

BOUNDARY EFFECTS IN QUANTUM FIELD THEORY

HADAMARD STATES WITH ROBIN BOUNDARY CONDITIONS

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Quantum Fields and Boundaries

Boundaries shape physical phenomena:

- Casimir effect and Casimir-Polder interaction
- Wave propagation at the interface of two materials
- Holographic setups: AdS/CFT correspondence

The Strategy

Starting point: **Dirichlet propagators** G_D^\pm built via the **method of images**

Robin-to-Dirichlet map²: $T_\kappa := \partial_z + \kappa \mathbb{I}$

⇒ distributional inverse: $T_\kappa^{-1} = \Theta(z)e^{-\kappa z}$

Intuition: Propagation Near Boundary

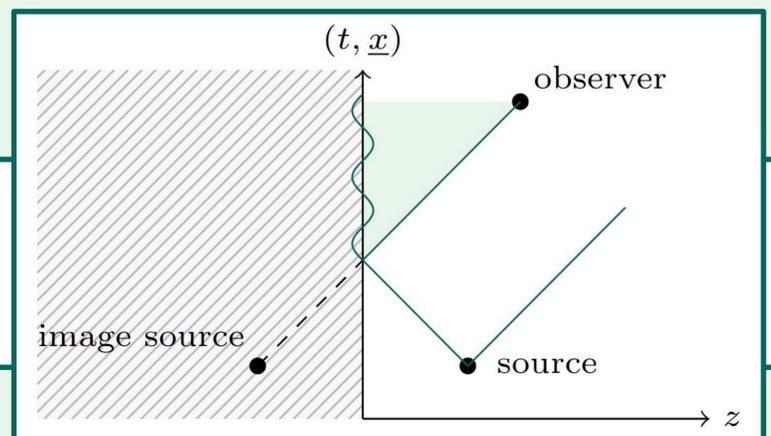
When waves meet boundary: **reflection** and **tangential propagation**

⇒ Modified propagation is encoded in the **advanced and retarded propagators**

Main Result³

Explicit characterisation of **Robin propagators**:

$$G_\kappa^\pm = (T_\kappa^{-1} \otimes \delta) \star G_D^\pm \circ (\mathbb{I} \otimes T_\kappa)$$



⇒ Proof of the **causal support property**
Construction of **Hadamard states from propagators**

The Model

Half-Minkowski: $\mathbb{H}^d := \{(t, \underline{x}, z) \in \mathbb{R}^d \mid z \geq 0\}$

Real scalar field: $\Phi : \mathbb{H}^d \rightarrow \mathbb{R}$ such that

$$\begin{cases} P\Phi := (\square + m^2)\Phi = 0, \\ (\partial_z + \kappa)\Phi = 0, \end{cases} \quad \kappa > 0.$$

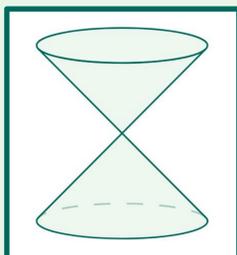
The Problem

Robin propagators $G_\kappa^\pm \in \mathcal{D}'(\mathbb{H}^d \times \mathbb{H}^d)$:

$$\begin{cases} (P \otimes \mathbb{I})G_\kappa^\pm = (\mathbb{I} \otimes P)G_\kappa^\pm = \delta, \\ \text{supp}(G_\kappa^\pm(f)) \subseteq J^\mp(\text{supp}(f)) \quad \forall f \in \mathcal{D}(\mathbb{H}^d), \quad (*) \\ [(\partial_z + \kappa) \otimes \mathbb{I}]G_\kappa^\pm|_{z=0} = 0. \end{cases}$$

J^\pm : **future/past light cone**

1. **Existence and uniqueness**¹
2. **Causal support property** (*)?



Outlook

- Compute observables, e.g. **stress-energy tensor**
- Extension to **curved backgrounds with boundary**

References

- [1] C. Dappiaggi, N. Drago, H. R. C. Ferreira, Ann. Henri Poincaré **18** (2017)
- [2] J.D. Bondurant, S.A. Fulling, J. Phys. A: Math. Gen., **38** (2005)
- [3] B. Costeri, C. Dappiaggi, B.A. Juárez-Aubry, R.D.S., ArXiv:2509.26035[math-ph]

