Discretize and Conquer

How to Access Fundamental Physics from Numerical Computations

Andrea Maestri, Barbara Pasquini, Simone Rodini, **Andrea Schiavi**

OUR GROUP

UNIVERSITÀ DI PAVIA

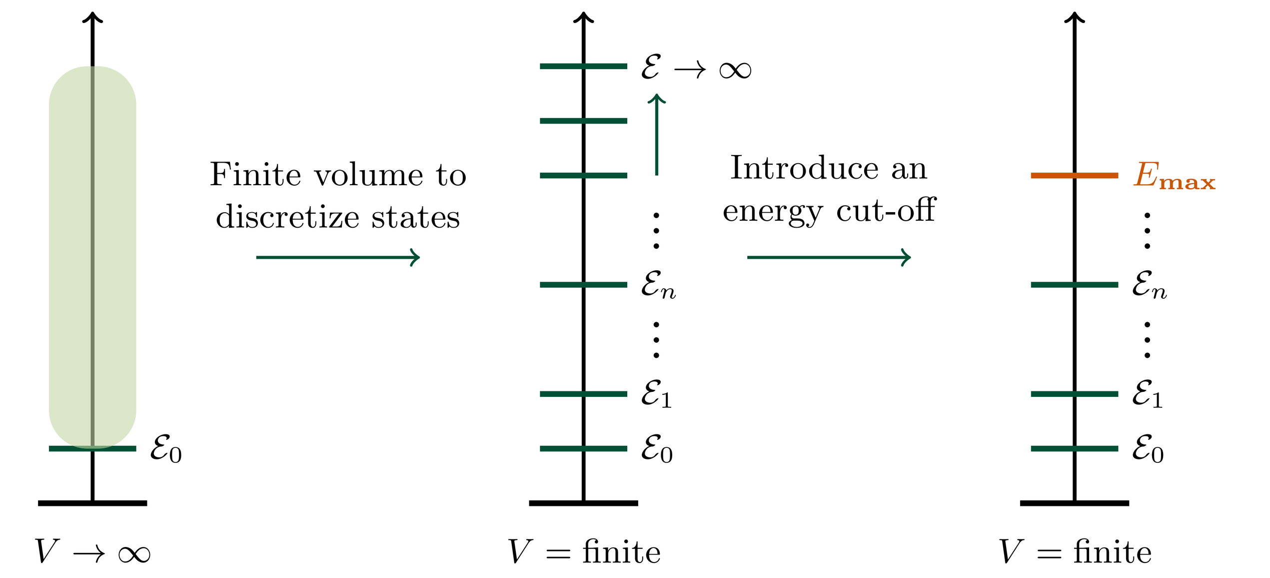
INFN

Abstract

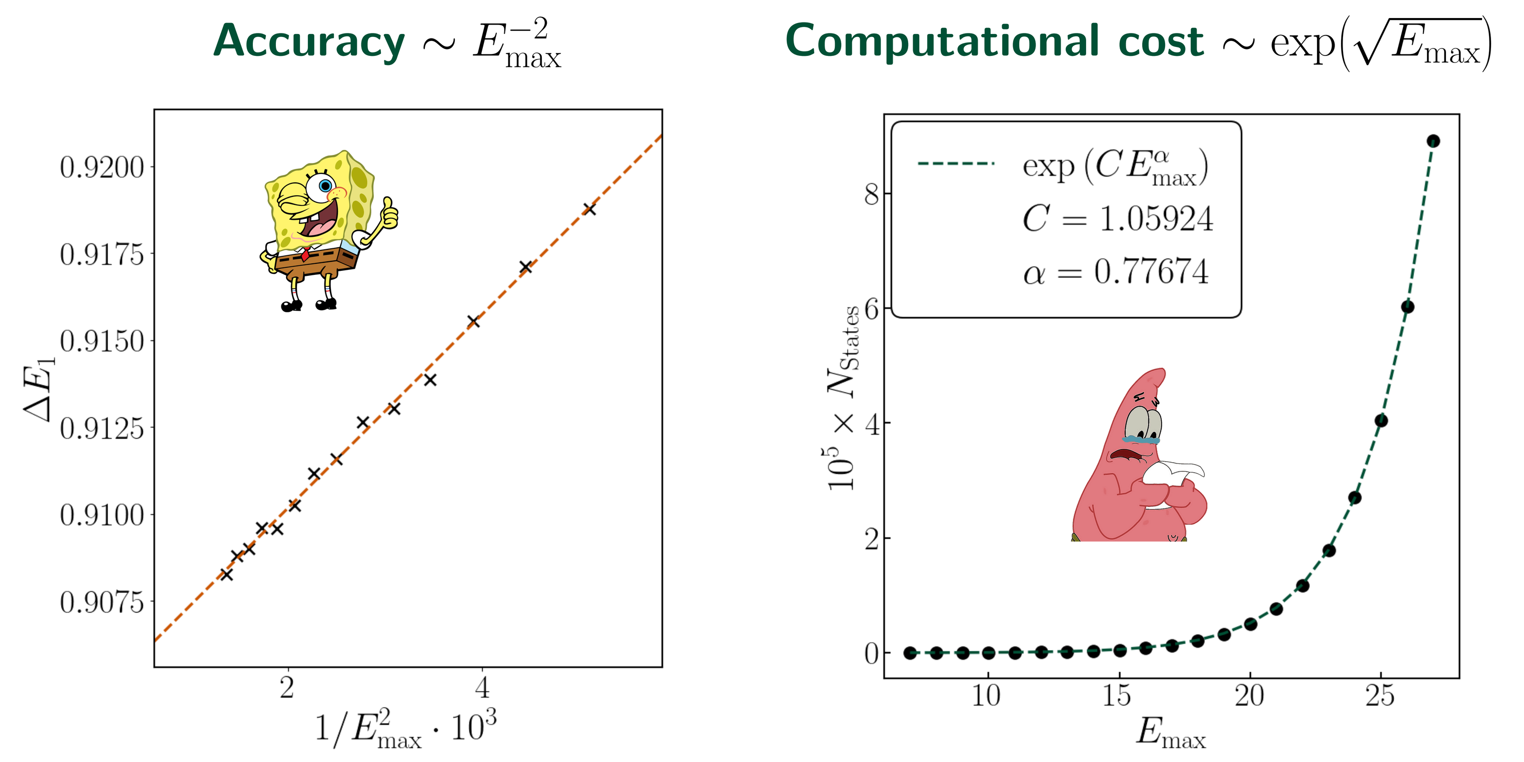
A number of numerical approaches has been developed to study quantum field theories in nonperturbative regimes. One consists in confining spacetime to a finite volume and in discretizing it, allowing for ab initio simulations. In the case of Quantum Chromodynamics (QCD), this is known as Lattice QCD. Another strategy is to numerically diagonalize the Hamiltonian of the theory, which becomes feasible by truncating its energy spectrum. For the former approach, we demonstrate how to extract from correlators computed in Lattice QCD the light-front wave functions of baryons — such as the proton — which encode information on the configurations of its elementary constituents. For the latter approach, we present our results for the $\lambda\phi^4$ theory in $1+1$ dimensions, where an effective theory can be constructed to mitigate the computational resources needed to achieve convergence.

Hamiltonian Truncation

$H = H_0 + V \longrightarrow H_0$ provides the set of unperturbed eigenstates: $H_0 |\mathcal{E}_i\rangle = \mathcal{E}_i |\mathcal{E}_i\rangle$

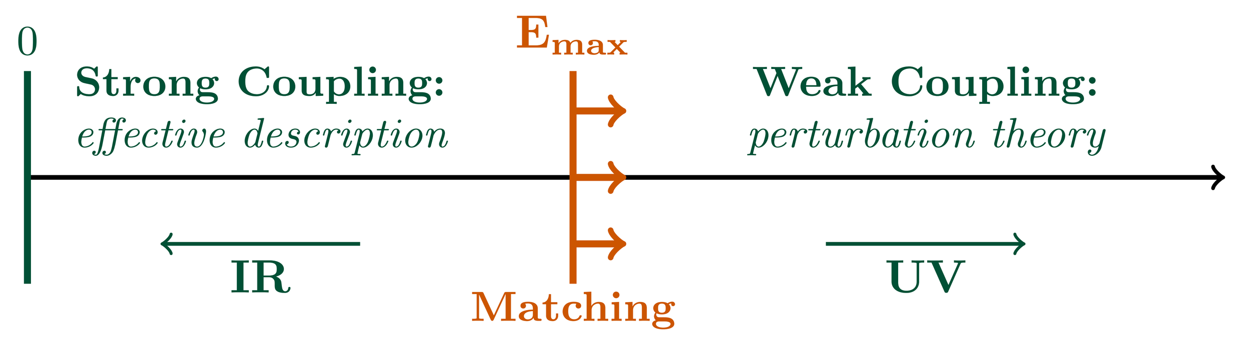


Accuracy Comes at a Cost

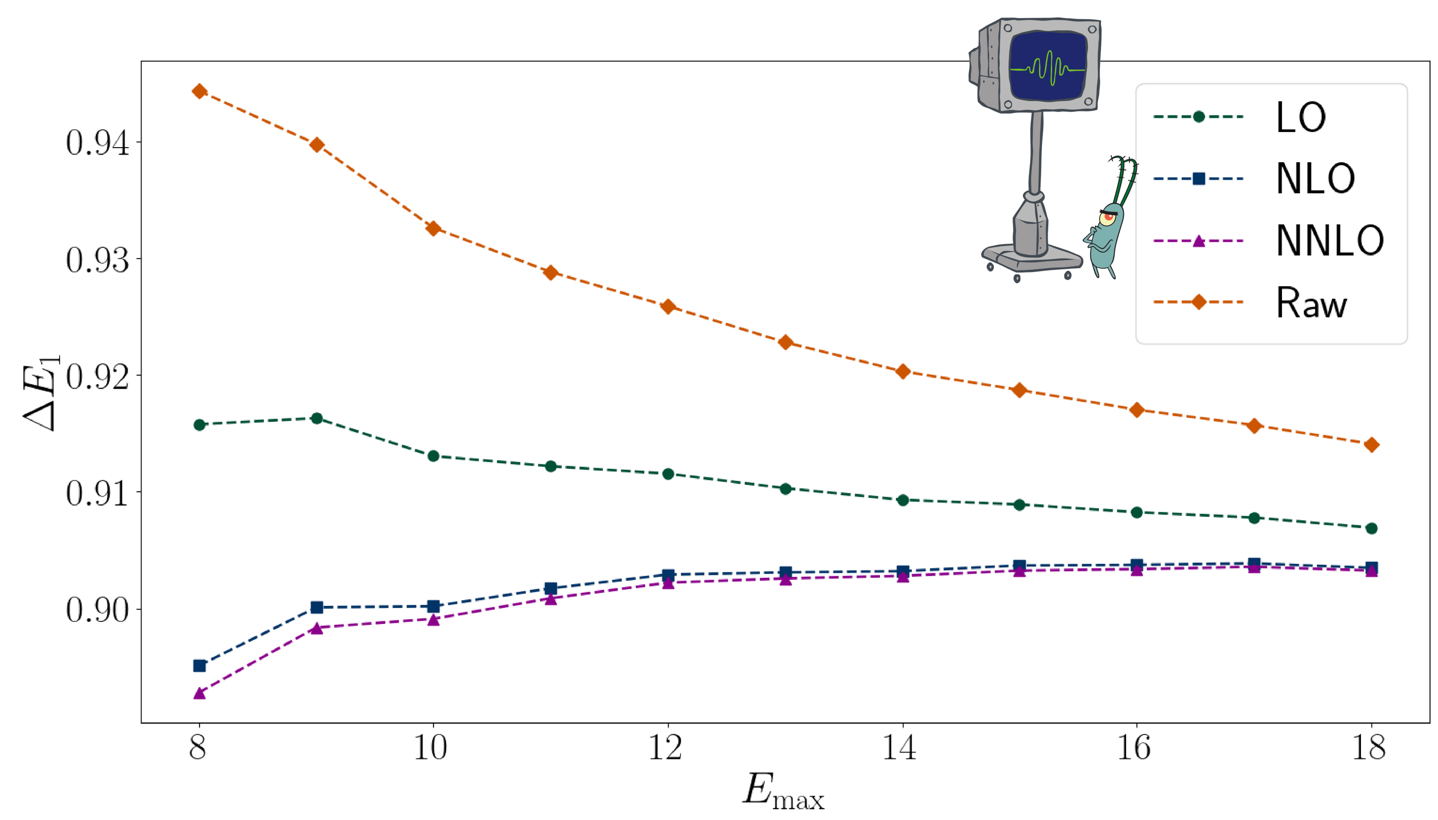


Effective Field Theory

Matching the **transition matrix**: $\lim_{t_f\to\infty} \langle f| U_{IP}(t_f,0) |i\rangle = \delta_{ij} + \dfrac{\langle \boldsymbol{f}|\boldsymbol{T}|\boldsymbol{i}\rangle}{E_{fi}+i\epsilon}$



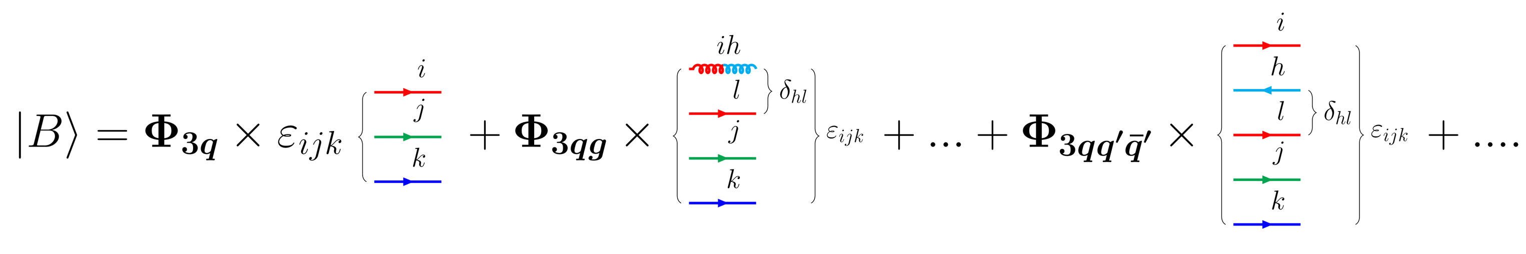
- **[1] Local Corrections** $\mathcal{O}(E_{\max}^{-2})$: coupling $\delta\lambda$ and mass δm^2
- **[2] Non-Local Corrections** $\mathcal{O}(E_{\max}^{-3})$: non-local operators, e.g. $H_0 :\phi^4:$
- **[3] Next Non-Local Corrections** $\mathcal{O}(E_{\max}^{-4})$: Second-order expansion of the nonlocal structure and on matching in infinite volume. New operatorial structures arise, e.g. $H_0 :\phi^2: H_0$ or $(\partial_t\phi)^2$



[1] *T. Cohen et al. (2022), SciPost Phys. 13.* [2] *E. Demiray et al. (2025), arXiv:2507.15941.*
[3] *Andrea Maestri, Barbara Pasquini, Simone Rodini, (2026) arXiv:2602.13019.*

Baryon Light-Front Wave Functions

The state of a baryon B, in **light-front quantization** [4], can be expressed as a series of quark and gluon configurations through light-front wave functions:



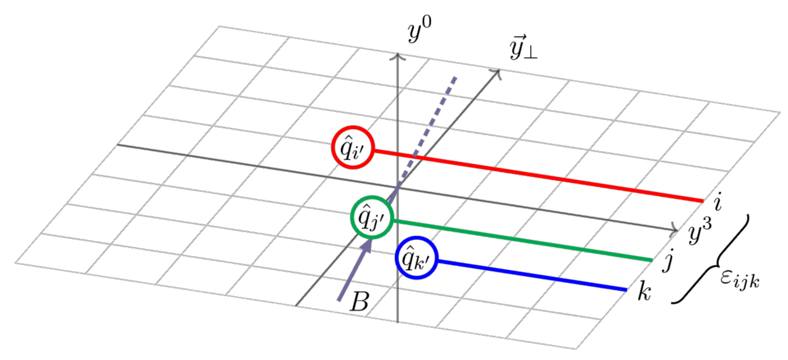
[4] *S. Brodsky et al. (1998), Phys. Rept. 301:299.*

Lattice Correlators

Lattice QCD studies **equal-time correlators**, e.g., for a 3-quark-singlet operator,

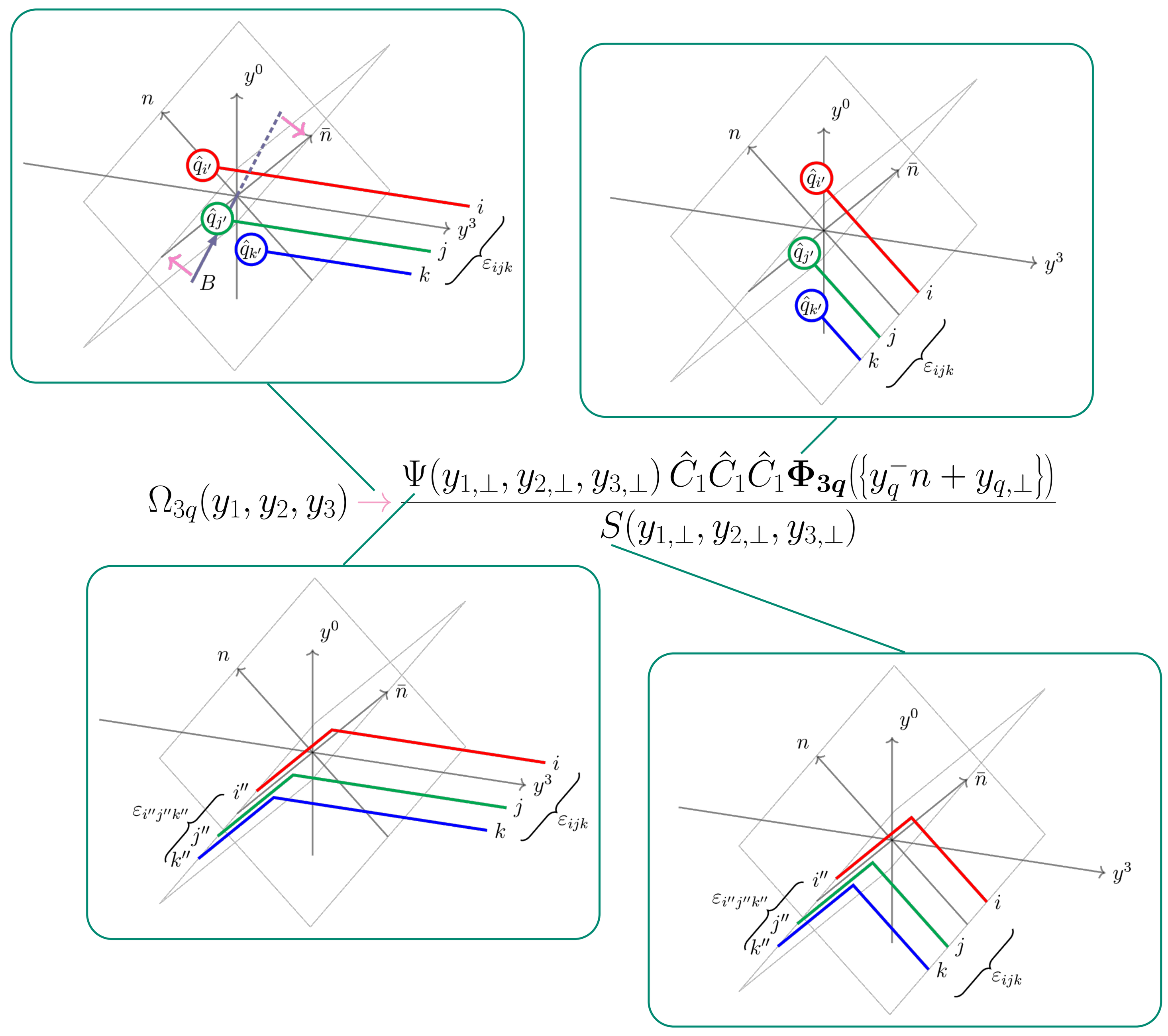
$$\Omega_{3q}(y_1,y_2,y_3) = \langle 0|\varepsilon_{ijk}\hat{q}_i(y_1)\,\hat{q}_j(y_2)\,\hat{q}_k(y_3)|B\rangle,$$

where implicit Wilson lines send the color charges to the boundary of the lattice:



Lattice Factorization

In the **relativistic limit**, the baryon tends to move along the lightlike direction $\bar{n}$. Then, applying the **background-field method** with two background fields [5, 6], we can extract Φ_{3q} from the lattice correlator:



$$\Omega_{3q}(y_1,y_2,y_3) \to \frac{\Psi(y_{1,\perp},y_{2,\perp},y_{3,\perp})\,\hat{C}_1\hat{C}_1\hat{C}_1\,\mathbf{\Phi_{3q}}\big(\{y_q^- n + y_{q,\perp}\}\big)}{S(y_{1,\perp},y_{2,\perp},y_{3,\perp})}$$

The coefficients $\hat{C}_1$ are loop corrections, each acting on one of the quark legs.

[5] *A. Vladimirov et al. (2022) JHEP 01:110.* [6] *S. Rodini and A. Vladimirov (2023) JHEP 09:117.*

Physical Factorization

- The **soft factor** S cancels the divergences created by the separation of Φ_{3q} from the residual lattice factor Ψ, including the new kind of **rapidity divergences**.
- The **physical $\mathbf{\Phi_{3q}}$ is renormalizable independently** of Ψ, and leg by leg up to next-to-leading power.
- The physical Φ_{3q} additionally depends on **one UV renormalization scale, plus one rapidity scale for each leg**, each with independent evolution [7].

[7] *Barbara Pasquini, Simone Rodini, and Andrea Schiavi (2026), in preparation.*