Precise calculations for the Muon Anomalous Magnetic Moment

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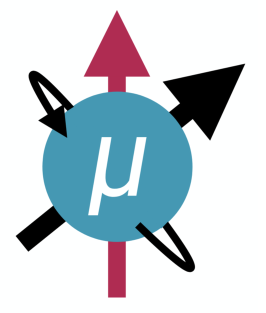
Summary

The anomalous magnetic moment of the muon, $a_\mu = (g-2)_\mu/2$, is a sensitive probe of the Standard Model of particle physics and a possible window to New Physics. Presently, the data-driven dispersive predictions for a_μ significantly disagree with the experimental world average. These calculations are based on the optical theorem and make use of the measurements of the hadron production cross sections in electron-positron annihilation at low energies. In this context, the process of pion pair production is the dominant channel. We present precise predictions for the processes $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-\gamma$ that have been recently implemented in the Parton Shower (PS) event generator BabaYaga and will be crucial for new measurements and data re-analysis.

The Muon Anomalous Magnetic Moment



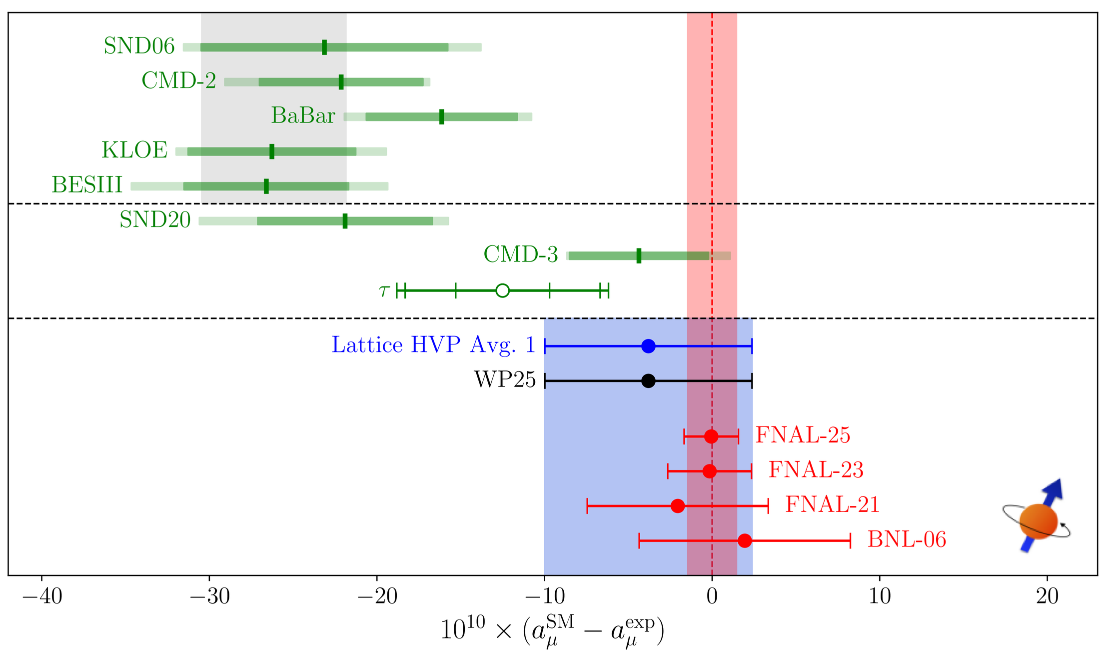
Muon g-2

The Muon anomaly

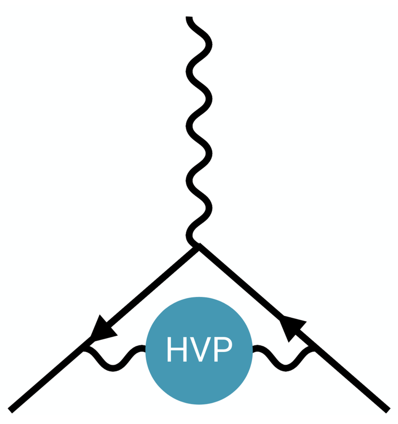
$$\vec{\mu} = -g\mu_B \vec{S}$$

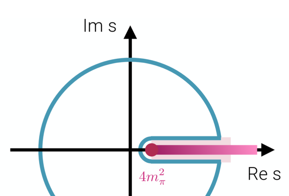
$$a_\mu = \frac{(g-2)_\mu}{2}$$

The current status of the theory–experiment comparison is the following:

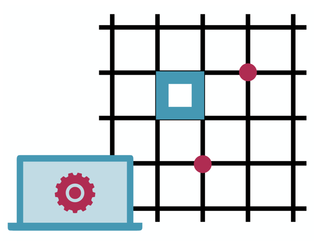


The dominant contribution to the theoretical uncertainty comes from the HVP-LO





Dispersive approach



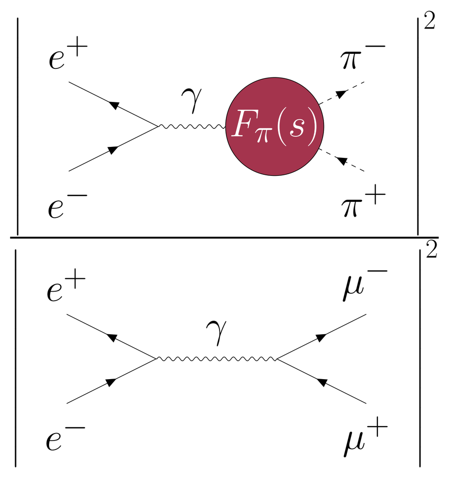
Lattice QCD

The dispersive approach is a data-driven formulation where

$$a_\mu^{\text{HVP-LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{m_{\pi_0}^2}^{\infty} ds \frac{1}{s^2}\mathcal{R}_0^{\text{had}}(s)\hat{\mathcal{K}}(s)$$

and

$\mathcal{R}_0^{\text{had}}(s) \propto$

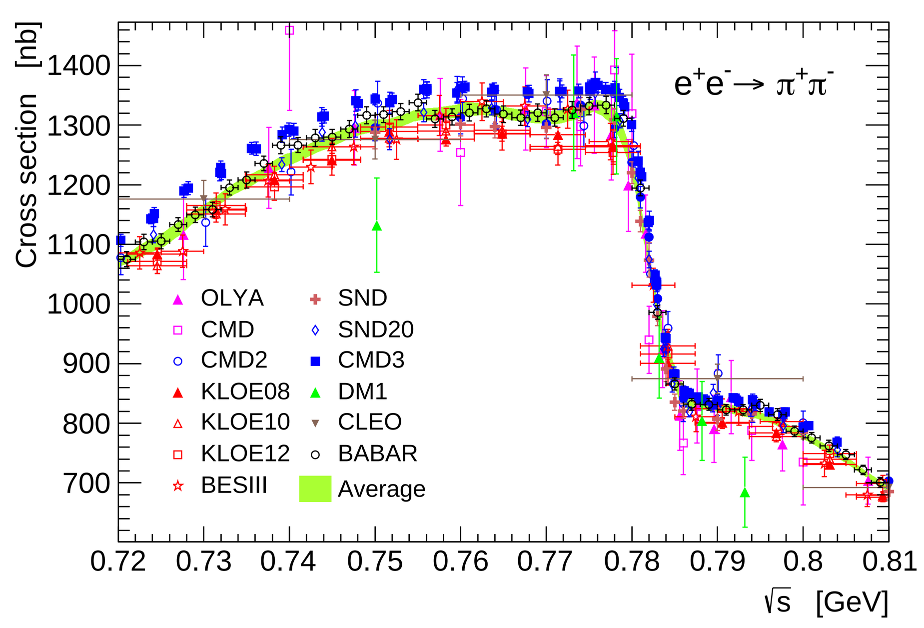


The pion form factor, defined as

Vector Pion Form Factor

$$\langle \pi^\pm(p')|j^\mu_{\text{em}}(0)|\pi^\pm(p)\rangle = \pm(p'+p)^\mu F_\pi\left((p'-p)^2\right)$$

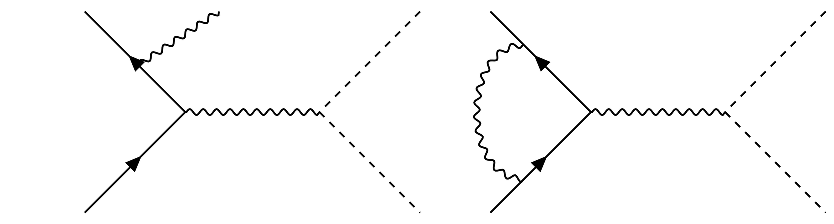
is extracted from data, showing tensions between different experiments.



Precision calculation

Fixed order calculation

The acronym Next-to-Leading Order (NLO) stands for the exact calculation of the scattering amplitude at $\mathcal{O}(\alpha)$ w.r.t. the lowest order Feynman diagram (LO). Namely we have to consider **exactly** the emission of one (**real** or **virtual**) photon



and the total cross section will be the sum of $\sigma_{2\to n}$ and $\sigma_{2\to(n+1)}$ defined as

$$\sigma_{2\to n} = \frac{1}{\mathcal{F}}\left\{\int d\Phi_n |\mathcal{M}_{\text{LO}}|^2 + \int d\Phi_n\, 2\,\text{Re}\left(\mathcal{M}_{\text{LO}}^\dagger \mathcal{M}_V\right)\right\}$$

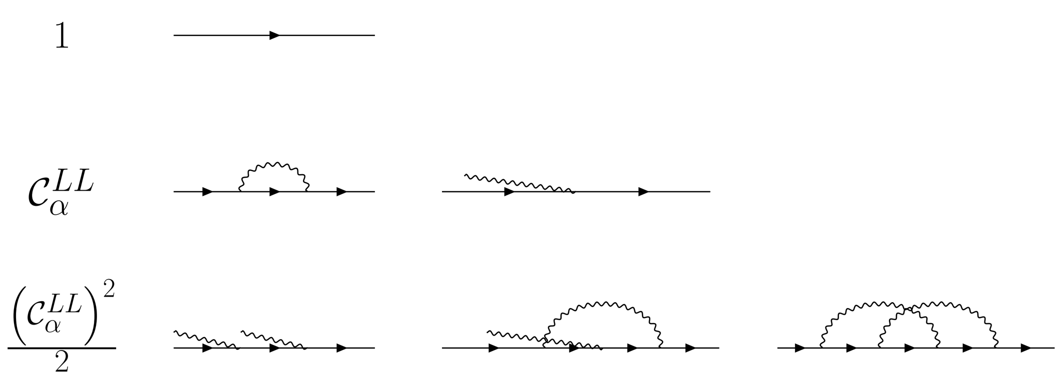
$$\sigma_{2\to(n+1)} = \frac{1}{\mathcal{F}}\left\{\int_{\lambda\le\omega\le\Delta E} d\Phi_{(n+1)}|\mathcal{M}_{(n+1)}|^2 + \int_{\omega>\Delta E} d\Phi_{(n+1)}|\mathcal{M}_{(n+1)}|^2\right\}.$$

Resummation techniques

In a realistic process, multiple photon emission can happen. This can be taken into account by resummation techniques, like the PS. We define the Sudakov Form Factor, that represent the probability of non-emission, namely to emit photon with energy smaller then $\varepsilon\sqrt{s}/2$

$$\Pi(\varepsilon, Q^2) = \exp\left\{\frac{\alpha}{2\pi}\int_0^{1-\varepsilon} dz\, P(z)\int d\Omega_k \sum_{i,j}\eta_i\eta_j \frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}k_0^2\right\} \doteq \exp\{\mathcal{C}_\alpha^{LL}\}$$

where $P(z)$ is the Altarelli-Parisi splitting function. The latter takes into account **approximately** Soft-Virtual emissions to all orders, namely



Matching PS and NLO

In order to achieve exact $\mathcal{O}(\alpha)$ accuracy and consistently exploit the resummation provided by the PS, the master formula is given by

NLO⊕PS accuracy

$$d\sigma_{\text{NLOPS}} = \Pi(\varepsilon, Q^2)\sum_{n=i}^{\infty} F_{SV}\frac{1}{n!}|\mathcal{M}_n^{\text{PS}}|^2 d\Phi_n$$

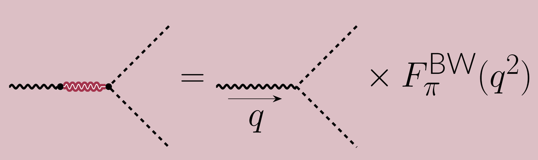
Energy scan

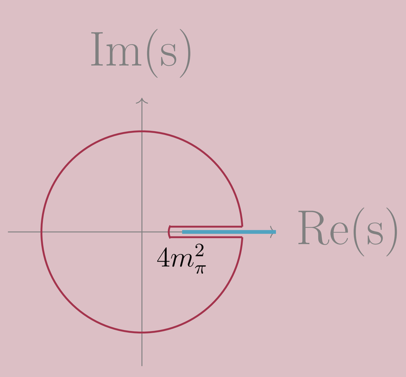
The **energy scan** method determines the pion form factor by varying the accelerator's center-of-mass energy. By measuring the $e^+e^- \to \pi^+\pi^-$ cross section as a function of $\sqrt{s'}$, $F_\pi(s')$ can be directly extracted.

Inclusion of the Pion Form Factor

The main technical challenge lies in integrating the pion form factor over the loop momentum in the calculation of the NLO amplitude. The latter is implemented in two different ways

GVMD	FsQED
$F_\pi^{\text{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\text{BW}}(q^2)$	$F_\pi(q^2) = 1 - \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\text{Im}\,F_\pi(s')}{q^2-s'}$
$F_{\pi,v}^{\text{BW}}(q^2) = \frac{c_v}{c_t}\frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$	$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\text{Im}\,F_\pi(s') = 1$





arXiv:2409.03469

arXiv:2601.19530

Energy scan

Numerical results

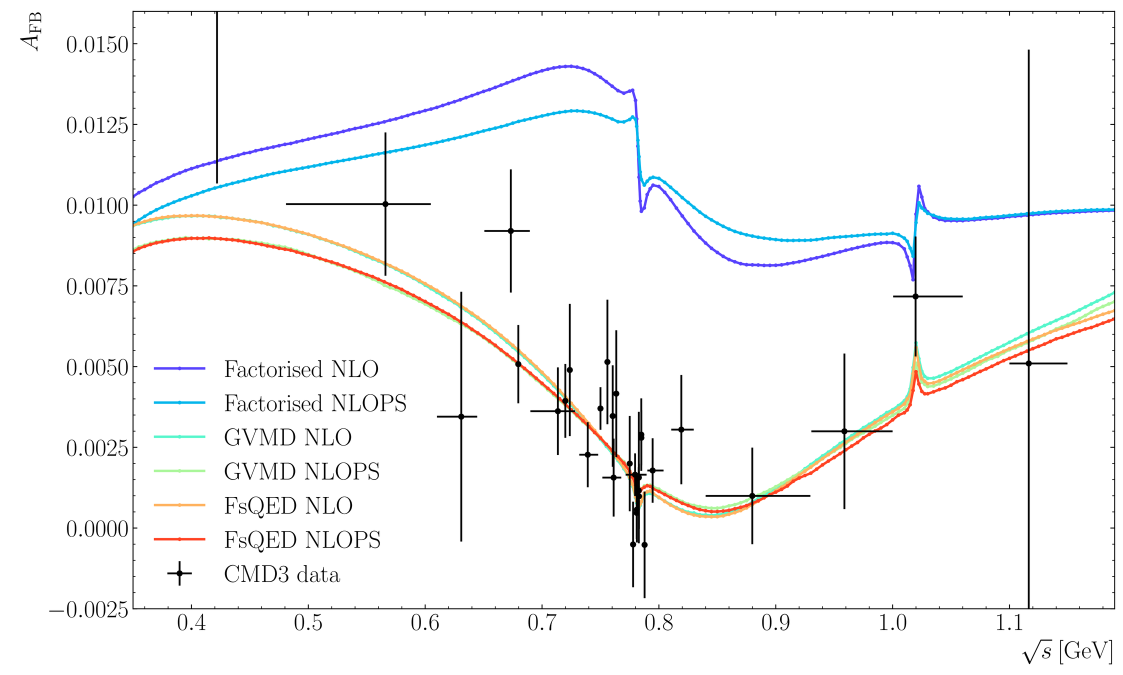
To obtain realistic numerical results, we apply event selection criteria inspired by the kinematic cuts used in CMD-3.

$$p^\pm \equiv |\boldsymbol{p}^\pm| > 0.45E\,, \qquad \vartheta_{\text{avg}} \equiv \tfrac{1}{2}(\pi - \vartheta^+ + \vartheta^-) \in [1, \pi-1]\,,$$

$$\delta\vartheta \equiv |\vartheta^+ + \vartheta^- - \pi| < 0.25 \qquad \delta\phi \equiv \left||\phi^+ - \phi^-| - \pi\right| < 0.15$$

The charge asymmetry serves as a test of the approach for describing F_π and is defined as

$$A_{\text{FB}}\left(\sqrt{s}\right) = \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}}$$



Radiative return

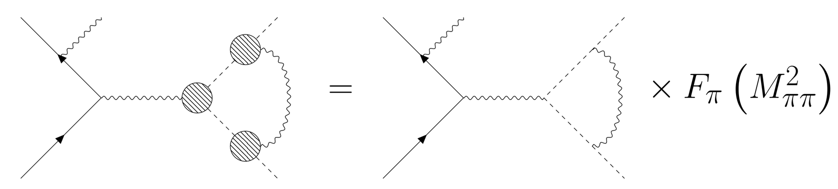
At fixed collider energy $\sqrt{s}$, lower-energy pion pairs can be studied via initial-state radiation:

$$e^+e^- \to \pi^+\pi^-\gamma, \qquad M_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2 < s$$

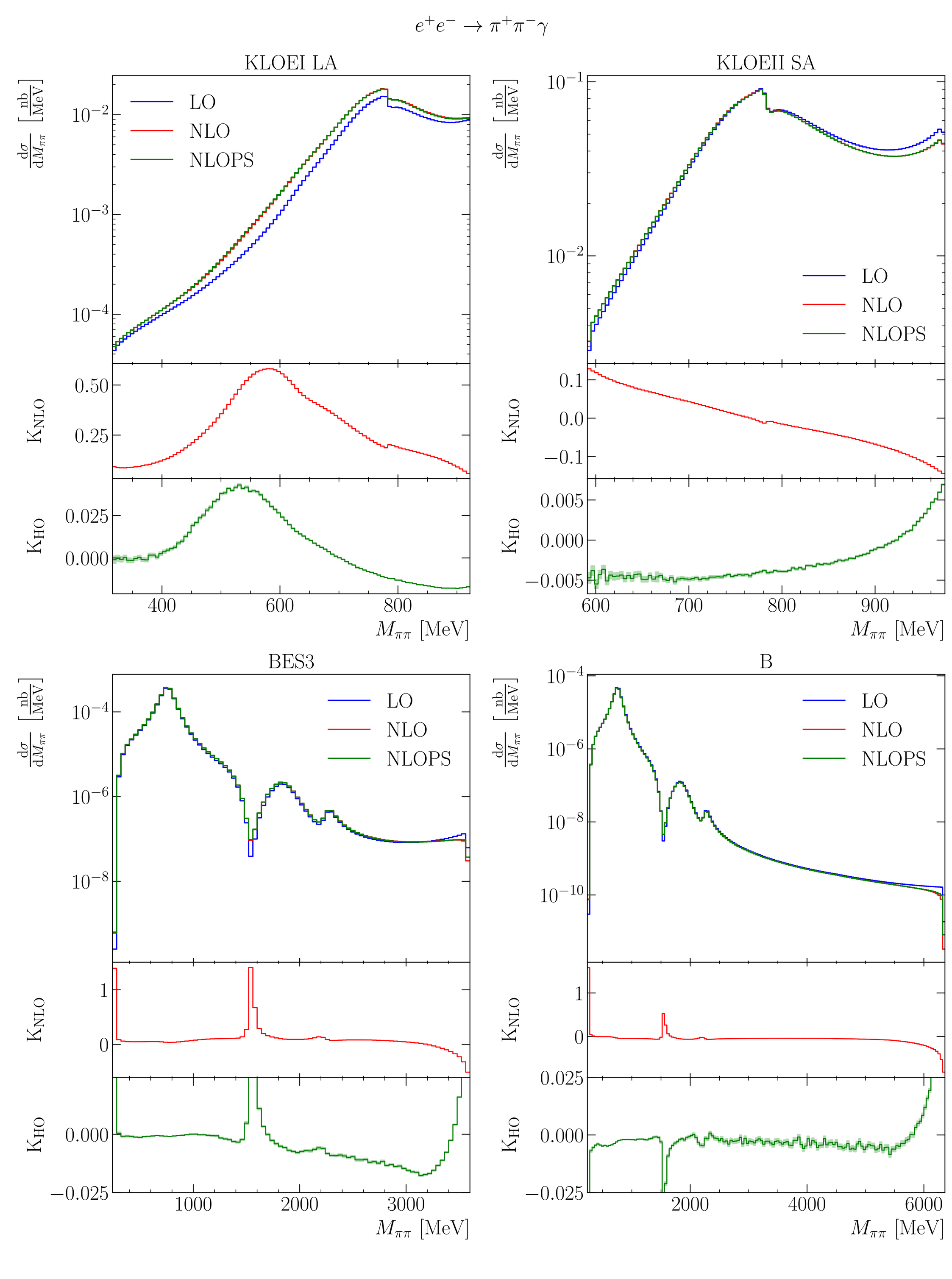
The measured differential cross section relates to the pion production cross section:

$$\frac{d\sigma_{\pi^+\pi^-\gamma}}{dM_{\pi\pi}} = H(s, M_{\pi\pi}^2)\,\sigma_{\pi^+\pi^-}(M_{\pi\pi}^2),$$

where $H(s, M_{\pi\pi}^2)$ is the calculable radiator function.
The pion form factor is included in the virtual calculation as follows



Numerical results for the $M_{\pi\pi}$ differential distribution are given in the following figure in four different experimental scenarios with different levels of accuracy.



Existing Monte Carlo generators for radiative return, such as Phokhara, are limited to NLO precision; to fill this gap, we have implemented radiative processes at NLOPS accuracy in BabaYaga@NLO, improving the description of multiple photon radiation effects.