The pion form factor through radiative return method

A journey from muon g-2 to the radiative return approach

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Supervisors: Prof. G. Montagna
Prof. F. Piccinini

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Outline

The Standard Model of particle physics

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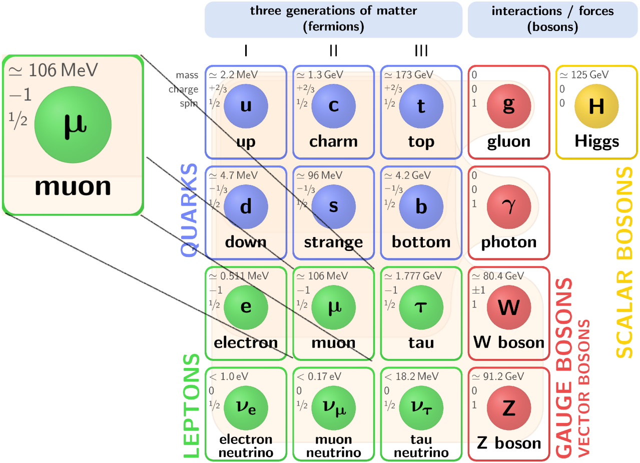
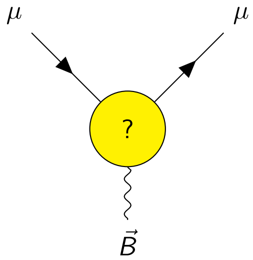


The muon $g-2$



Muon magnetic moment

$$\vec{\mu} = -g_\mu \mu_B \vec{S} = -g_\mu \mu_B \frac{\vec{\sigma}}{2}$$

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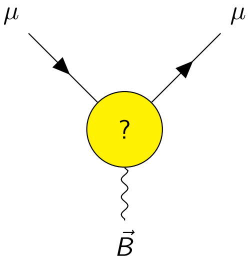


The muon anomaly

$$a_\mu = \frac{g_\mu - 2}{2}$$

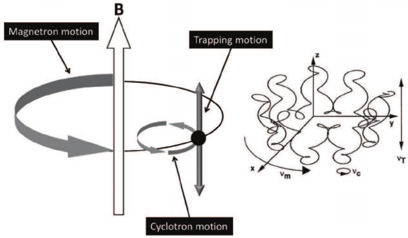
Muon magnetic moment

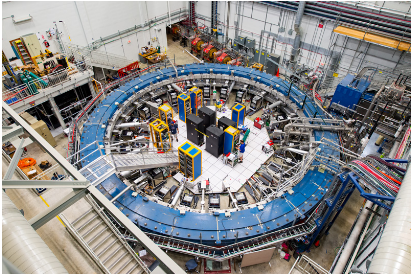
$$\vec{\mu}_s = -g_\mu \mu_B \vec{S} = -g_\mu \mu_B \frac{\vec{\sigma}}{2}$$

Theoretical predictions:

Dirac	$g_\mu = 2$
Schwinger	$g_\mu \neq 2$

The muon anomaly a_μ - Experiment





- Circular motion

https://arxiv.org/pdf/2104.03201

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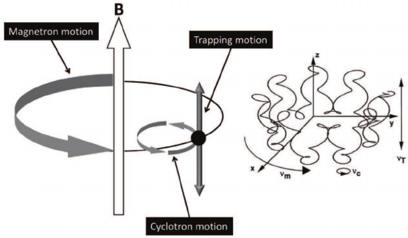
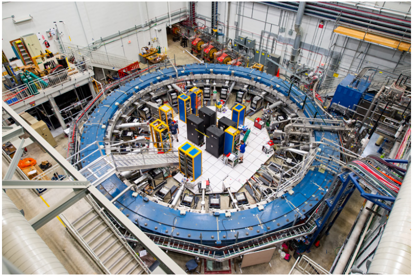


- Circular motion
- Cyclotron frequency ω_c



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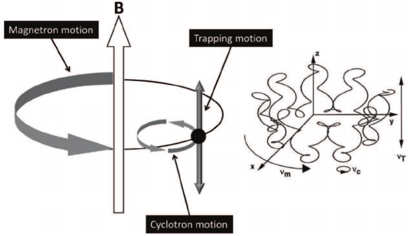
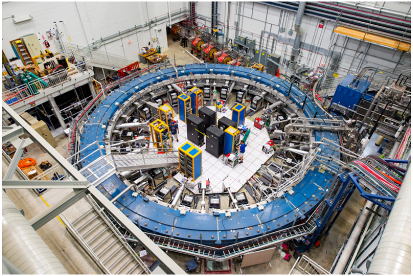


- Circular motion
- Cyclotron frequency ω_c
- Spin precession frequency ω_s

$$\mathcal{H}_s = -\vec{\mu}_s \cdot \vec{B}$$



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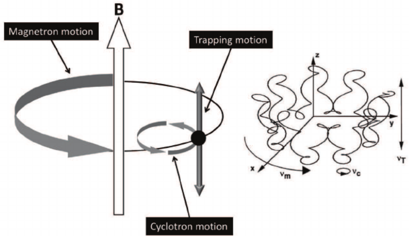

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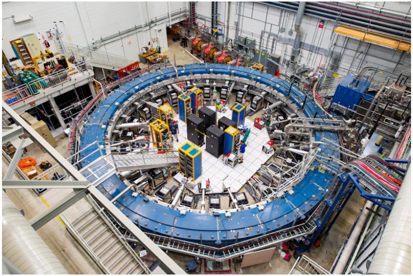

The muon anomaly can be extracted from

$$a_\mu^{exp} \propto \omega_c - \omega_s$$

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The muon anomaly a_μ - Experiment

A recent update in the measurement gives:

$$a_\mu^{exp} = 116\ 592\ 071.5(14.5) \times 10^{-11}$$

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Experimental error

$$\frac{\delta a_\mu}{a_\mu} = 127\,\text{ppb}$$

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The muon anomaly a_μ - Theory

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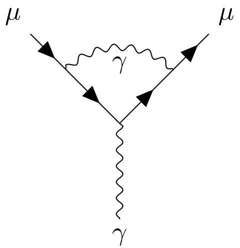
In the SM we can write:

$$a_\mu = \underbrace{a_\mu^{\text{QED}}}_{>99.99\%} + a_\mu^{\text{EW}} + \underbrace{a_\mu^{\text{had}}}_{Non-perturbative}$$



N^{5}LO QED

$$a_\mu^{\text{QED}} = 116584718.8(2) \cdot 10^{-11} \quad .$$

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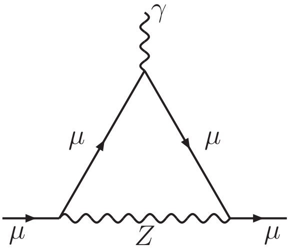

(a)

N^{3}LO EW

$$a_\mu^{\text{EW}} = 154.4(4) \cdot 10^{-11}$$

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https://arxiv.org/abs/hep-ph/0212229

Hadronic contributions

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$$a_\mu^{\text{had}} = \underbrace{a_\mu^{\text{HVP-LO}}}_{\mathcal{O}(7\cdot 10^{-8})} + \underbrace{a_\mu^{\text{HLbL}}}_{\mathcal{O}(10^{-9})} + \underbrace{a_\mu^{\text{HVP-NLO}}}_{\mathcal{O}(10^{-9})} + \underbrace{a_\mu^{\text{HVP-NNLO}}}_{\mathcal{O}(10^{-10})}$$

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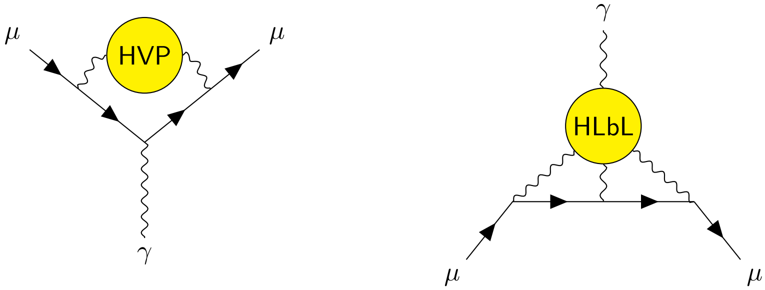

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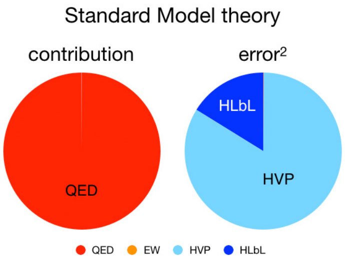

$$a_\mu^{\text{HVP-LO}} \simeq (5 \cdot 10^{-5})\, a_\mu$$
$$\delta(a_\mu^{\text{HVP-LO}}) \simeq (8 \cdot 10^{-1})\, \delta(a_\mu)$$

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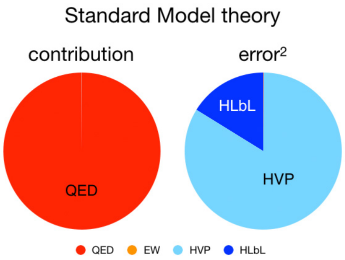

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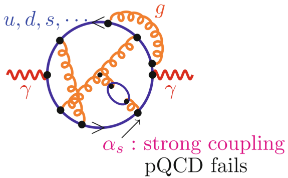
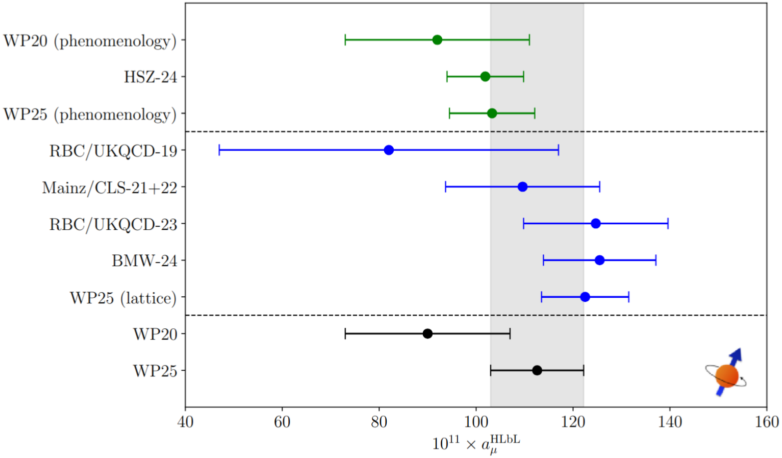


HVP and HLbL are non-perturbative:

- Lattice QCD
- Data-driven approach

HLbL contribution



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Lattice QCD (in a nutshell)

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The partition function is defined through the *Path Integral* formulation

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S[U]}$$

and any observable can be defined as

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, \mathcal{O}[U] \, e^{-S[U]}$$

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If we generate

$$U_i \sim e^{-S[U_i]}$$

than the Monte Carlo method (MC) allows to write:

$$\langle \mathcal{O} \rangle \approx \sum_i \mathcal{O}[U_i]$$

Lattice QCD (in a nutshell)

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- *Ab initio* calculation, no data-driven approach.

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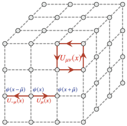

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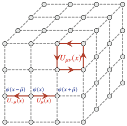

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Time-like approach to HVP

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$$a_\mu^{\text{HVP-LO}} = \frac{\alpha}{3}\left(\frac{m_\mu}{\pi}\right)^2 \int_{m^2_{\pi_0}}^{\infty} \frac{ds}{s^2} \,\text{Im}\, \hat{\Pi}'^{\text{had}}_{\gamma}(s) \hat{\mathcal{K}}(s)$$

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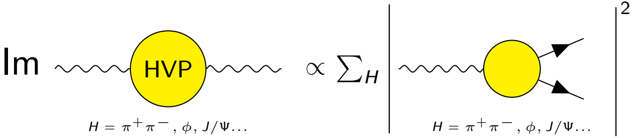

The precise relation reads:

$$\text{Im}\, \hat{\Pi}_\gamma^{\prime\text{had}}(s) = \frac{\alpha}{3} \frac{\sigma_0(e^+e^- \to \gamma \to \text{hadrons})}{\sigma_0(e^+e^- \to \mu^+\mu^-)} \mathcal{R}_\mu(s) = \frac{\alpha}{3} \mathcal{R}_0^{\text{had}}(s)$$

$e^+e^- \to \pi^+\pi^-$ channel

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$$a_\mu^{\text{HVP-LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{m_{\pi_0}^2}^{\infty} ds \frac{1}{s^2} \mathcal{R}_0^{\text{had}}(s) \hat{\mathcal{K}}(s)$$

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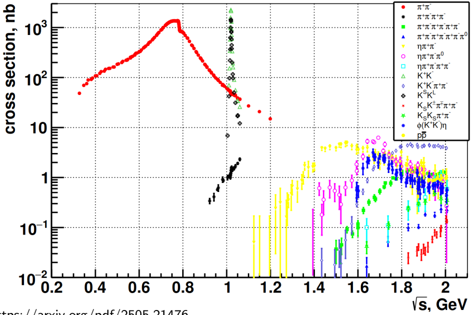

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$e^+e^- \to \pi^+\pi^-$

73% of $a_\mu^{\text{HVP-LO}}$

71% of $\delta(a_\mu^{\text{HVP-LO}})$

How to evaluate $\mathcal{R}_0^{\text{had}}(s)$?

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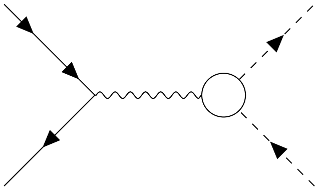
The quantity $\mathcal{R}_0^{\text{had}}(s)$ is the hadronic ratio of Leading Order cross sections. Namely this means that we must be able to compute the Lowest-Order (in α) Feynman diagram:



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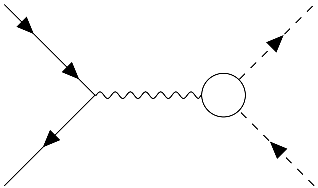

NOT EASY!!!

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- The measured cross section $\sigma\left(e + e^- \to \pi^+\pi^-\right)$ IS NOT $\sigma_0\left(e + e^- \to \pi^+\pi^-\right)$.

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WHY?

How to evaluate $\mathcal{R}_0^{\text{had}}(s)$?

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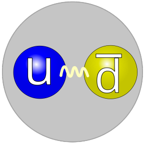
Nature knows nothing of the perturbative order

How to treat the pion?

Parametrize the pion interaction to the photon field in a suitable way.



- **Model**

$$\underbrace{\text{sQED}}_{\text{pert}} \oplus \underbrace{F_\pi(q^2)}_{\text{non-pert}}$$

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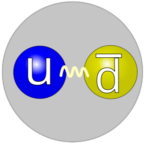

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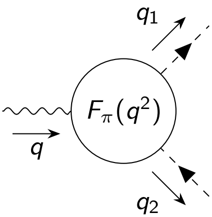

$$\langle \pi^+(q_2)\pi^-(q_1)|J^\mu_\pi(0)|0\rangle = e(q_1 - q_2)^\mu F_\pi\left(q^2\right)$$

What can we do then?

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The best we can:

- Perform the perturbative calculation at the highest possible order (Next-To-Leading-Order at the state of the art)

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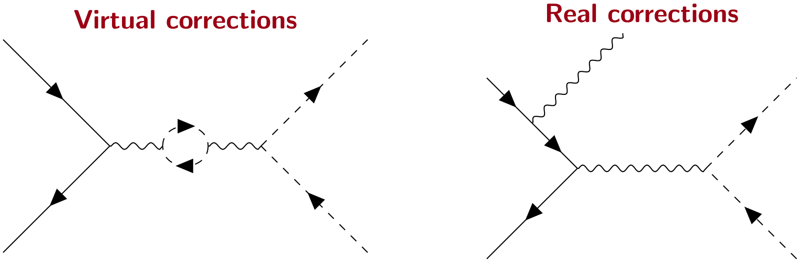
$$\sigma_{\text{NLO}} = \sigma_0(\alpha^2) + \sigma_1(\alpha^3)$$



- Consider (approximately) the emission of any number of additional photons through resummation technique (*e.g.* PS).

The pion form factor

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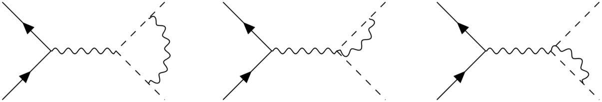
What seen above is not enough to solve all the ambiguities. What about loops diagram?



https://arxiv.org/abs/2409.03469
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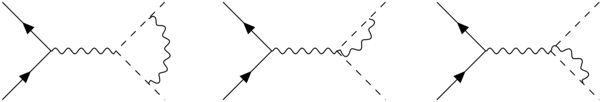

- Multiply the point-like amplitude to $F_\pi\left(Q^2\right)$

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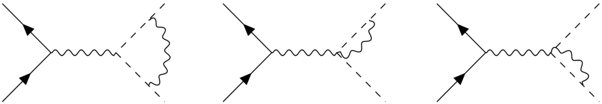
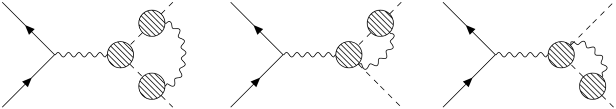


- Multiply the point-like amplitude to $F_\pi\left(Q^2\right)$
- Include $F_\pi\left(Q^2\right)$ into loop integration.



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Finally what we see is that:

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The pion form factor

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$e^+e^- \to \pi^+\pi^-$

$e^+e^- \to \pi^+\pi^-\gamma$

The radiative return

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$$e^+e^- \to \pi^+\pi^-\gamma$$

The radiative return

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How to link

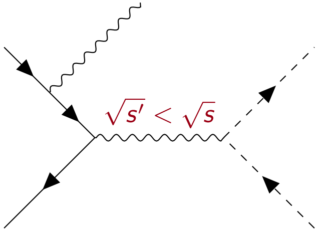
$$\sigma(e^+e^- \to \pi^+\pi^-, Q^2) \iff \sigma(e^+e^- \to \pi^+\pi^-\gamma)$$

The radiative return

$$e^+e^- \to \pi^+\pi^-\gamma$$

How to link

$$\sigma(e^+e^- \to \pi^+\pi^-, Q^2) \iff \sigma(e^+e^- \to \pi^+\pi^-\gamma)$$



The radiative return

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Since QED is well know, we can describe Initial-State-Radiation effects at the best we can through a function, called **radiator**

$$H(s')$$

and finally we can link the two processes through

$$s' \frac{d\sigma_{\pi^+\pi^-\gamma}}{ds'} = \sigma_{\pi^+\pi^-}(s') \times H(s')$$

https://arxiv.org/abs/2006.04822
https://arxiv.org/abs/1507.08188
https://arxiv.org/abs/1205.2228
https://arxiv.org/abs/2110.00520

The radiative return - Preliminary results

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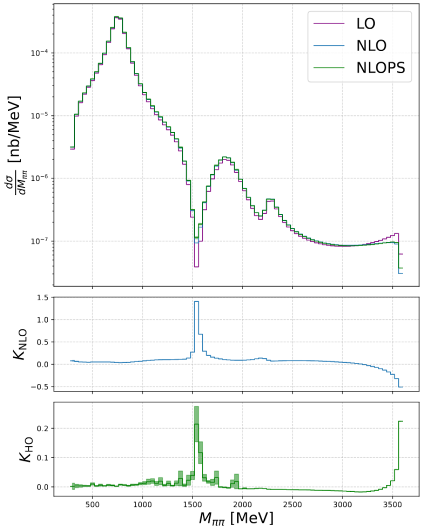

Budassi et al. (WIP)

The final picture

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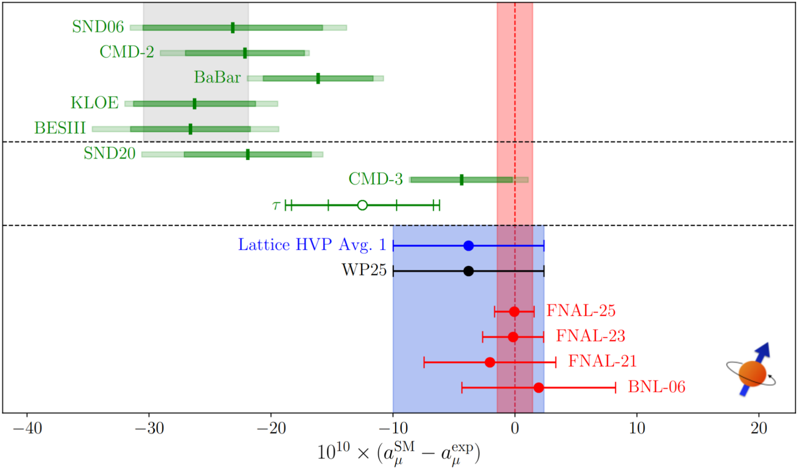

Conclusions

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- The official theory prediction (LQCD) is now consistent with experimental results.

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Careful review of all data analysis through new MC generators (WIP)

Backup

Pion Form Factor inconsistency

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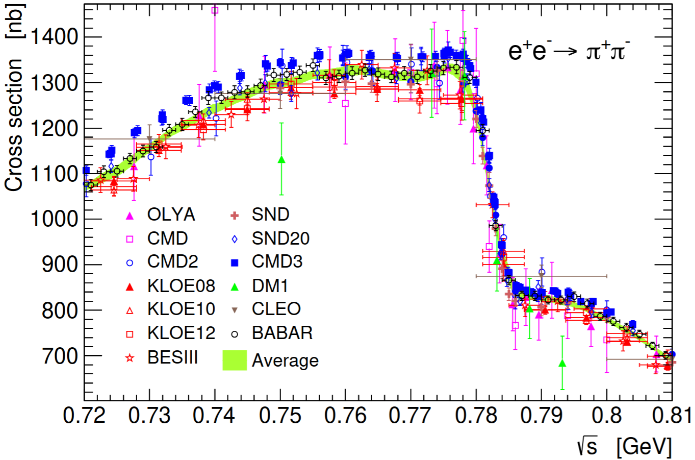

https://arxiv.org/pdf/2312.02053

Space-like approach

$$a_\mu^{\text{HVP-LO}} = \frac{\alpha}{3}\left(\frac{m_\mu}{\pi}\right)^2 \int_{m_{\pi_0}^2}^{\infty} \frac{ds}{s^2} \text{Im}\,\hat{\Pi}_\gamma'^{\text{had}}(s)\hat{\mathcal{K}}(s)$$

where

$$\hat{\mathcal{K}}(s) = \int_0^1 dx\ f(x,s)$$

https://arxiv.org/abs/1504.02228
https://arxiv.org/abs/1609.08987

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Switching the integration order

$$s \longleftrightarrow x$$

we can rewrite the master formula as:

$$a_\mu^{\text{HVP-LO}} = \frac{\alpha}{\pi}\int_0^1 dx\;\, (x-1)\;\hat{\Pi}_\gamma^{\prime\text{had}}[t(x)] \qquad t(x) \le 0$$

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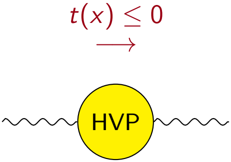
Space-like approach



$H = \pi^+\pi^-, \phi, J/\Psi \ldots$

https://arxiv.org/abs/1504.02228
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Space-like approach

$t(x) \leq 0$

$\longrightarrow$

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?

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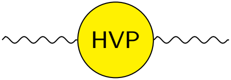
Space-like approach

$$t(x) \leq 0 \quad \longrightarrow$$



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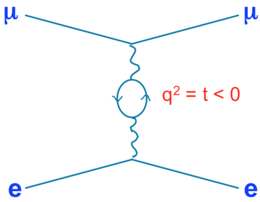

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