Dirac Fields and Boundary Conditions on AdS_n

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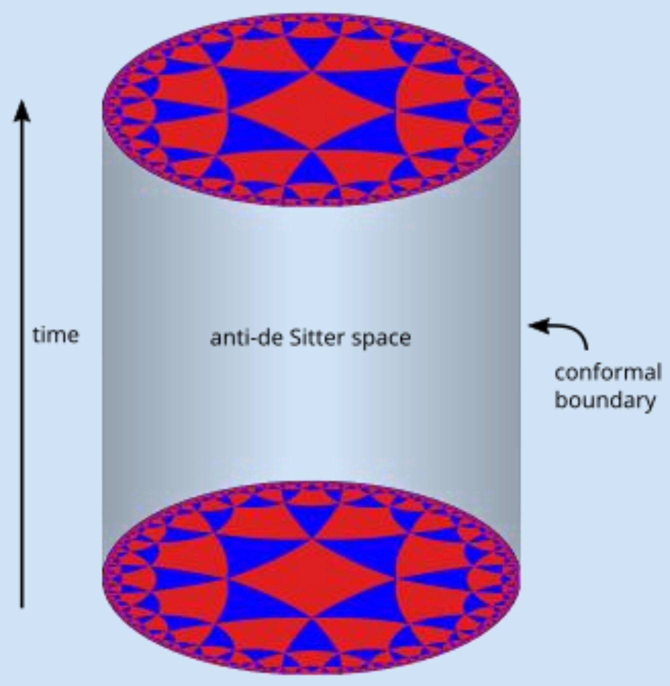
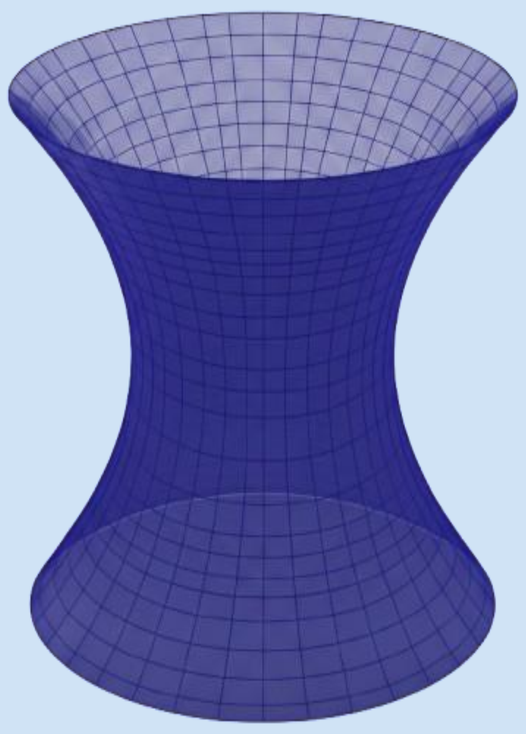
Motivations

QFT on AdS = particle in a “gravitational box”. It is the playground of the AdS/CFT correspondence → QFT-gravity interplay.
Goal: first systematic classification of spinor boundary problems.





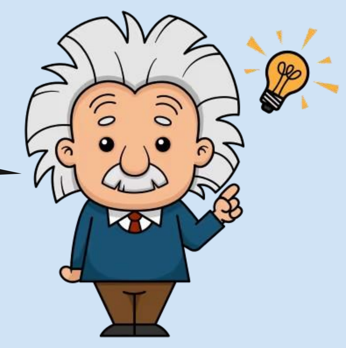
Dirac Equation on anti-de Sitter spacetime

- $AdS_n \cong (0,+\infty) \times \mathbb{R}^{n-1}$ → Hyperboloid in $\mathbb{R}^{n+1}$
- $ds^2_{Ads} = z^{-2}\left(-dt^2 + \delta_{ij} dx^i dx^j\right) = z^{-2} ds^2_{\mathbb{H}}$ (Poincaré chart)

Calling $\phi = z^{\frac{1-n}{2}}\psi$ one has $\left(\Upsilon^\mu \nabla_\mu - m\right)\psi = 0 \Leftrightarrow \left(\gamma^\mu \partial_\mu - \frac{m}{z}\right)\phi = 0$

Boundary conditions = well posedness!

Quantum physics = self-adjoint Hamiltonian!



Generalized MIT-bag boundary conditions

Self-adjoint boundary conditions are the solution we re looking for. They descend from the theory of deficiency indices.

$$m < 1/2: \; (\mathbb{I} - U)\psi^{(1)}_{|z=0} + i(\mathbb{I} + U)\hat{\gamma}^0\psi^{(2)}_{|z=0} = 0$$

$$m \geq 1/2: \; \text{no boundary conditions required}$$

These results establish the first systematic classification of a spinor boundary value problem!
They are consistent with previous results[1] on AdS_2 and MIT-bag boundary conditions[2].

Example 1: proper MIT-bag boundary conditions

$\gamma^1\psi_{|z=0} = \psi_{|z=0}$ → spectrum $(-\infty, -\omega_{min}) \cup (\omega_{min}, +\infty)$.
The Hilbert space is generated by eigenfunctions $\{u_1^\omega, u_2^\omega\}$ and the causal propagator is $u_1(x)\bar{u}_1(y) + u_2(x)\bar{u}_2(y)$.

Example 2: upper-half boundary conditions

$\psi^{(1)}_{|z=0} = 0$ → $(-\infty, -\omega_{min}) \cup (\omega_{min}, +\infty) \cup \{0\}$.
$\{v_1^\omega, v_2^\omega, w^0\}$ generate the Hilbert space and the causal propagator is $v_1(x)\bar{v}_1(y) + v_2(x)\bar{v}_2(y) + w(x)\bar{w}(y)$.

Future outlooks

- Hadamard states for generalized boundary conditions
- QFT with dynamical boundary conditions
- Interacting fermions

References

[1]D. S. Blanco, J. Math. Phys. 64, 032301 (2023), arXiv:2208.08252.

[2]K. Johnson, Acta Phys. Pol. B 6, 865 (1975).