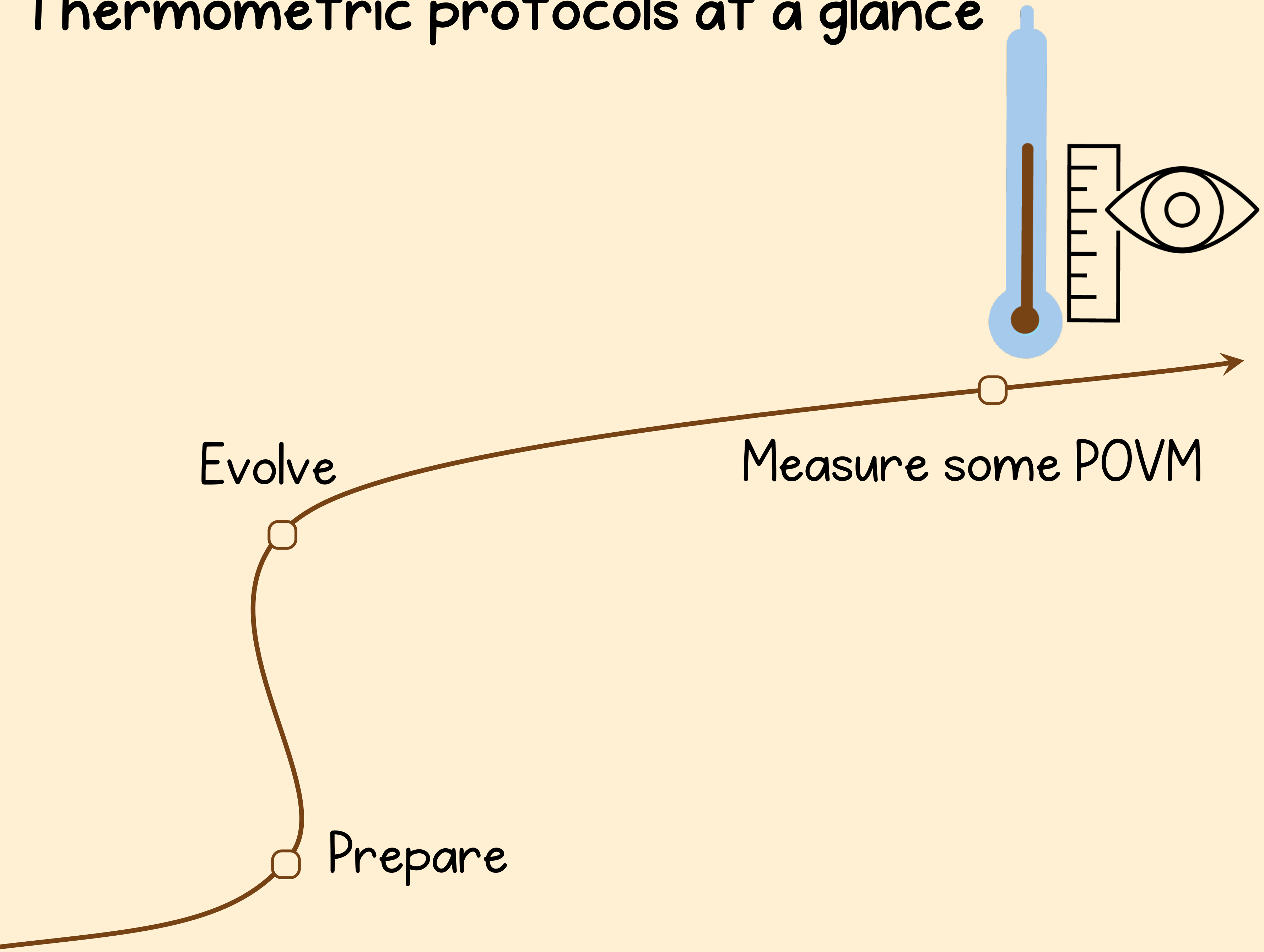
Thermometric protocols at a glance

$\mathcal{E}_T$:
A quantum channel that imprints info on T

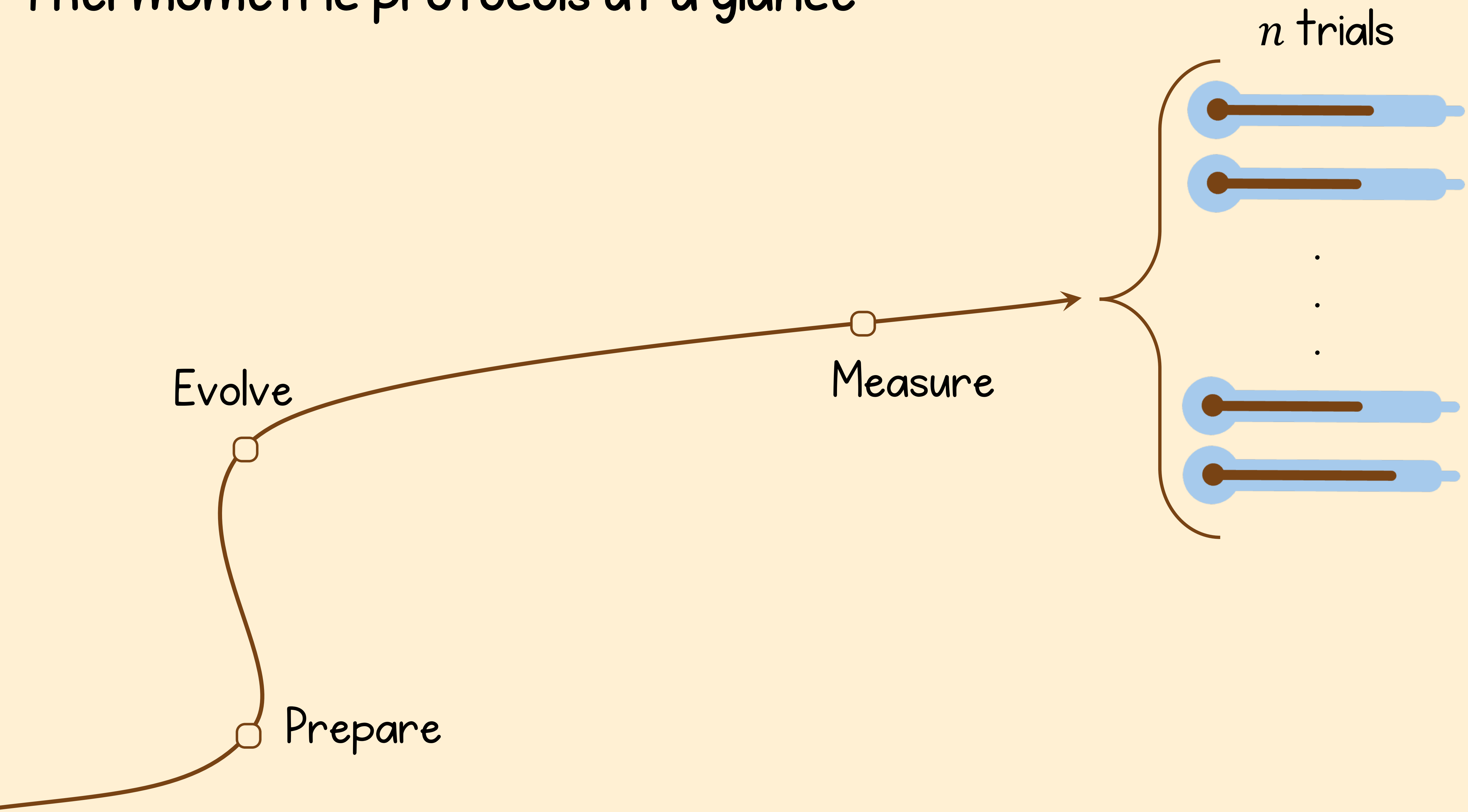
Evolve

Prepare

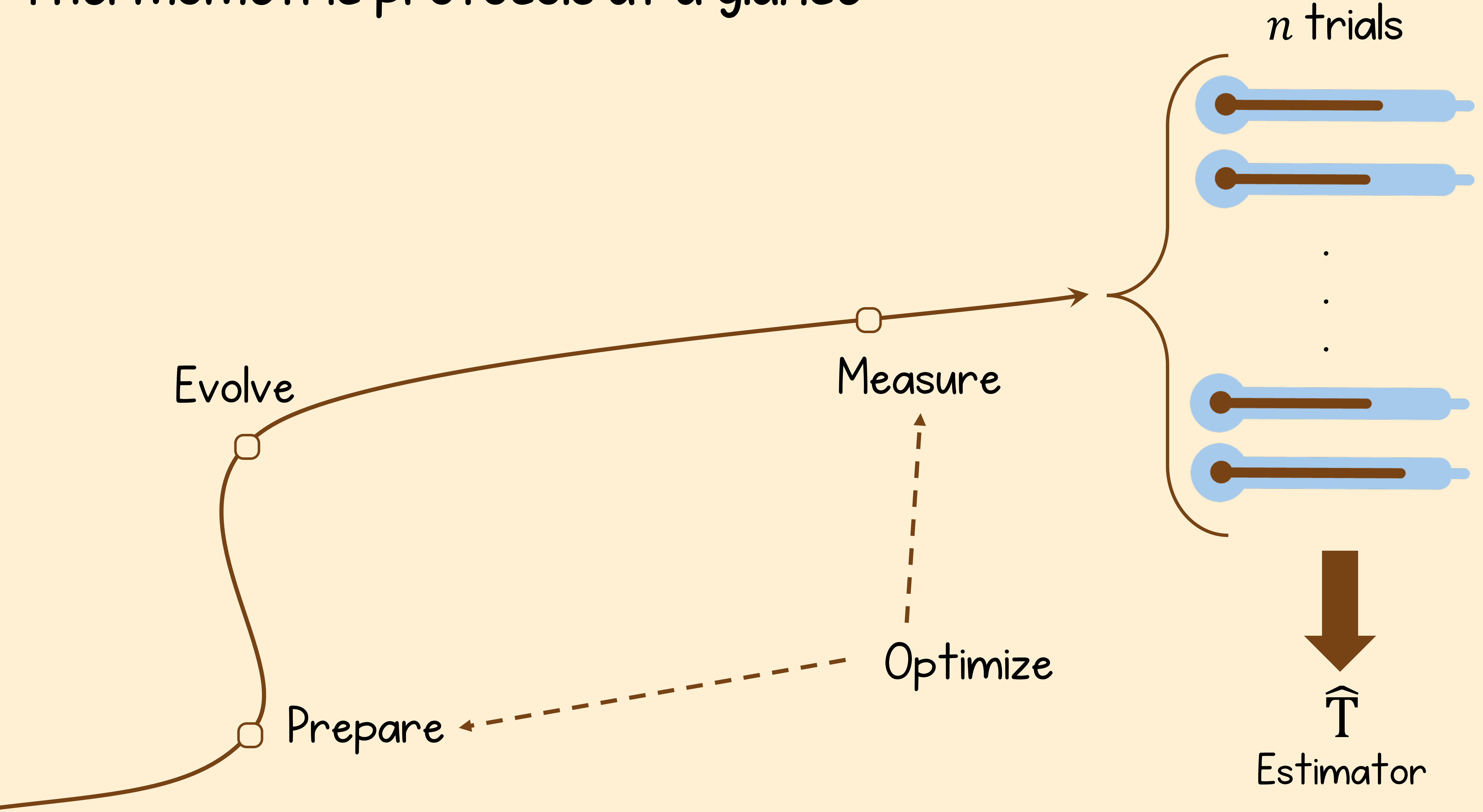
Thermometric protocols at a glance



Thermometric protocols at a glance



Thermometric protocols at a glance



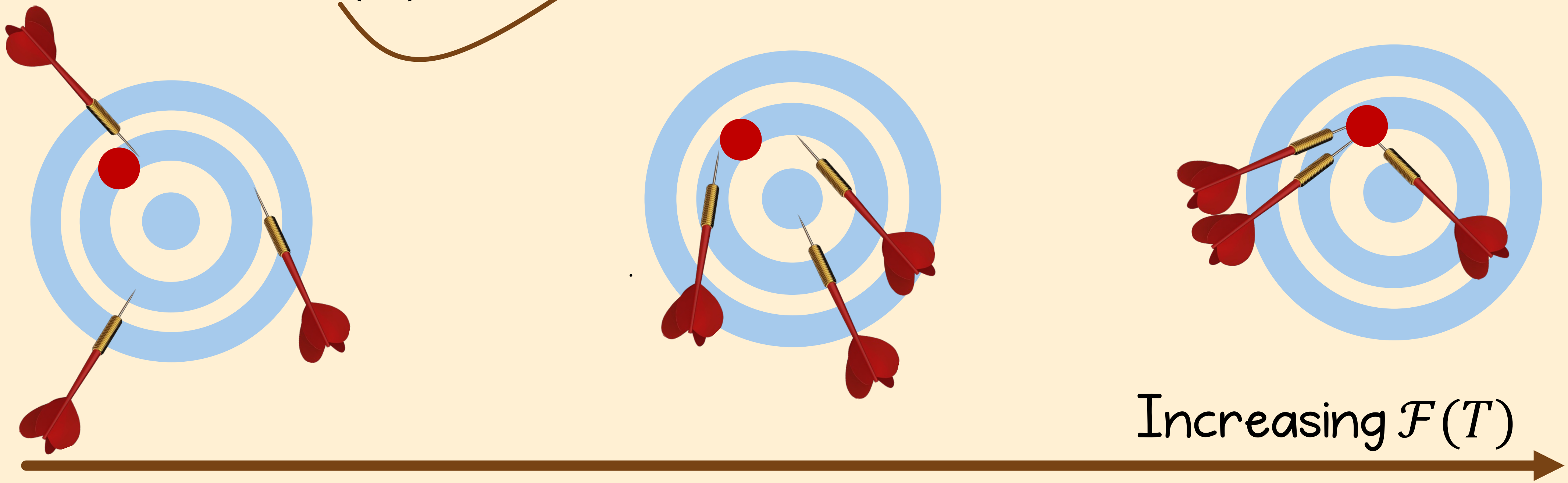
Quantifying precision

Quantifying precision

The precision of our estimate is governed by the (quantum) Cramer-Rao bound

$$\mathrm{Var}[\,\hat{T}\,] \geq \frac{1}{n\,\mathcal{F}(T)}$$

Quantum Fisher information (QFI)



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		Regimes	
		Classical	Quantum
Scaling	with n	Constant	n
	with t	At best linear	t^2

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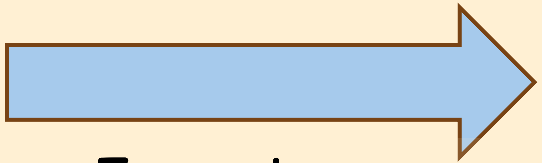
Understanding the QFI

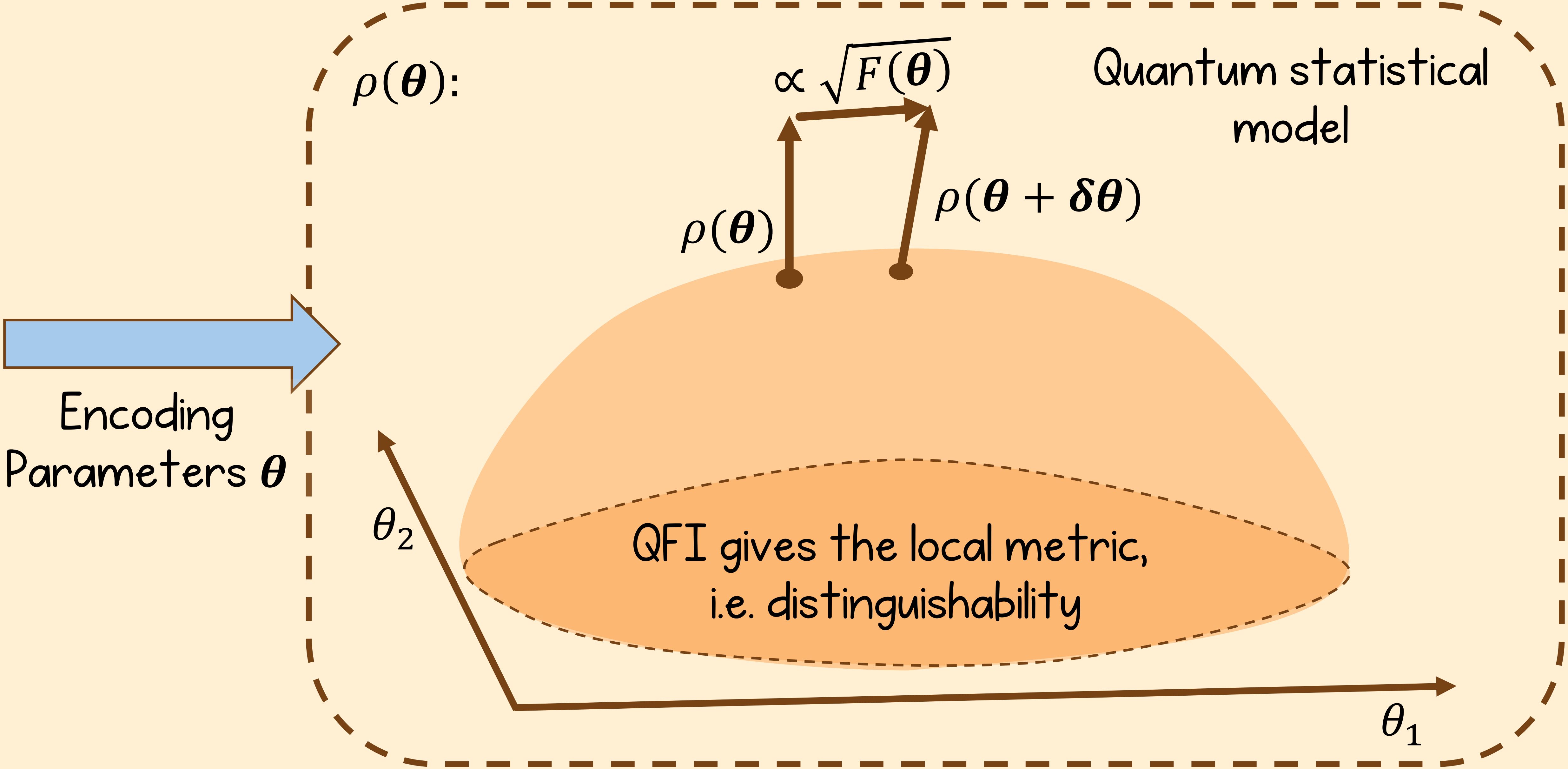
S.L. Braunstein et Al., Phys. Rev. Lett. 72, 3439 (1994).
T. Shitara et Al., Phys. Rev. A 94, 062316 (2016).



Encoding
Parameters $\boldsymbol{\theta}$

Understanding the QFI

S.L. Braunstein et Al., Phys. Rev. Lett. 72, 3439 (1994).
T. Shitara et Al., Phys. Rev. A 94, 062316 (2016).



What is the most “classical” thermometric protocol?

Equilibrium

Equilibrium thermometry:

L. A. Correa et Al., *Phys. Rev. Lett.* 114, 220405 (2015).

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L. A. Correa et Al., *Phys. Rev. Lett.* 114, 220405 (2015).

Evolve

Prepare

Thermalization dynamics

Dissipative interaction acting on characteristic timescale

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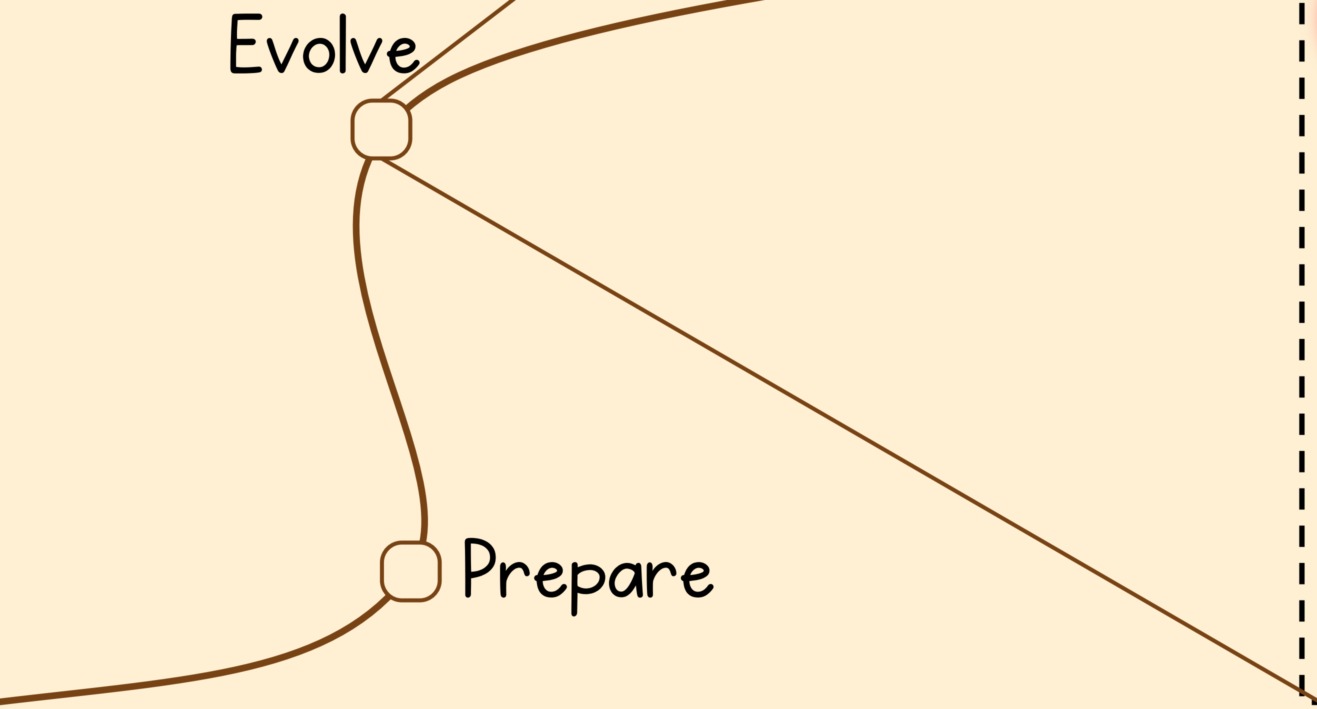
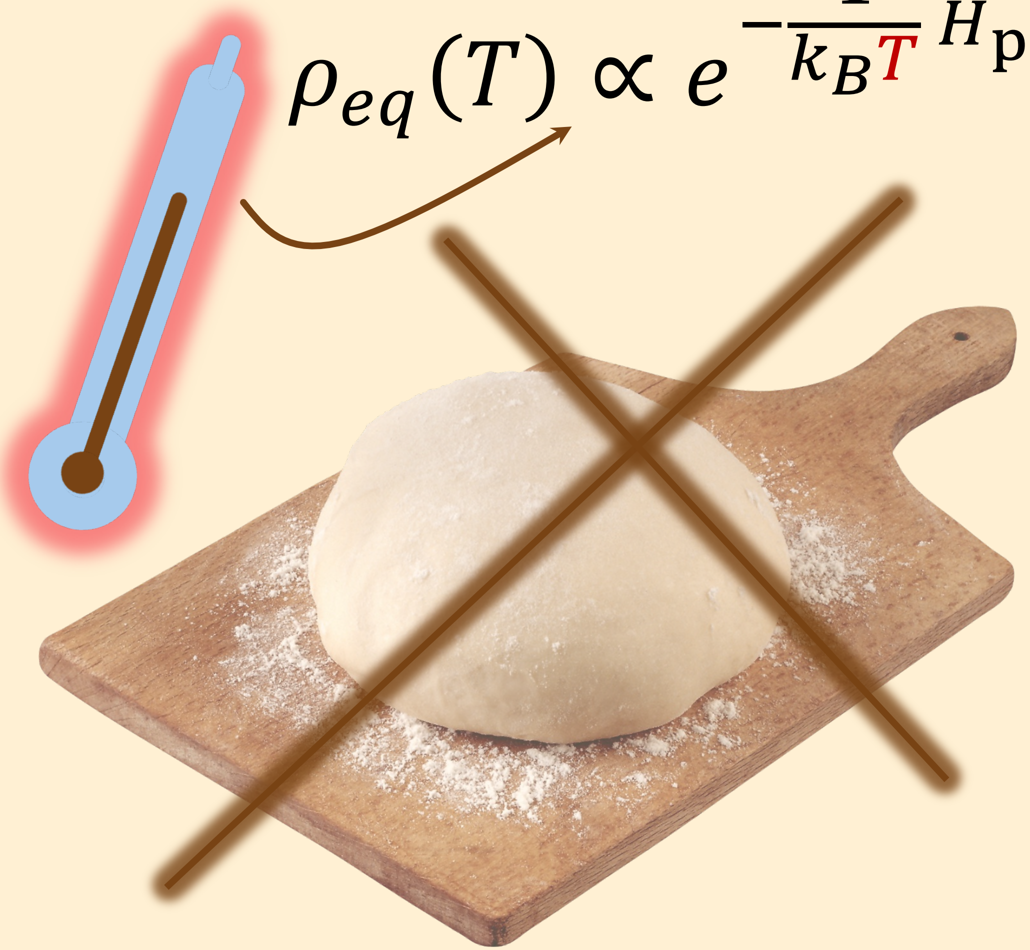


Thermalization dynamics

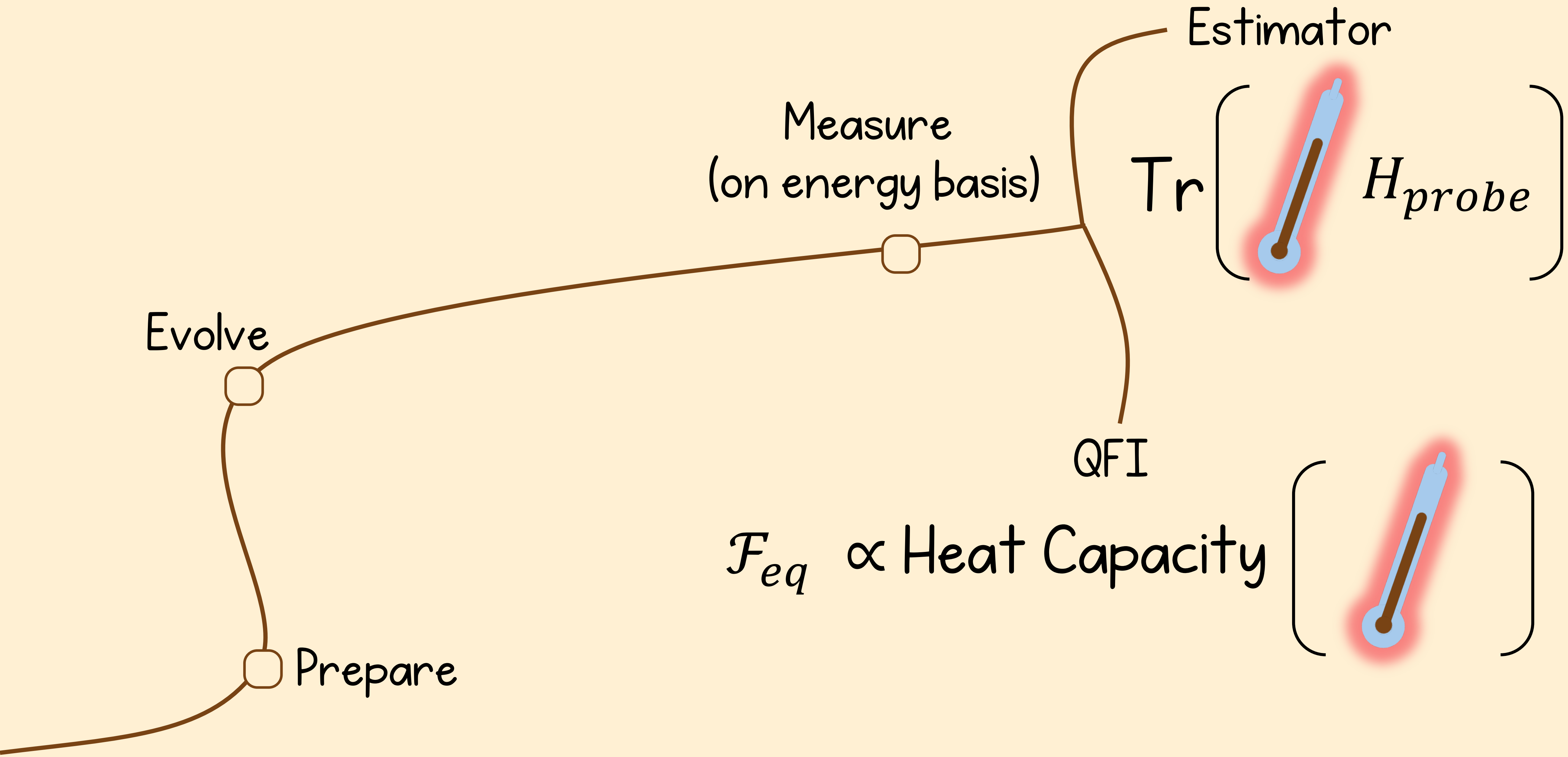
$$\rho_{eq}(T) \propto e^{-\frac{1}{k_B T} H_{\text{probe}}}$$



Tracing out the bath gets us a thermal state for the probe

Equilibrium thermometry:

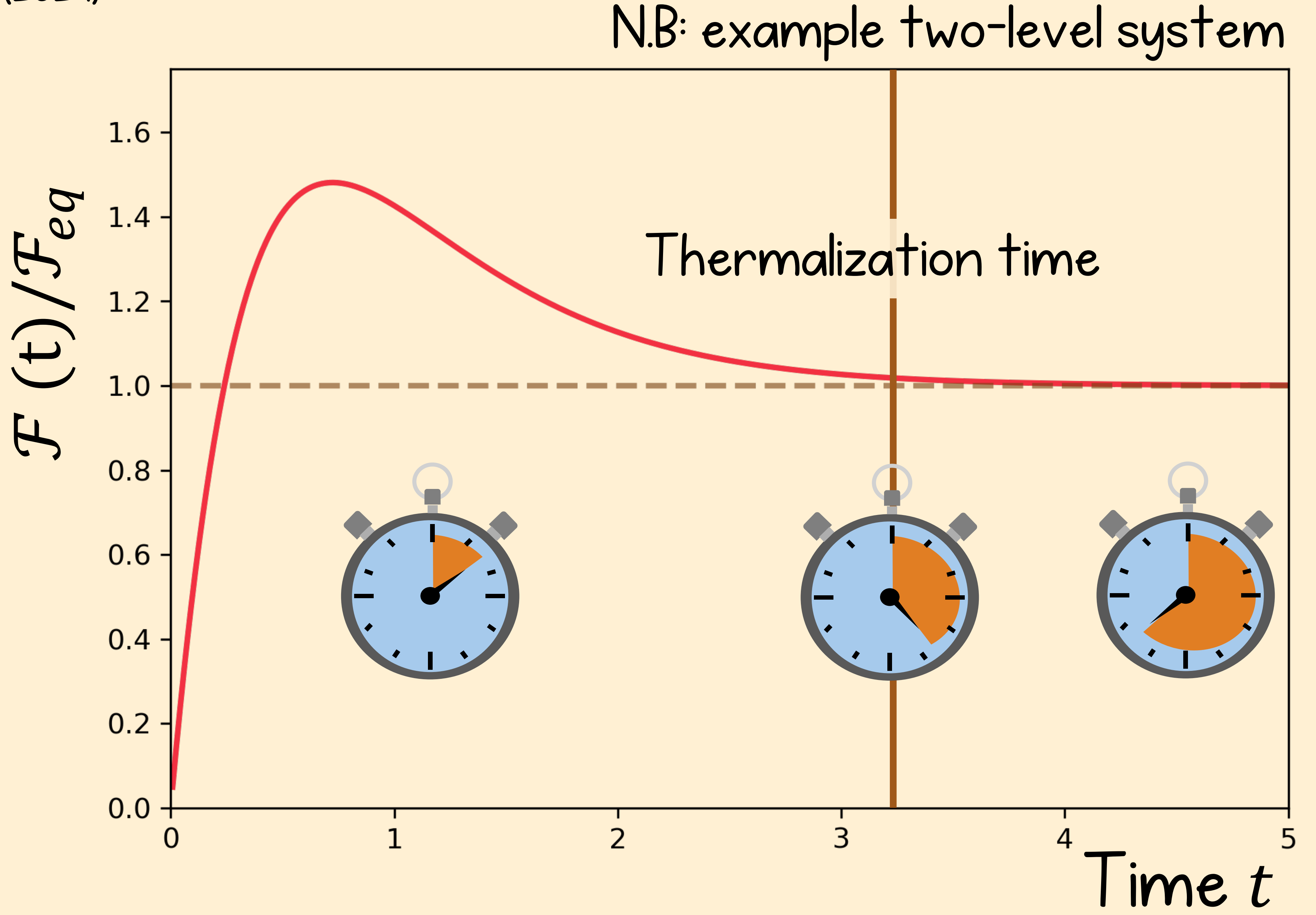
L. A. Correa et Al., *Phys. Rev. Lett.* 114, 220405 (2015).



Limits of the equilibrium approach?

G. Frazão et al., Entropy 26, 568 (2024)

- "Too classical"



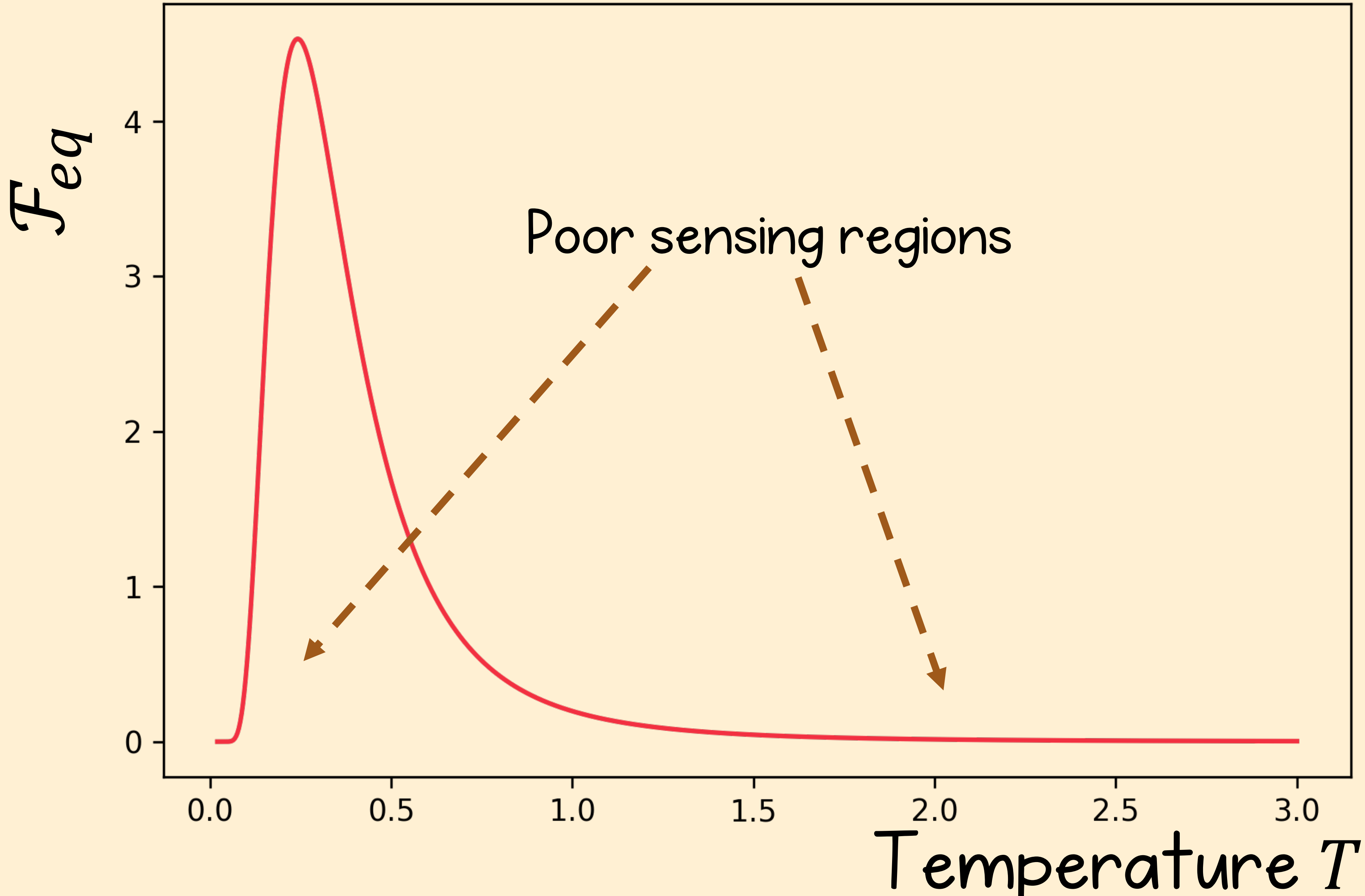
Limits of the equilibrium approach?

G. Frazão et al., Entropy 26, 568 (2024)

N.B: example two-level system

- "Too classical"

- Vanishing Heat capacity



Can we do better?

Can we do better?

Enter driven thermometry

The state of the literature

Quantum Thermometry for Ultra-Low Temperatures Using Probe and Ancilla Qubit Chains

by Asghar Ullah [1,*], Vipul Upadhyay [2] and Özgür E. Müstecaplıoğlu [1,3]

Enhanced precision bound of low-temperature quantum thermometry via dynamical control

Victor Mukherjee [1,2,3*], Analia Zwick [4], Arnab Ghosh [1,2,5], Xi Chen [1,6] & Gershon Kurizki [2]

Optimal adaptive control for quantum metrology with time-dependent Hamiltonians

Shengshi Pang [1,2] & Andrew N. Jordan [1,2,3]

The state of the literature

Quantum Thermometry for Ultra-Low Temperatures Using Probe and Ancilla Qubit Chains

Exploring the role of asymmetric-pulse modulation in quantum thermal machines and quantum thermometry

Saikat Mondal, Sourav Bhattacharjee,* and Amit Dutta

Enhanced precision bound of low-temperature quantum thermometry via dynamical control

Vict

Floquet Engineering to Overcome No-Go Theorem of Noisy Quantum Metrology

Si-Yuan Bai[1] and Jun-Hong An[1,*]

Optimal adaptive contr... with time-dependent Hamiltonians

Shengshi Pang[1,2] & Andrew N. Jordan[1,2,3]

The state of the literature

Quantum Thermometry for U
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Exploring the role of asymmetric-puls
quantu

Saikat Mondal, Sourav Bhattacharjee,[*] and Amit

Enhanced p
quantum th

Enhancement of sensitivity in low-temperature quantum thermometry via reinforcement learning

Qing-Shou Tan[1,*], Xulin Liu[1], Lan Xu[2,†], Wei Wu[3], and Le-Man Kuang[4,5,‡]

Low-temperature quantum thermometry boosted by coherence generation

Asghar Ullah,[1,*] M. Tahir Naseem,[1,2,†] and Özgür E. Müstecaplıoğlu[1,3,‡]

Thermometry of ultracold atoms via nonequilibrium work distributions

T. H. Johnson[1,2,3,*], F. Cosco[2,4,5], M. T. Mitchison[2,6], D. Jaksch[1,2,3], and S. R. Clark[2,3,7,8]

Theorem of Noisy Quantum Metrology

Jun-Hong An[1,*]

with time-dependent Hamiltonians

Bending the rules of low-temperature thermometry with periodic driving

Jonas Glatthard and Luis A. Correa

The state of the literature

Driving can enhance sensitivity, recover "quantum scaling", shift sensing regions

BUT:

The state of the literature

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- Only model-dependent studies

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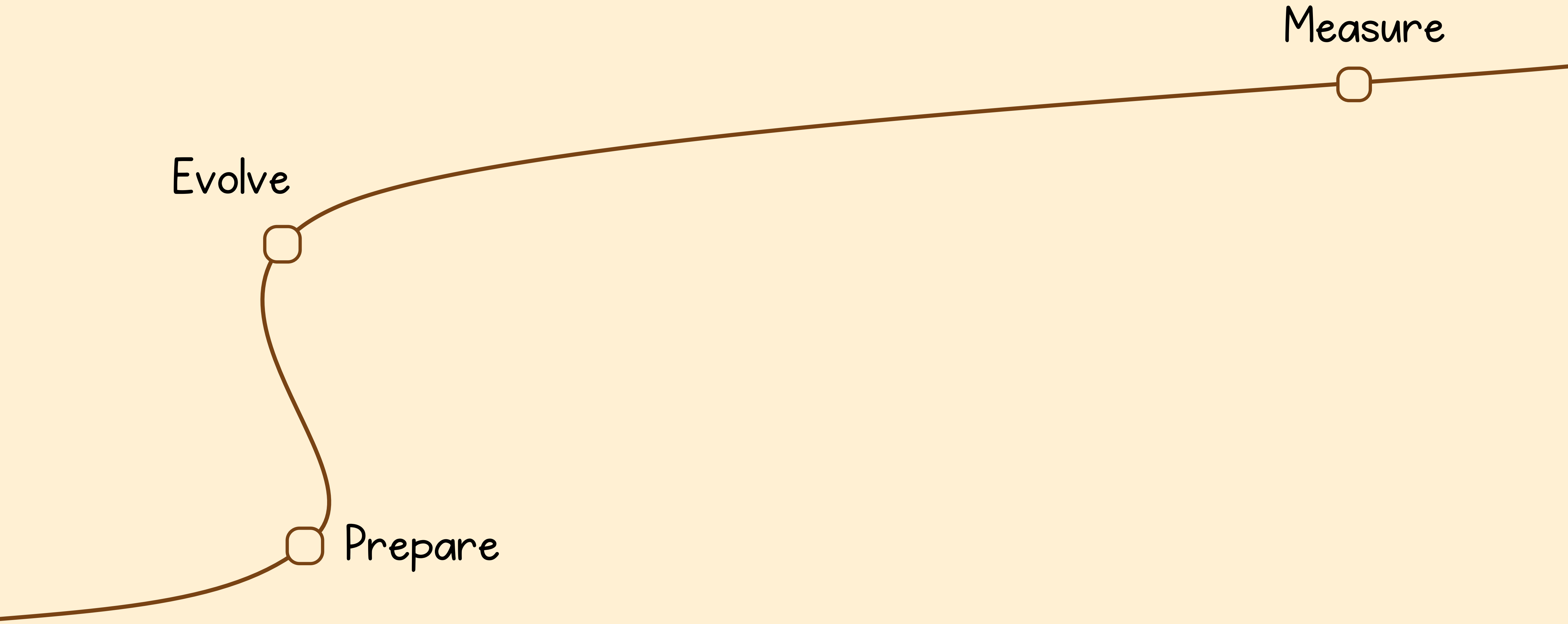
Our question:

How much can driving, in principle, enlarge the QFI for temperature? Is it always an enhancement?

Driven thermometry - shake it 'till you make it

Consider the equilibrium protocol, but with a slight modification



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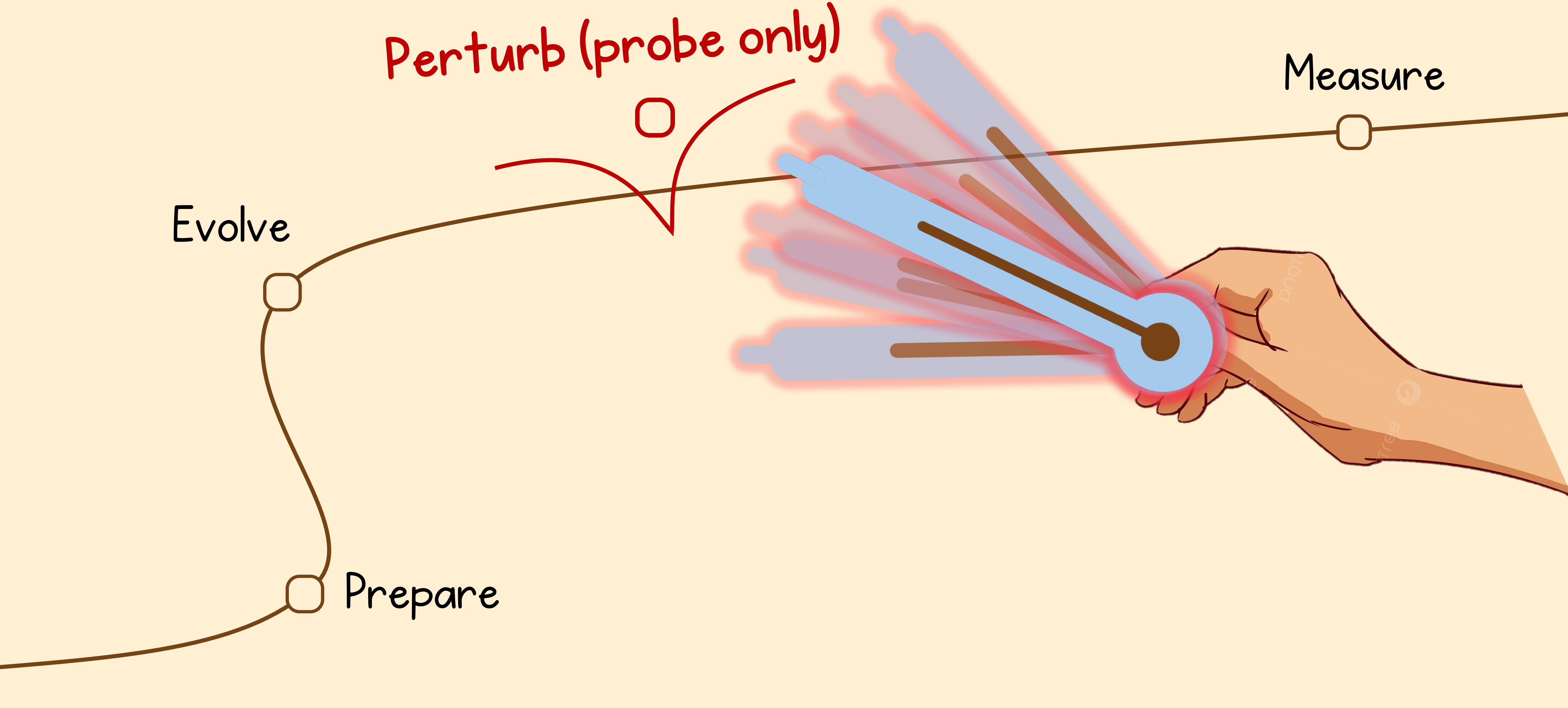
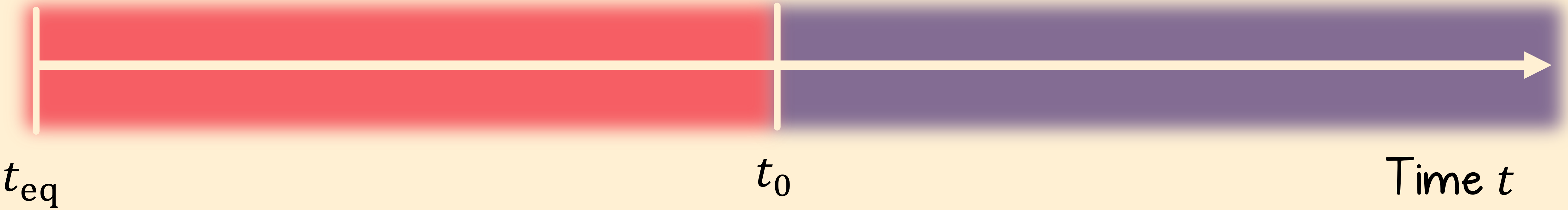


Unitary driving

The probe Hamiltonian is modified via a time-dependent perturbation



$$H_{probe}(t < t_0) = H_0$$

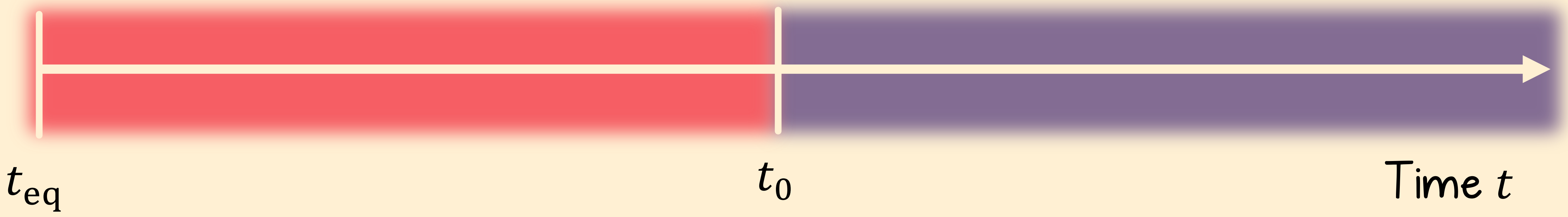
$$H_{probe}(t > t_0) = H_0 + \boldsymbol{\lambda(t, t_0, T)V}$$

$\lambda(t, t_0, T)$ → Driving profile

V → perturbation

Unitary driving

The probe Hamiltonian is modified via a time-dependent perturbation



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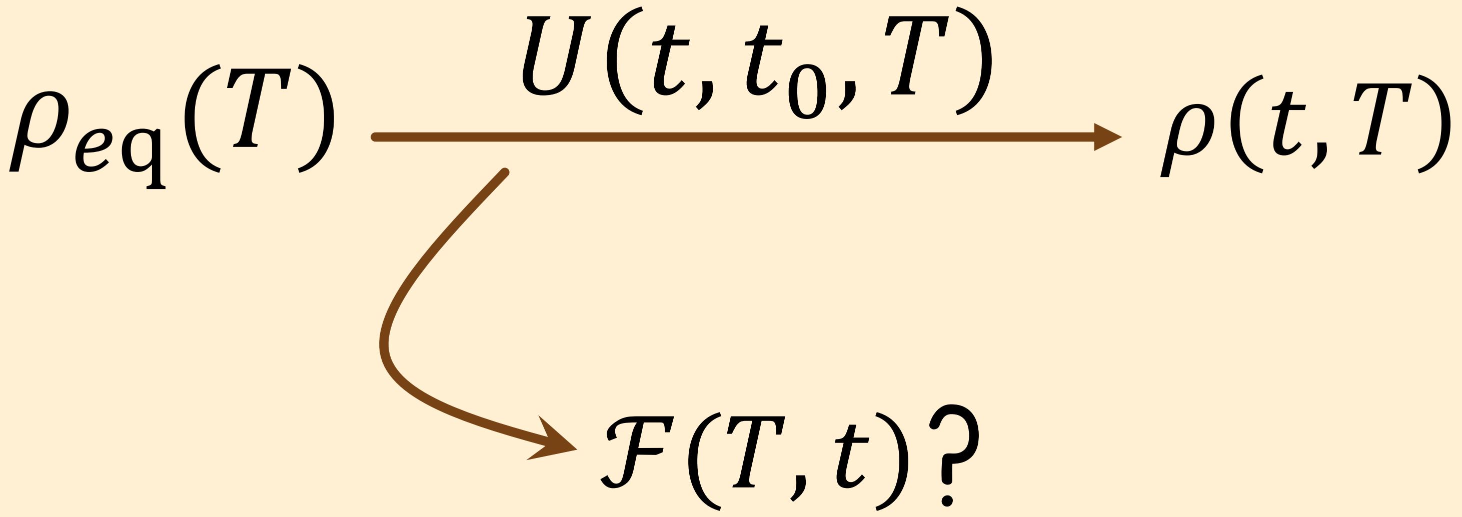


The road so far

Results: QFI enhancement

$$\mathcal{F}(T, t > t_0) = \mathcal{F}_{eq}(T) + \mathcal{F}^{'}(T, t)$$

Results: QFI enhancement

Equilibrium baseline

$$\mathcal{F}(T, t > t_0) = \mathcal{F}_{eq}(T) + \mathcal{F}'(T, t)$$

Driving correction

Results: QFI enhancement

Equilibrium baseline

$$\geq 0$$

$$\boldsymbol{\mathcal{F}(T, t > t_0) = \mathcal{F}_{eq}(T) + \mathcal{F}^{'}(T, t)}$$

Driving correction

Independently of dynamics details

Results: QFI enhancement

$$\geq 0$$

$$\mathcal{F}(T, t > t_0) = \mathcal{F}_{eq}(T) + \mathcal{F}'(T, t)$$

$$\int_{t_0}^{t} du \int_{t_0}^{t} ds \; \partial_T \lambda(u, t_0, T)\, \partial_T \lambda(s, t_0, T)\, Tr\left[J_V(u)\, J_V(s)\, \rho_{eq}(T) \right]$$

Results: QFI enhancement

- Driving profile: No T-dependency, no party

$$\int_{t_0}^{t} du \int_{t_0}^{t} ds \; \underline{\partial_T \lambda(u, t_0, T)\, \partial_T \lambda(s, t_0, T)} \; Tr\left[J_V(u)\, J_V(s)\, \rho_{eq}(T) \right]$$

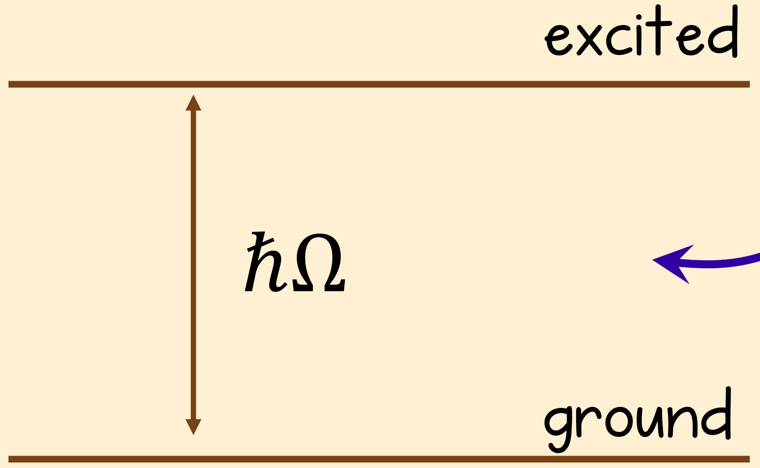
Results: QFI enhancement

Quantumness

- Driving profile: No T-dependency, no party
- $J_V(u) = f([V(u), \rho_{eq}])$ - "information current operator":
  No coherence, even less party

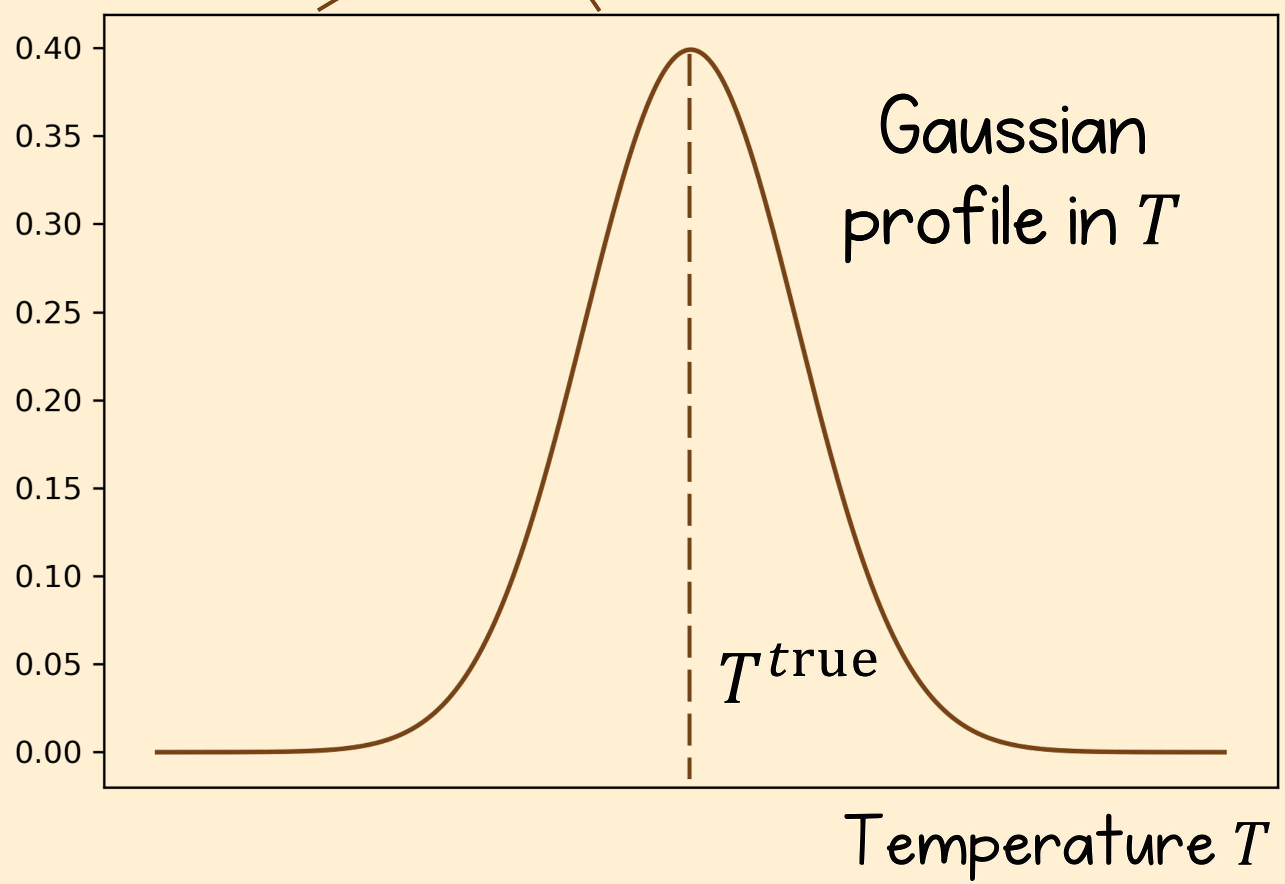
$$\int_{t_0}^{t} du \int_{t_0}^{t} ds \; \partial_T \lambda(u, t_0, T)\, \partial_T \lambda(s, t_0, T)\, \underline{\mathrm{Tr}\left[J_V(u)\, J_V(s)\, \rho_{eq}(T) \right]}$$

Toy model - single two level system

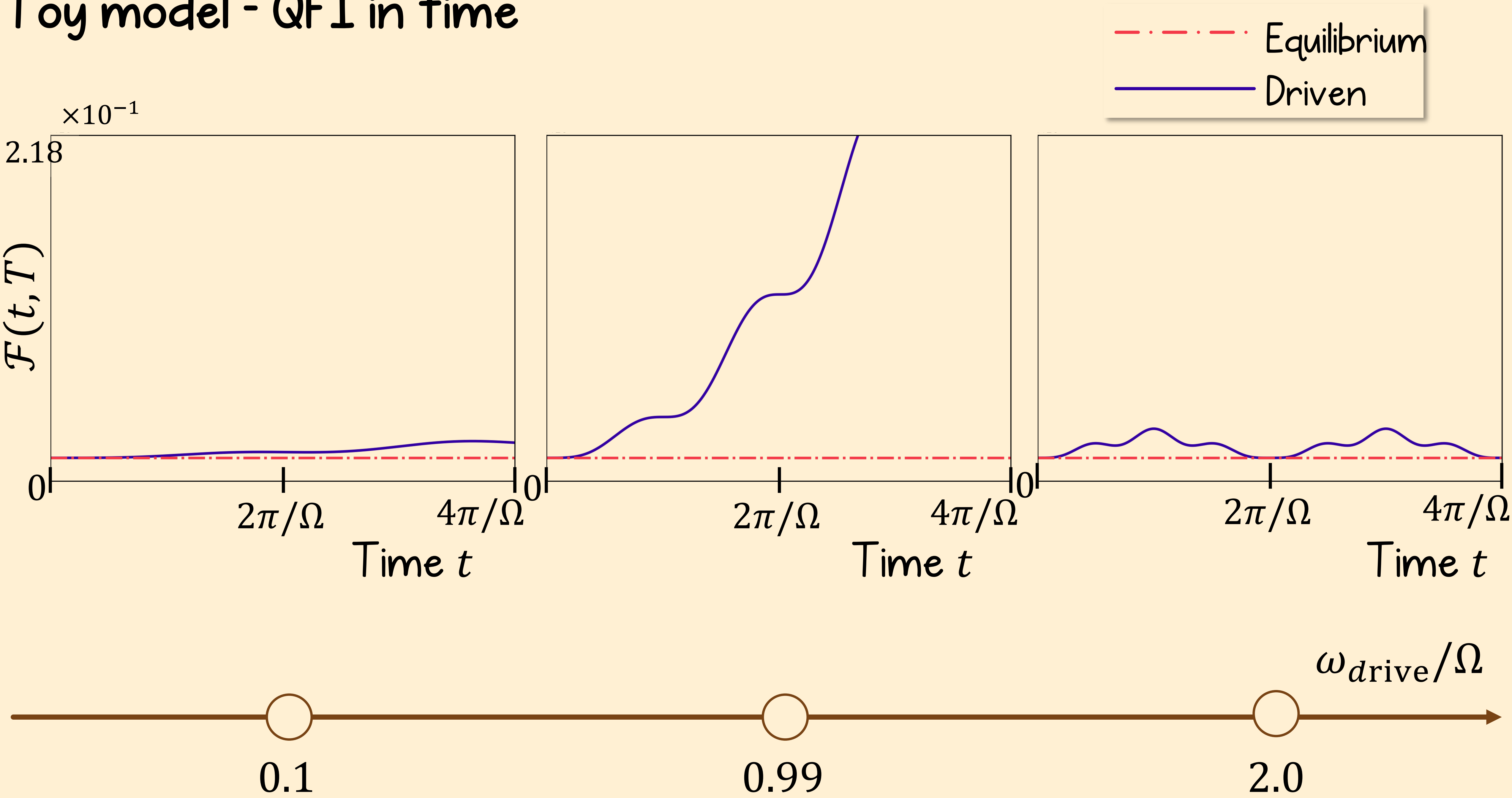


$$\lambda_0 f(T)\, sin(\omega_{drive} t + \phi)\sigma_x$$

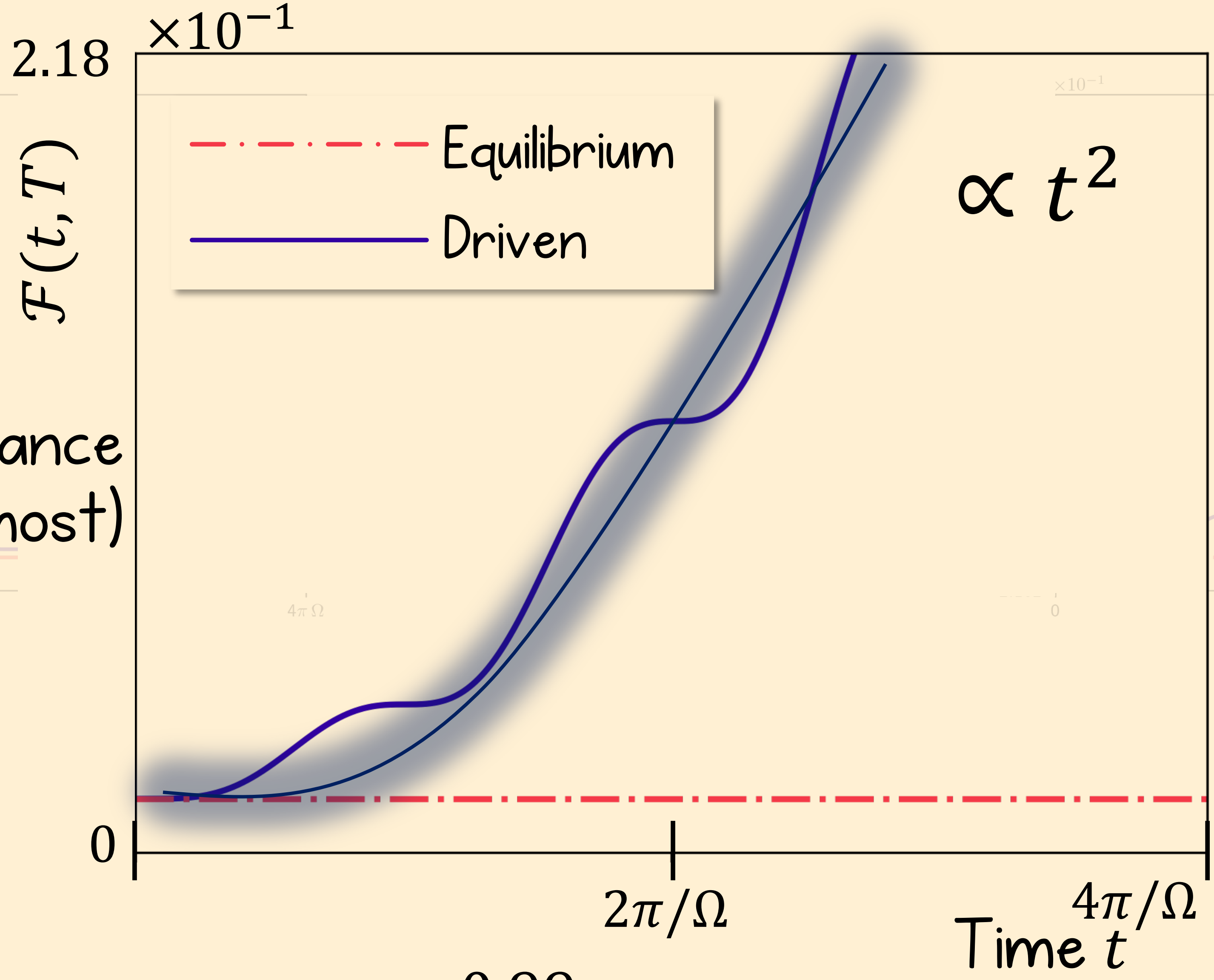
$$H_0 = \frac{\hbar}{2}\Omega\sigma_z$$



Toy model - QFI in time



Toy model - QFI in time

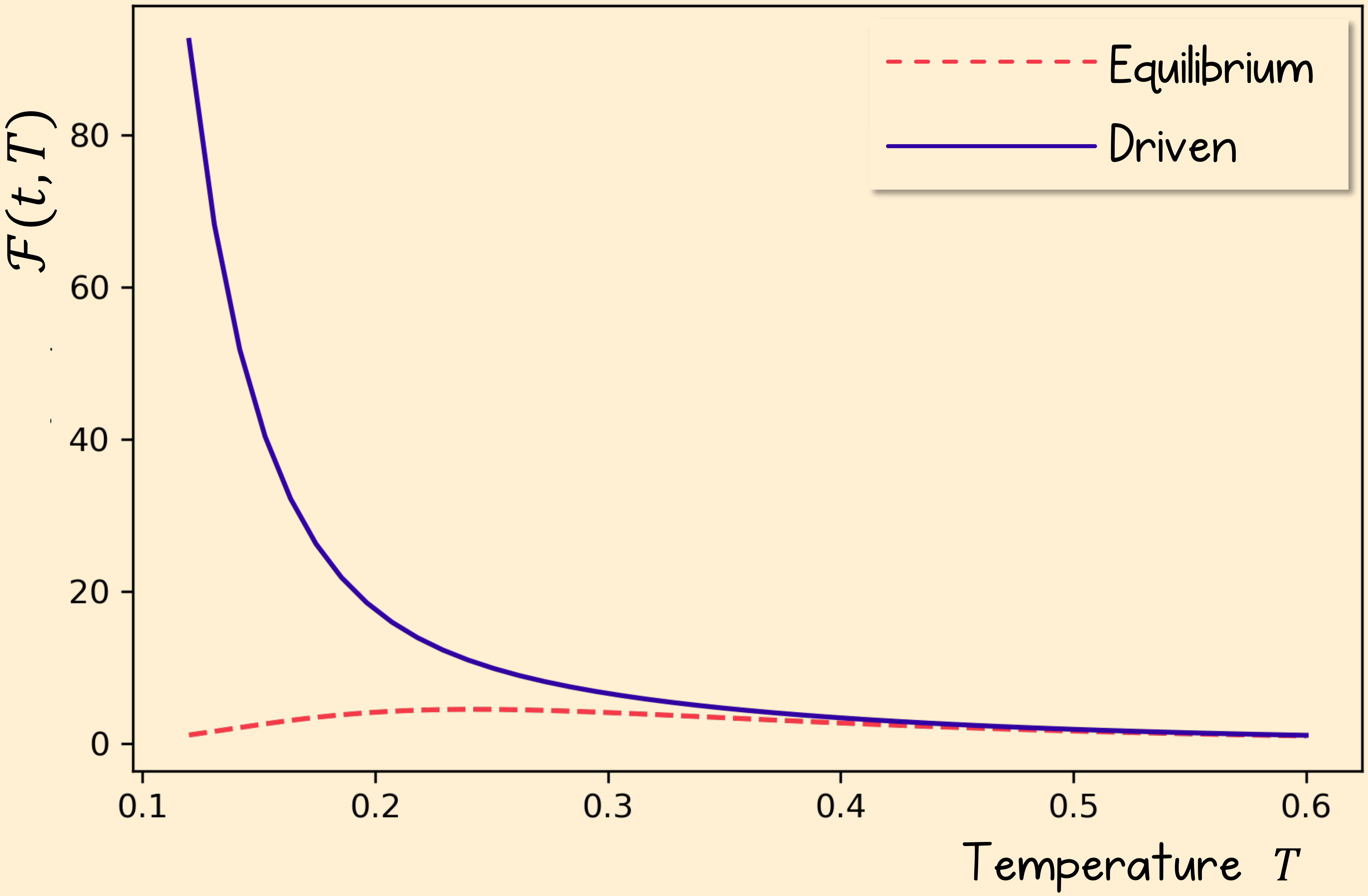


Driving at resonance recovers an (almost) t^2 scaling

ω_{drive}/Ω

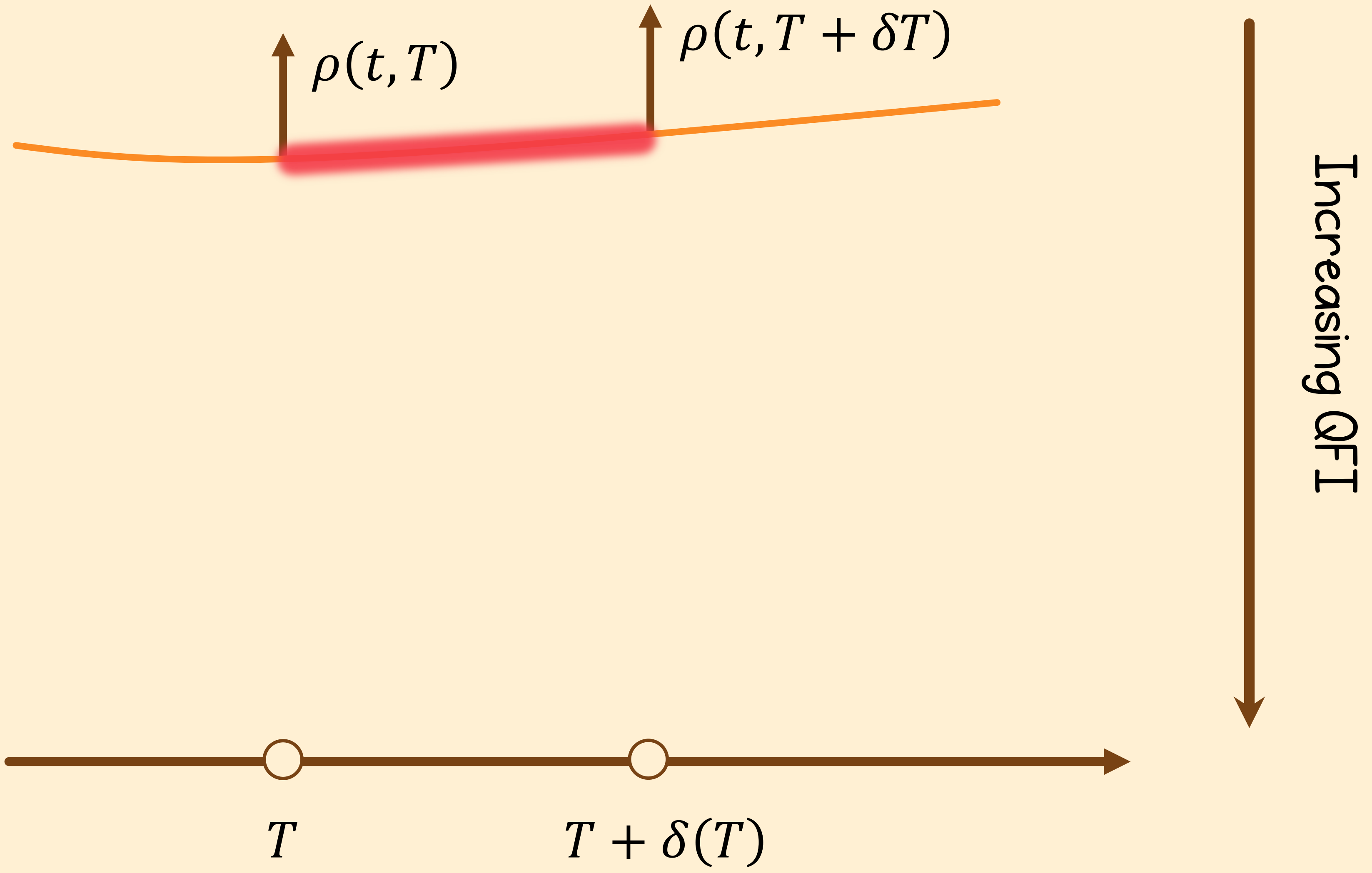
0.99

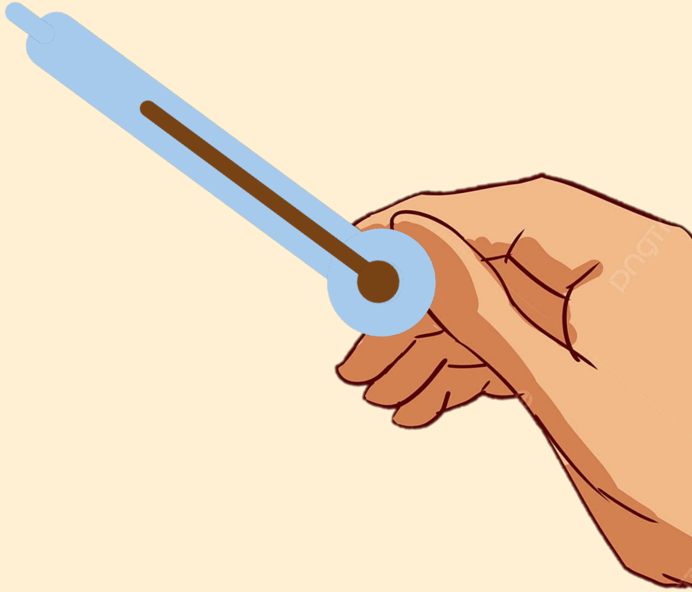
Toy model - QFI vs Temperature



Drive engineering allows to shift optimal sensing region

Results: geometric interpretation





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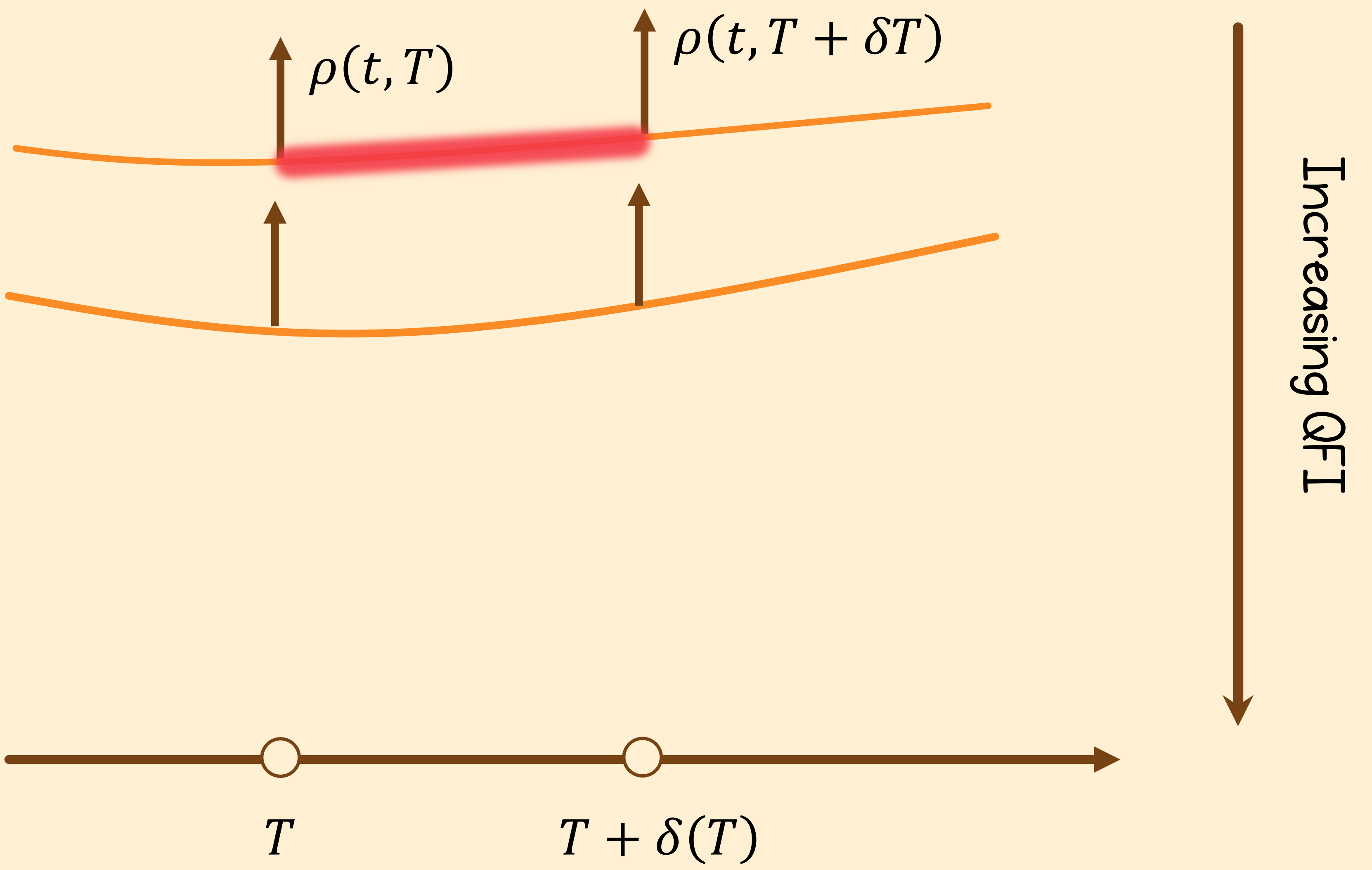

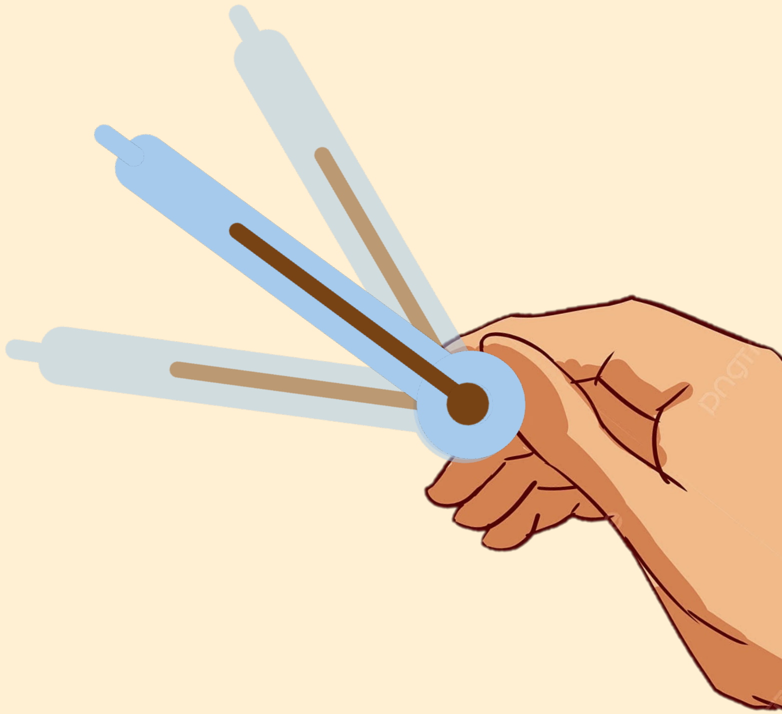

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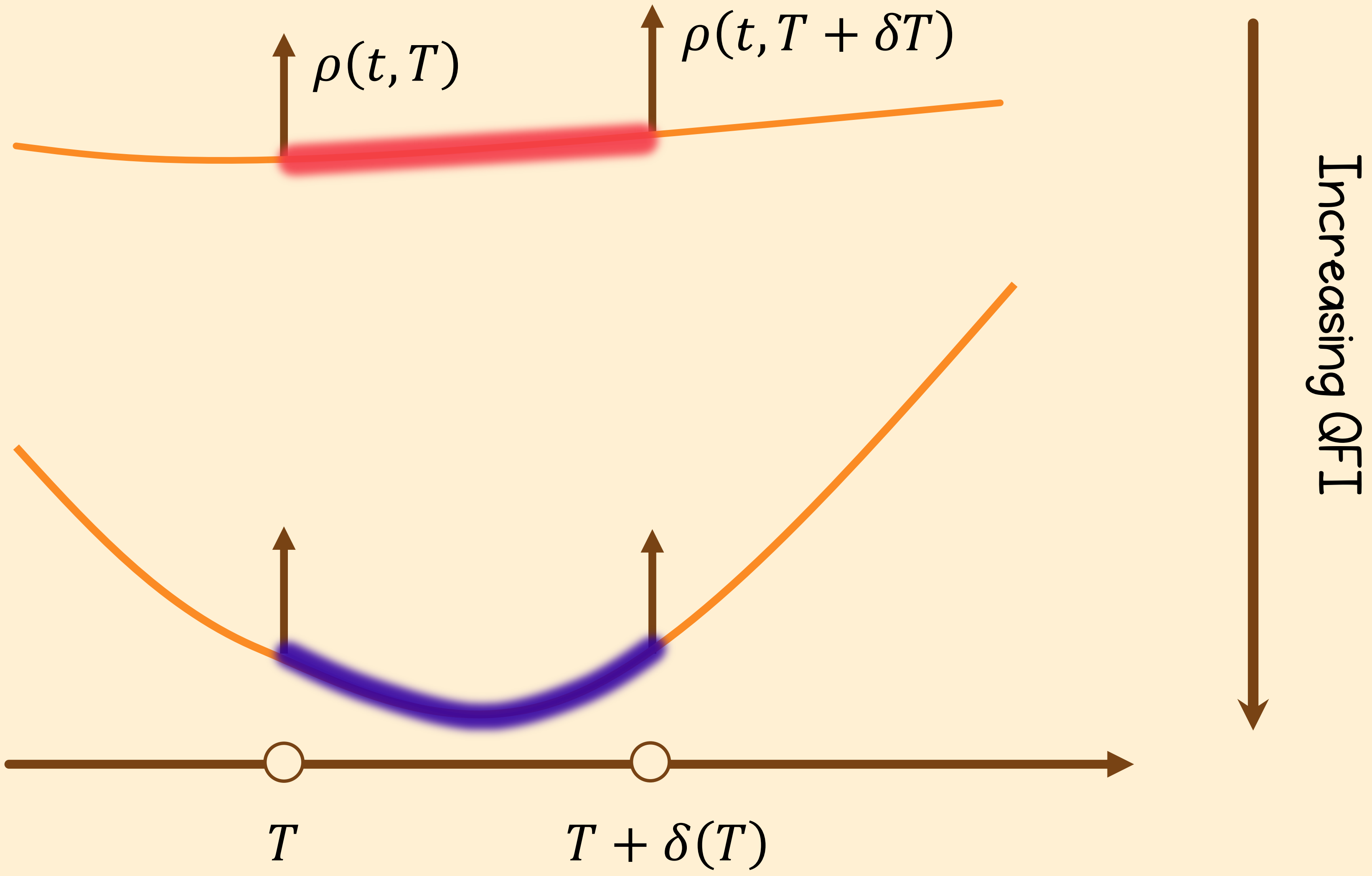
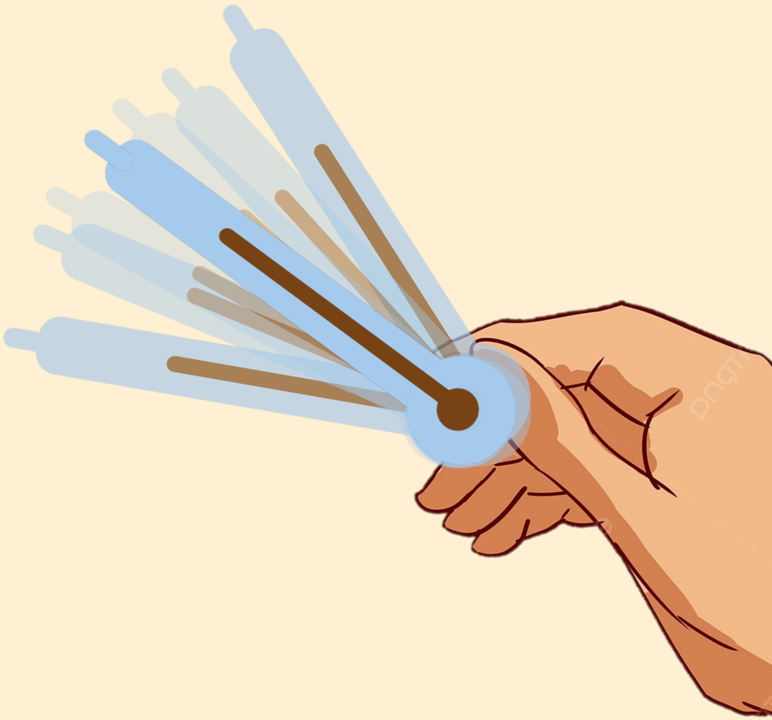
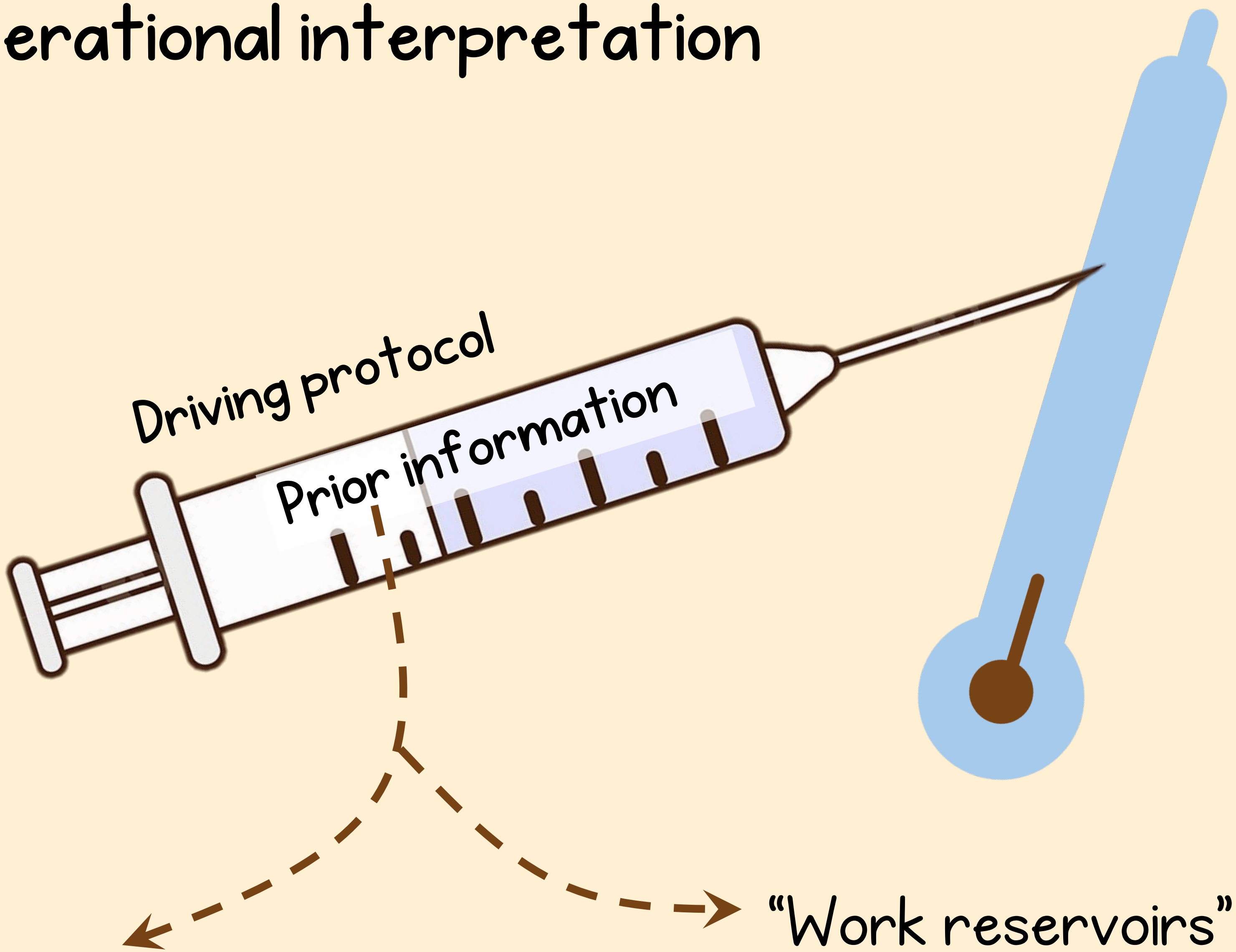




Results: operational interpretation



"Work reservoirs"

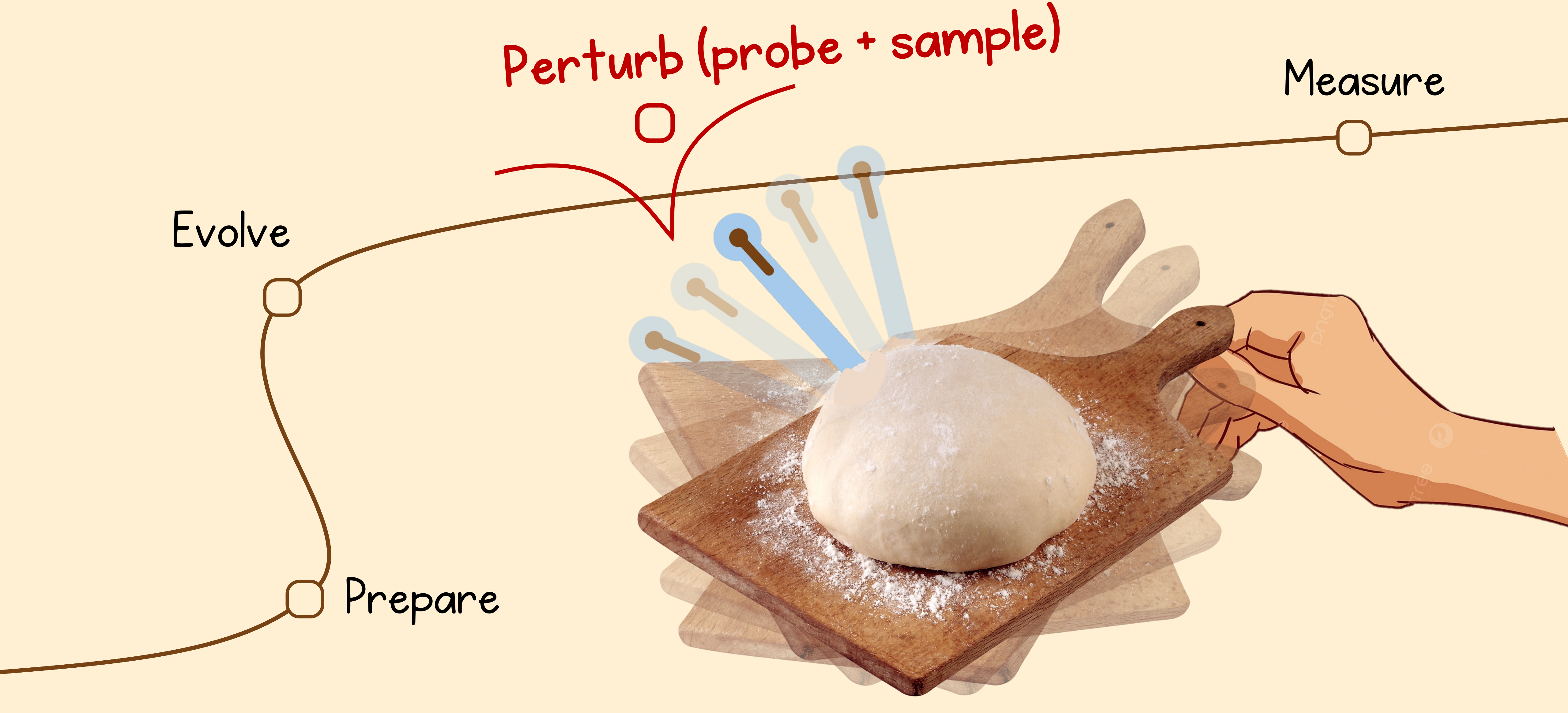
A. Colla et al., Quantum Sci. Technol. 10, 015047 (2024)

Adaptive protocols

M. Mehboudi et al., Phys. Rev. Lett. 128, 130502 (2022

What about non-unitary driving?

Driving an open system:



Driving an open system:

$$\rho(t, T) = U(\rho_{eq})U^{\dagger}$$

Driving an open system:

$$\rho(t,T) = \underline{\Lambda(t,t_0,T)}[\rho_{eq}]$$

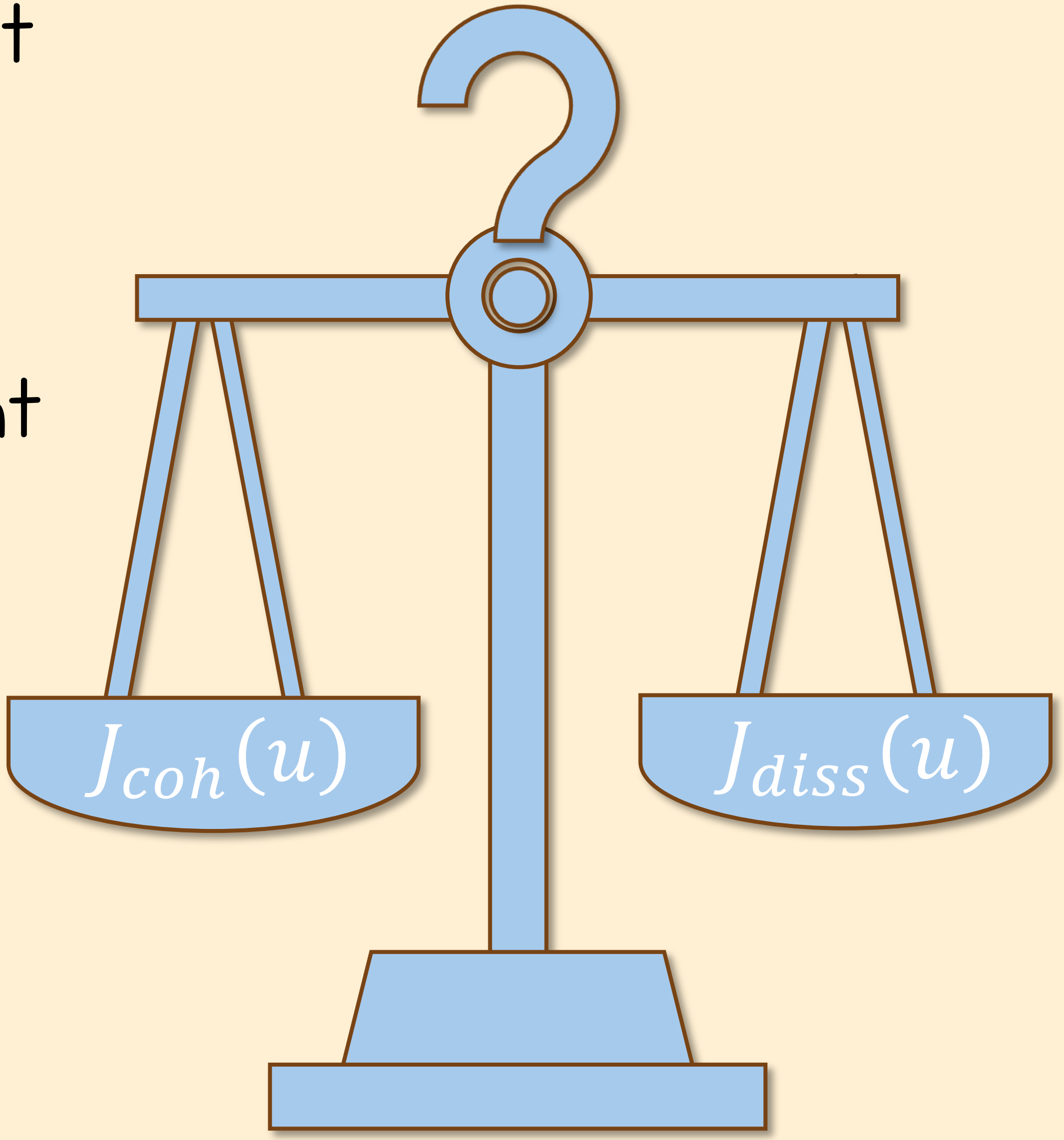
CPTP map from Master Equation dynamcis

Coherent + Dissipation

Driving an open system:

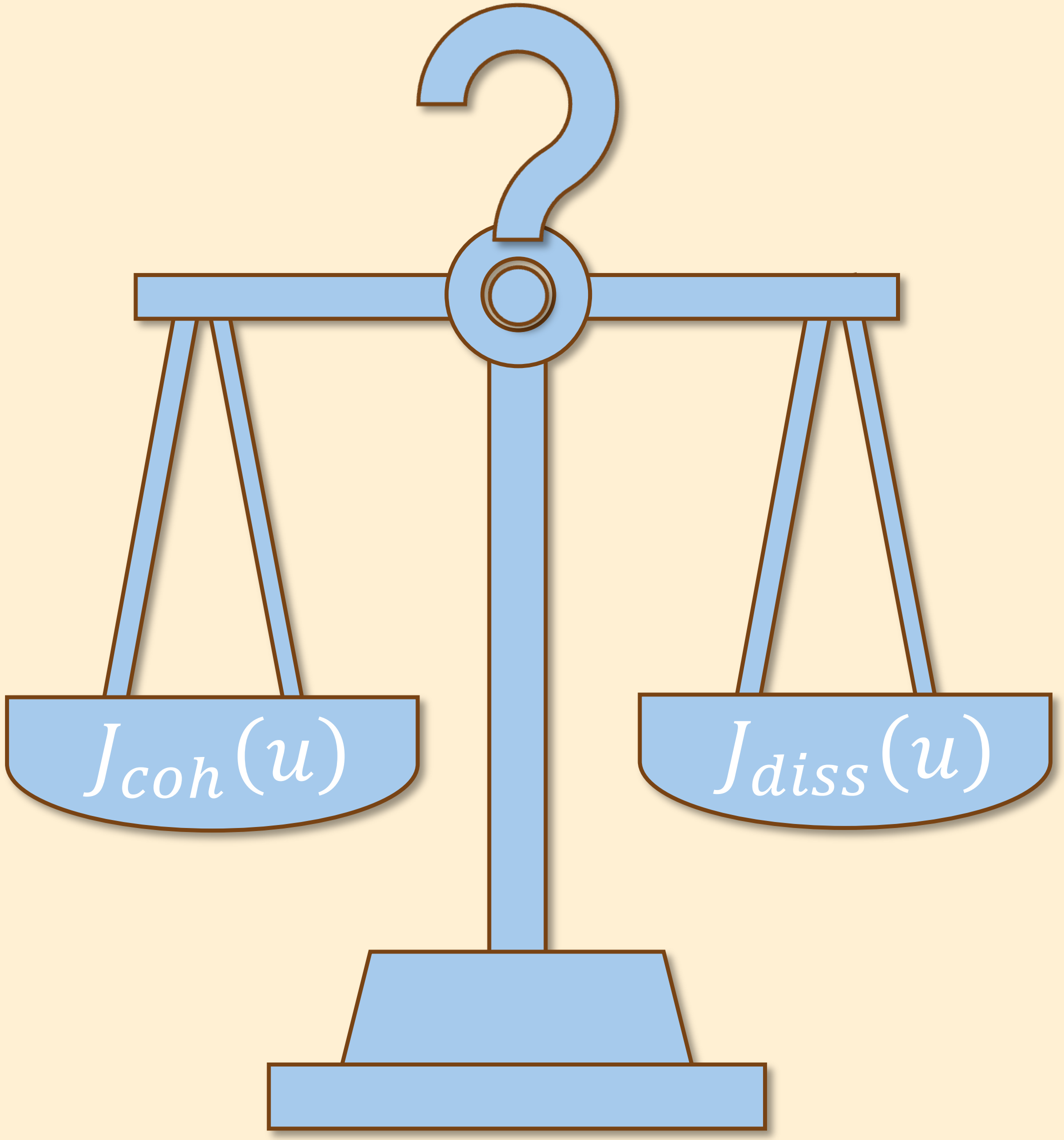
$J_V(u)$

- $J_{coh}(u)$ coherent current increases QFI
- $J_{diss}(u)$ dissipative current decreases QFI

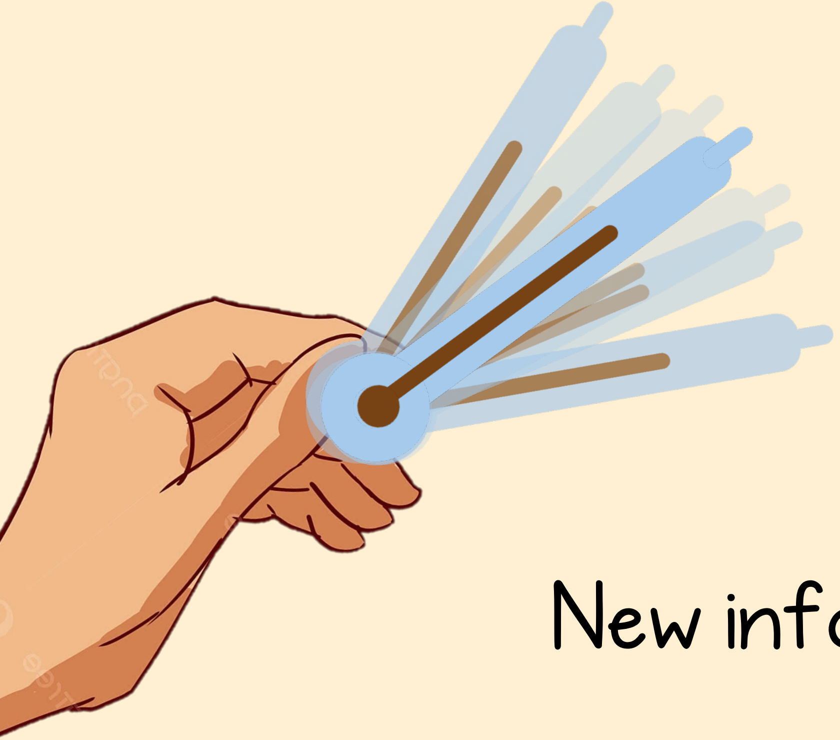


Current directions

- Realistic model to capitalize on driving enhancement
- Comparison with other non-equilibrium schemes
- Investigating different metrics in state space



Conclusions



(Unitarily) shaking your thermometer always helps

New information is stored in coherences: Purely quantum effect

Possibility for enhancement even with dissipation

Sometimes all you need is a little push

Don't be afraid to be shaken

Backup

Equilibrium thermometry

Zero-th law of thermodynamics

«If C is in thermal equilibrium with A and B, then A and B are in thermal equilibrium with one another»