The impact of quantum signatures in Quantum Thermodynamics close-to-equilibrium

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1. Quantum fluctuating work

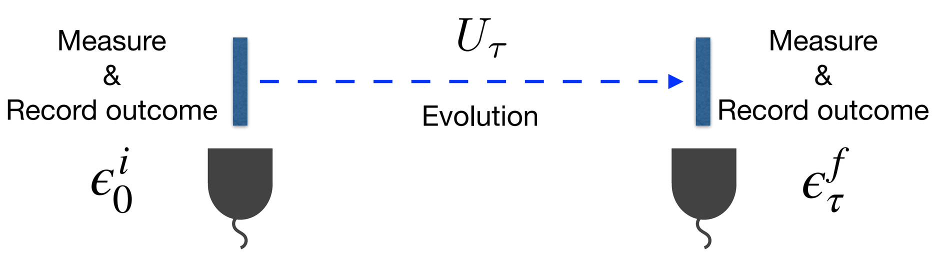
- Work is done when driving a system out of equilibrium via a time-dependent protocol $H_t = \sum_{\epsilon_t} \epsilon_t |\epsilon_t\rangle\langle\epsilon_t|$

$\pi_0 := e^{-\beta H_0}/\mathcal{Z}_0$ — Initial thermal state

$U_\tau = \overleftarrow{\mathcal{T}}\exp(i/\hbar \int_0^\tau dt' H_{t'})$ — Closed=Unitary evolution

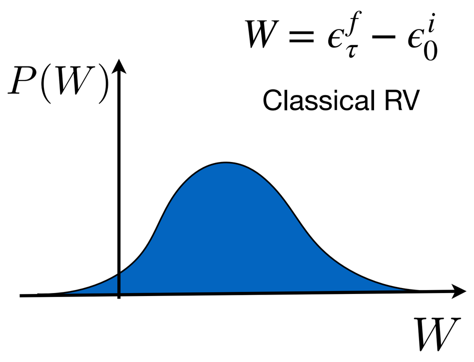
$\rho_\tau = U_\tau \pi_0 U_\tau^\dagger$ — System state after driving

- Work is defined as the difference between the outcomes of two energy projective measurements



$W = \epsilon_\tau^f - \epsilon_0^i$

Classical RV



$$p(W) = \sum_{\epsilon_0^i, \epsilon_\tau^f} \delta\left[W - (\epsilon_\tau^f - \epsilon_0^i)\right] p(\epsilon_\tau^f, \epsilon_0^i)$$

Joint probability for obtaining the two outcomes from Born's rule

- **CUMULANT GENERATING FUNCTION** $K_W(\eta) = \ln\langle e^{-\eta W}\rangle = \ln \int dW\, e^{-\eta W} p(W)$ $\qquad \kappa_W^n = \partial^n K_W(\eta)/\partial\eta^n|_{\eta=0}$

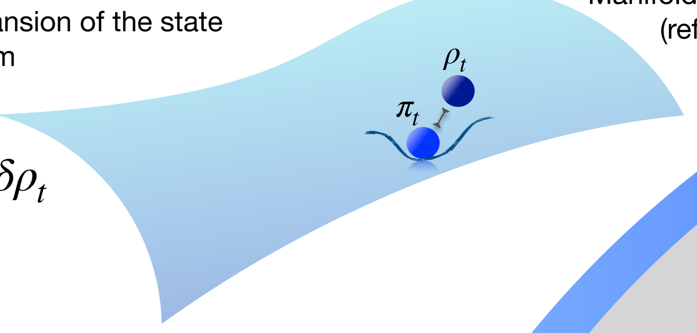
2. Close to equilibrium processes

Perturbative expansion of the state around equilibrium

$\rho_t \simeq \pi_t + \epsilon\, \delta\rho_t$

$\epsilon \ll 1$

Manifold of equilibrium (reference) states



- **SLOW-DRIVING (quasi-adiabatic) REGIME**

ϵ is the ratio between thermalization timescale and driving speed; typically assumes Lindblad

- **LINEAR RESPONSE REGIME**

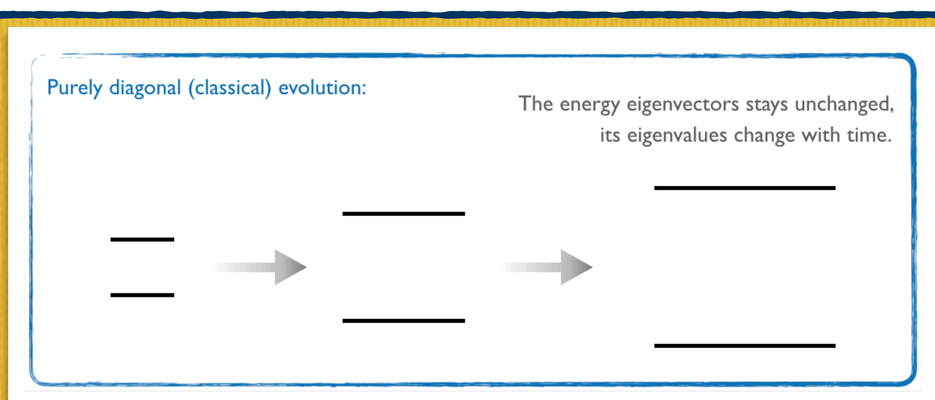
ϵ is the amplitude of the driving

3. What is genuinely "quantum"?

- Time-dependent Hamiltonian through external work protocol $H_t = \sum \epsilon_t |\epsilon_t\rangle\langle\epsilon_t|$

$[\dot{H}_t, H_t] = 0$

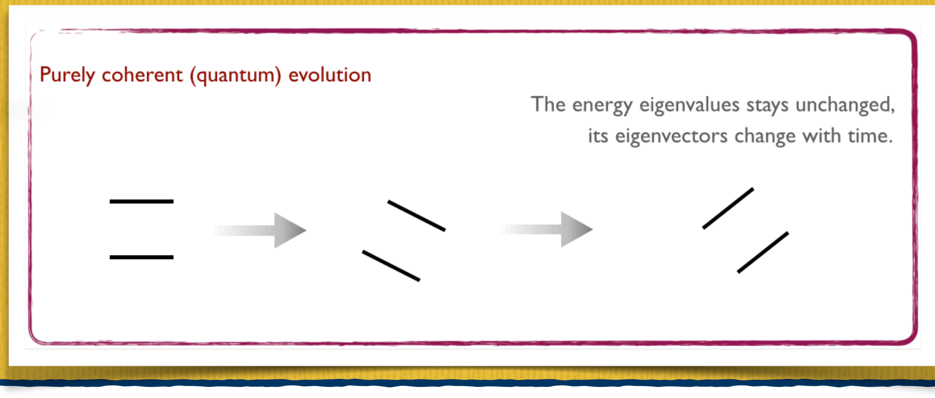
Energy fluctuations='thermal'



QUANTUM FRICTION

$[\dot{H}_t, H_t] \neq 0$

Energy fluctuations='thermal' +'coherent'



How does Quantum Friction affect the work statistics?

Work statistics close-to-equilibrium

FCS version of Landauer's principle

$$K(\eta) = \ln\langle e^{-\eta\beta W_{\rm diss}}\rangle = (\eta - 1) S_\eta(\pi_t || \rho_t)$$

G.G., et al. **PRE** 99, 050101(R)

Dissipated work quantifies irreversibility

$W_{\rm diss} \equiv W - \Delta F$

α-Renyi divergence quantifies distinguishability

$S_\alpha(\rho||\sigma) := (\alpha - 1)^{-1} \ln \mathrm{Tr}\,(\rho^\alpha \sigma^{1-\alpha})$

For close-to-equilibrium processes...

Geometric optimisation through geodesic equation

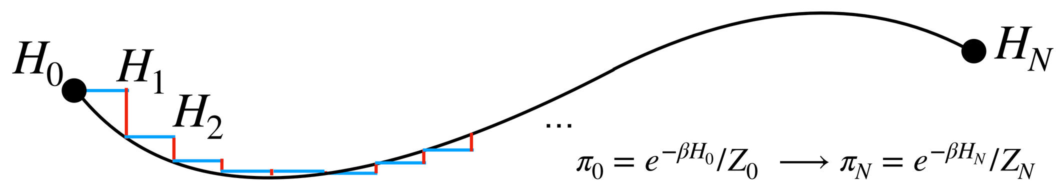
$$S_\eta(\pi_t + \epsilon\,\delta\rho_t || \pi_t) = S_\eta(\pi_t || \pi_t + \epsilon\,\delta\rho_t)$$

...which induces a metric

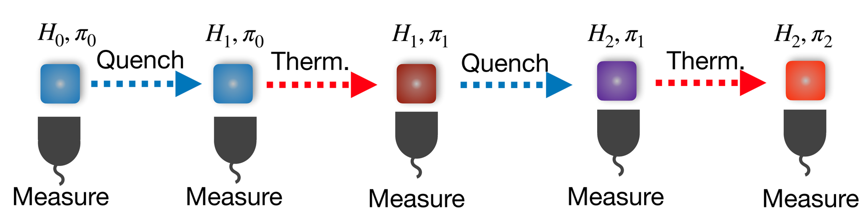
MAIN TAKE-HOME MESSAGE

For close-to-equilibrium processes, quantum friction results in positive and experimentally measurable corrections to all statistical cumulants of the work distribution

4. Slow - driving regime results



Alternating sequence of N quenches and thermalization steps



$$K(\eta) = -\frac{\beta^2}{2N}\int_0^1 dt \int_0^\eta dx \int_x^{1-x} dy\, \mathrm{Cov}^y_{\pi_t}(\dot{H}_t, \dot{H}_t)$$

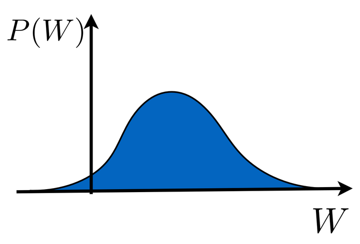
$$\mathrm{Cov}^y_\rho(A, B) \equiv \mathrm{Tr}\left[\left(A - \mathrm{Tr}\left[A\rho\right]\right)\rho^y \left(B - \mathrm{Tr}\left[B\rho\right]\right)\rho^{1-y}\right]$$

- **For 'classical' processes (no quantum friction)**

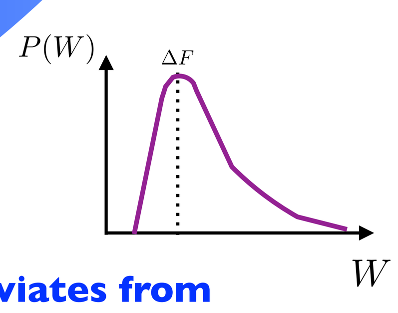
$$K(\eta) = \frac{\beta^2(\eta^2 - \eta)}{2N}\int_0^1 dt\, \mathrm{Var}(\dot{H}_t) \equiv K^{(cl)}(\eta)$$

Then the diss. work distribution is Gaussian

$$P(W_{\rm diss}) \propto \exp\left(-\beta\frac{(W_{\rm diss} - \langle W_{\rm diss}\rangle)^2}{4\langle W_{\rm diss}\rangle}\right)$$



- **For 'quantum' processes (quantum friction)**



$$K(\eta) = K^{(cl)}(\eta) + \frac{\beta^2}{2N}\int_0^1 dt \int_0^\eta dx \int_x^{1-x} dy\, I^y(\pi_t, \dot{H}_t)$$

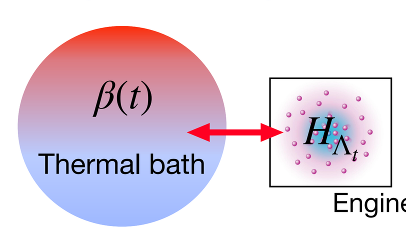
"Quantum - Stochastic" family of second laws: $\kappa_W^{(k)} \geq 0\ \forall k$

Work statistics distribution deviates from a Gaussian and becomes right-skewed!

M. Scandi, et al. **PRR** 2, 023377; H. Miller, G.G. et al, **PRL** 125, 160602

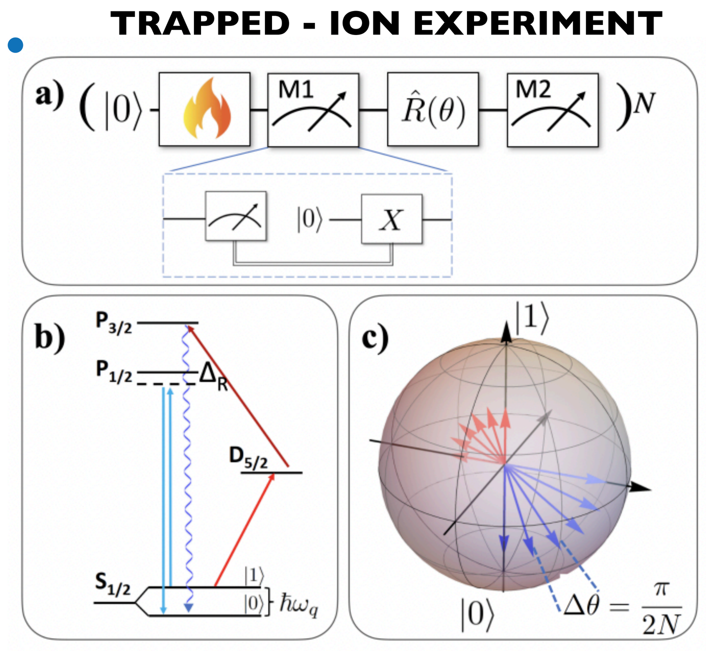
- **Quantum Thermodynamic-Uncertainty Relations**

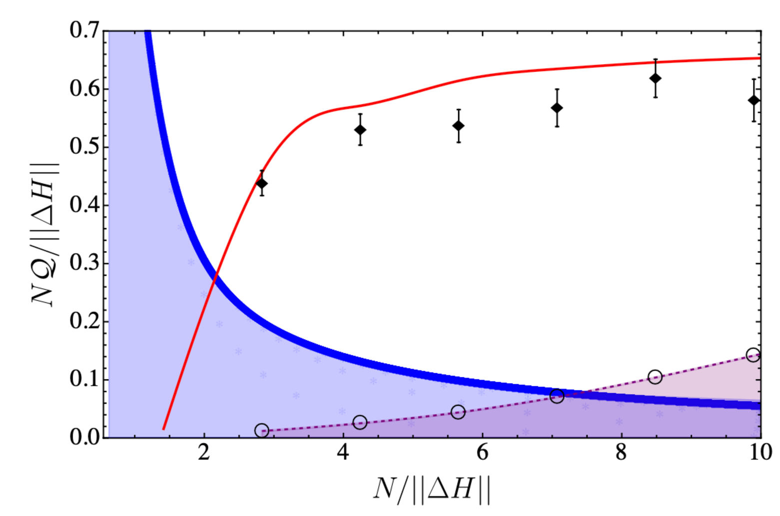
$$\eta \leq \frac{\eta_C}{1 - 2\theta_W T_c \left[P/(\mathrm{Var}(P) - \mathcal{Q})\right]}$$



H. Miller, …, G.G. **PRL** 126, 210603 ; **PRE** 103, 052138

- **TRAPPED - ION EXPERIMENT**

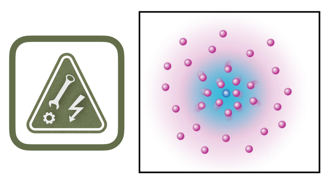




$$\langle W_{\rm diss}\rangle = \frac{\beta}{2}\left[\mathrm{Var}(W) - \mathcal{Q}\right] \qquad \mathcal{Q} = \frac{1}{N}\int_0^1 dt \int_0^1 dy\, I^y(\pi_t, \dot{H}_t) \geq 0$$

O. Onishchenko & G. G., et al. **Nat. Comm.** 15 (1), 6974

5. Linear Response Regime



$H(t) = H_0 + \epsilon\lambda_t V$

- Two equivalent expressions for the CGF

$$K(\eta) = -\beta^2 \int_0^\tau dt \int_0^t dt'\, \dot{\lambda}_t \dot{\lambda}_{t'} \int_0^\eta dx \int_x^{1-x} dy\, \mathrm{Cov}^y_{\pi_0}(V(t), V(t'))$$

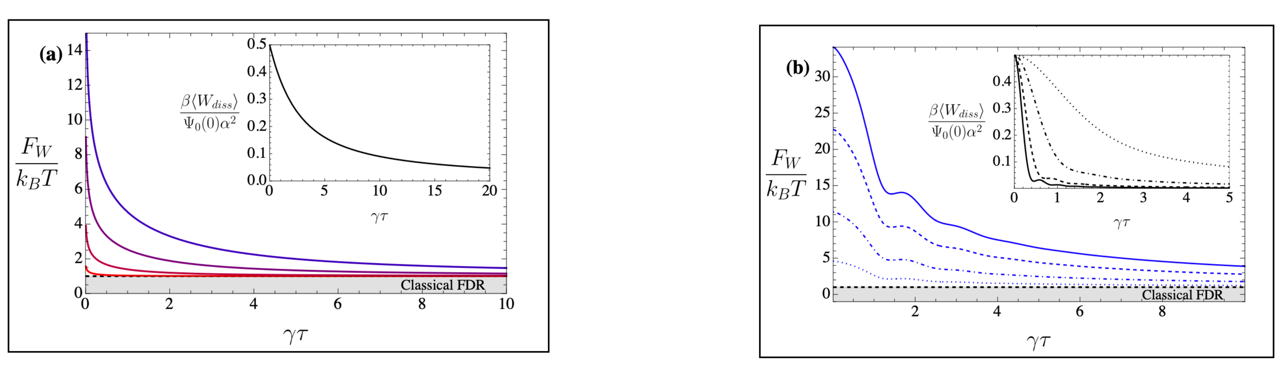
$$K(\eta) = -\int_0^\tau dt \int_0^\tau dt'\, \dot{\lambda}_t \dot{\lambda}_{t'} \left[g_\eta * \Psi_0\right](t - t')$$

Universal geometric factor, independent of specific process

Kubo's relaxation function, contains all the microscopic details

$$\Psi_0(t) := \beta\int_0^\beta ds\, \langle V(-i\hbar s)V(t)\rangle_0 - \beta^2\langle V\rangle_0^2$$

The knowledge of Ψ_0 allows to access the corrections to all cumulants!



Overdamped model: $\Psi_0(t) := \Psi_0(0)e^{-\gamma|t|}$

Dispersion monotonically decays to classical prediction in long time limit

Underdamped model: $\Psi_0(t) := \Psi_0(0)e^{-\gamma|t|}\left(\cos(\nu t) + (\gamma/\nu)\sin(\nu|t|)\right)$

Non-monotonic decay due to memory effects

G.G. , J. Eisert, H. Miller , **PRL** 133, 070405

References

G.G., et al. **PRE** 99, 050101(R)

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H. Miller, …, G.G. **PRL** 126, 210603 ; **PRE** 103, 052138

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