A Local Measurement Theory for Quantum Cellular Automata

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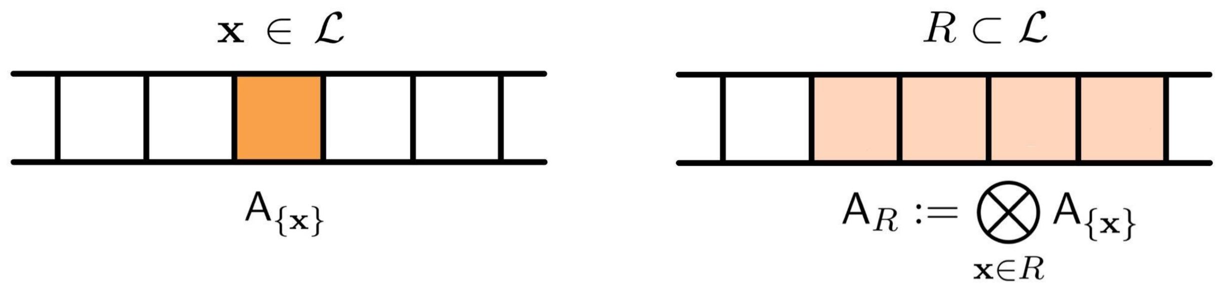
Why Quantum Cellular Automata?

A *Quantum Cellular Automaton* (QCA) is the most general **discrete–time, reversible, locality–preserving** quantum dynamics on a lattice of finite–dimensional quantum systems.



Algebras of **Local Observables**
$\mathsf{A}_{\{x\}} \simeq \mathsf{M}(d,\mathbb{C})$

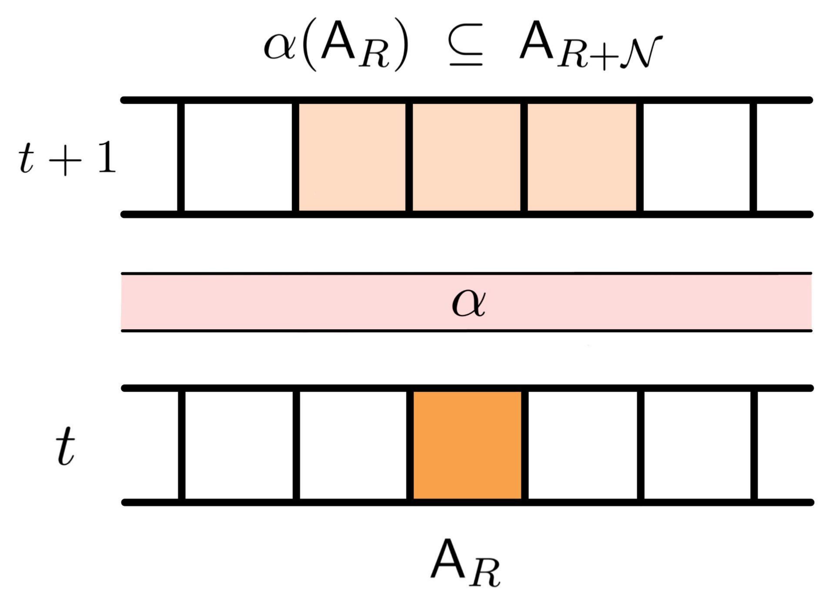
Mathematically,

Neighbourhood Scheme

$$\alpha(\mathsf{A}_R) = U^{\dagger}_{R+\mathcal{N}} \left(\mathsf{A}_R \otimes \mathbf{I}_{(R+\mathcal{N})\setminus R} \right) U_{R+\mathcal{N}} \subset \mathsf{A}_{R+\mathcal{N}}$$

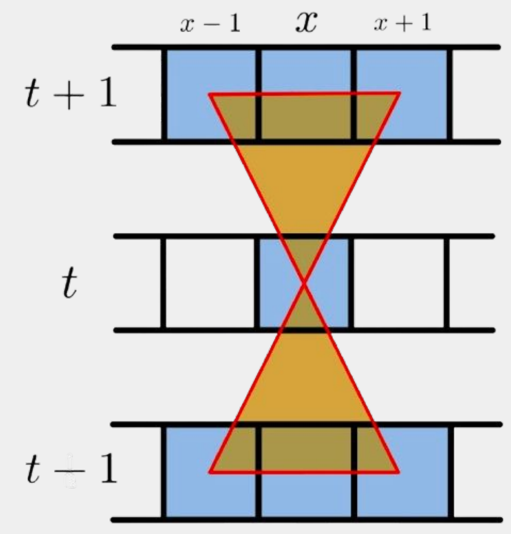
for some unitary $U_{R+\mathcal{N}} \in \mathsf{A}_{R+\mathcal{N}}$.

Locality is operational: information propagates at finite speed, fixed by the neighbourhood scheme.



Cellular Automata in Operational Probabilistic Theories, Paolo Perinotti (2021)
A Review of Quantum Cellular Automata, Terry Farrelly (2020)

The Causal Structure of QCAs



Locality implies a microscopic, discrete notion of causality:

$$\alpha^t(\mathsf{A}_R) \subset \mathsf{A}_{R+t\mathcal{N}}$$

The causal structure of a QCA is **intrinsic to the dynamics**.

QCAs provide a mathematically consistent framework for relativistic quantum dynamics on a *discrete spacetime*.

The Bell Protocol in One Dimensional Qubit QCAs

We consider a chain of qubits labelled by $\mathbb{Z}$. The local algebras are $\mathsf{A}_{\{x\}} \simeq \mathsf{M}(2,\mathbb{C})$.

The nearest–neighbour QCA dynamics is implemented by alternating layers of

- *controlled–phase gates*, generating correlations across adjacent cells:

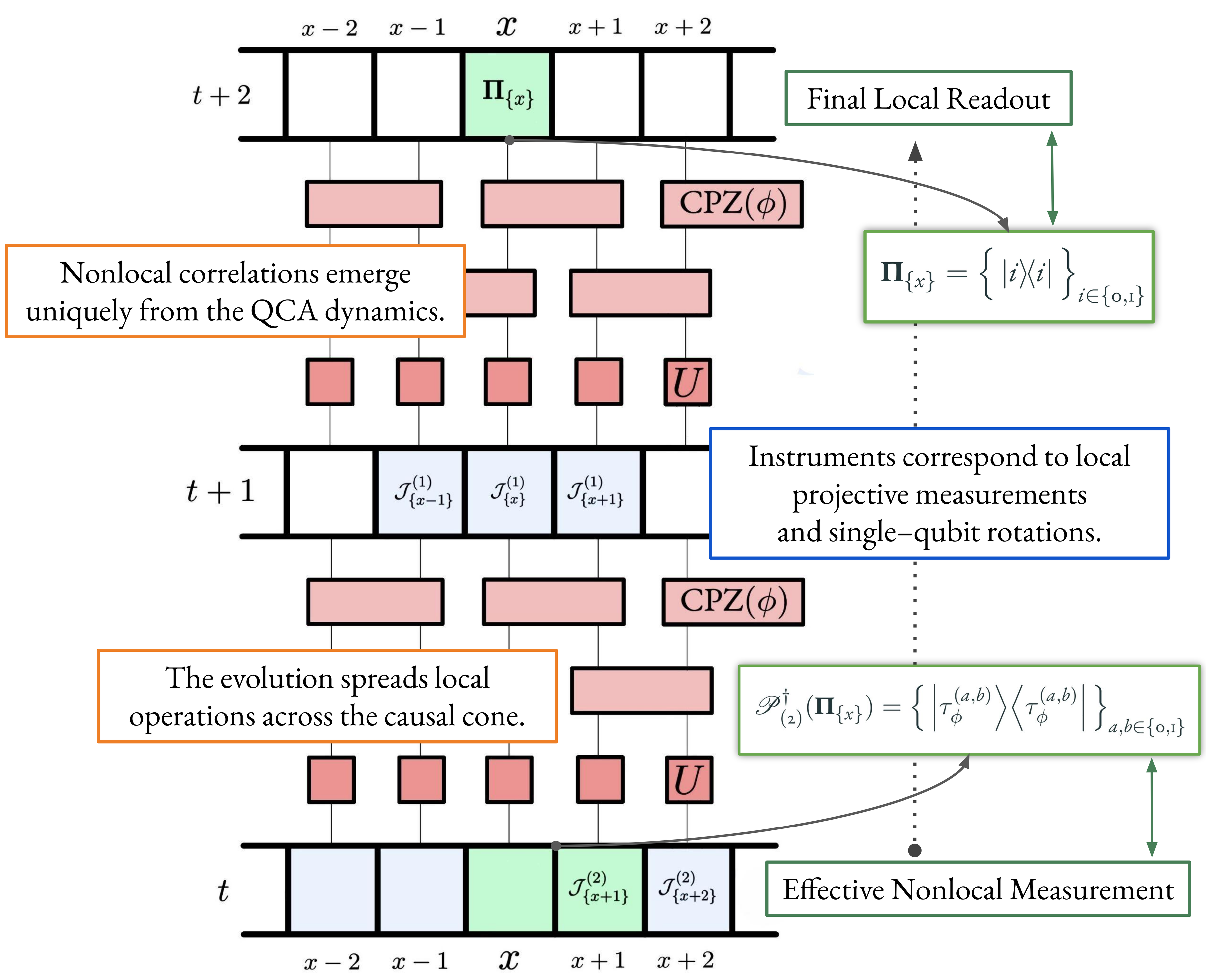
$$\mathrm{CPZ}(\phi)_{\{x-1,x\}} = \sum_{k\in\{0,1\}} |k\rangle\langle k|_{\{x-1\}} \otimes R_Z(k\phi)_{\{x\}};$$

- and *local unitaries* U.

We built a protocol that reconstructs the statistics of a **Bell measurement** without any nonlocal action.

A Bell measurement projects the joint system of two qubits onto the *Bell states*, a set of maximally entangled states. For this reason, a Bell measurement is **intrinsically nonlocal**.

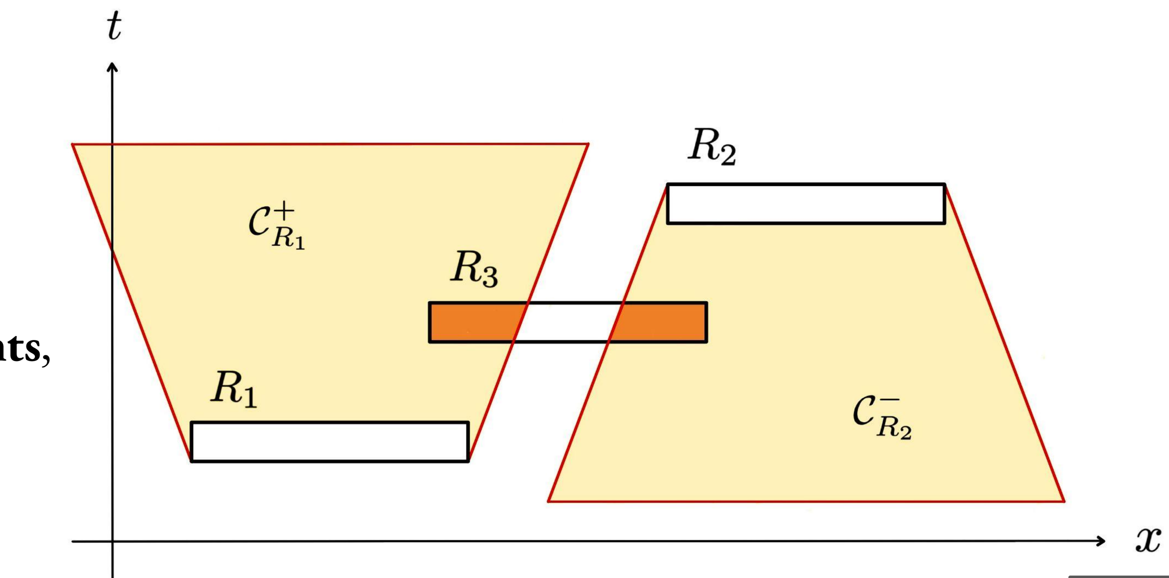
$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left[\, |00\rangle + |11\rangle \,\right]$$



The Problem of (Non)Local Measurements in QFT

In Quantum Field Theory, **microcausality** prevents superluminal signalling for strictly local measurements.

However, **nonlocal measurements**, when modelled as instantaneous projections, can violate causality, leading to signalling between spacelike–separated agents.



Impossible Measurements

Impossible Measurements on Quantum Fields, Rafael D. Sorkin (1993)

Toward a Measurement Theory in QFT [...], Nicolas Gisin, Flavio Del Santo (2023)

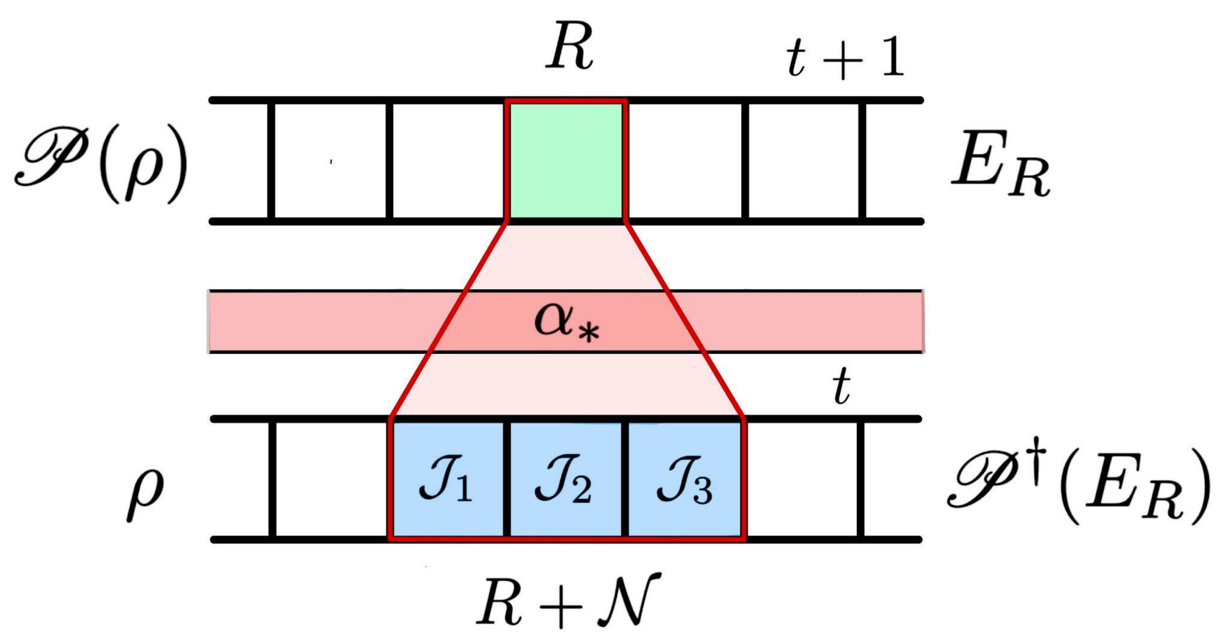
In relativistic settings, the notion of measurement cannot be defined independently of its *physical implementation*.

Nonlocal Measurements as Causal Protocols

Since quantum measurements on spatially distributed systems cannot, in general, be physically implemented as instantaneous global projections, we model them as causally compatible **operational protocols**.

A measurement protocol consists of a finite sequence of **local instruments**, interleaved with the QCA evolution.

$$\mathscr{P}(\rho) := \Big\{ \underbrace{\alpha_*}_{\text{QCA Evolution}} \circ \underbrace{\bigotimes_i \mathcal{J}_i}_{\text{Local Instruments}} \Big\}(\rho)$$



Since the protocols consist of strictly local operations, and correlations propagate only through the QCA dynamics, **compatibility with the causal structure is ensured by construction.**

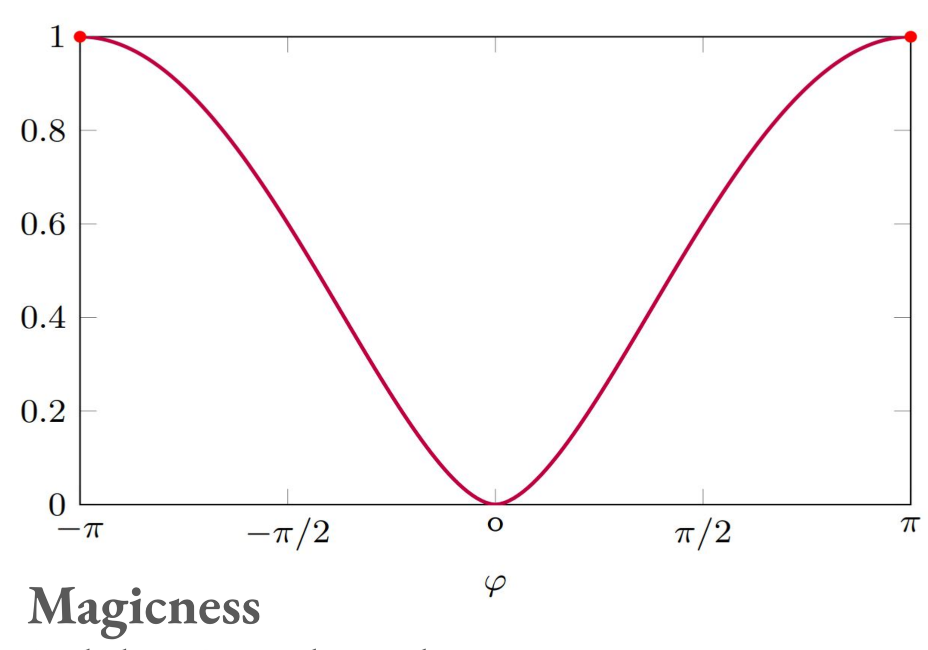
The statistics of nonlocal observables are reconstructed through a protocol of local instruments distributed in time and constrained by the causal dynamics.

$$\mathscr{P}(\rho)\Big(\underbrace{E_R}_{\text{Local}}\Big) = \rho\Big(\underbrace{\mathscr{P}^{\dagger}(E_R)}_{\text{Nonlocal}}\Big)$$

Quantum Measurements as Computational Resources

The measurement protocols induce effective nonlocal observables whose statistical properties encode relevant computational resources.

An Introduction to Measurement Based Quantum Computation, Richard Jozsa (2005)



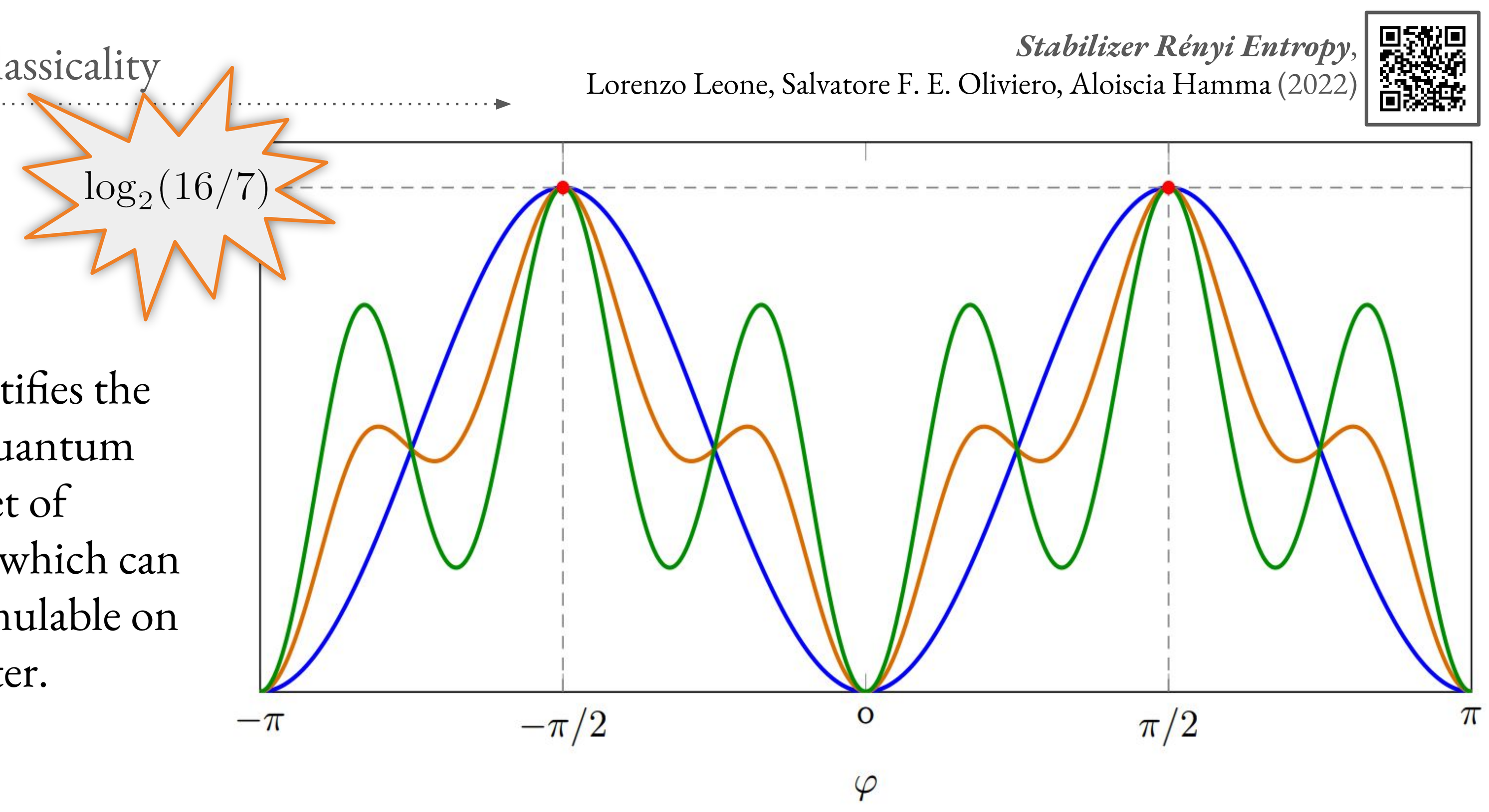
These properties depend on the structure of the protocols and on the parameters defining the underlying quantum dynamics.

This operational framework provides a controlled way to generate and quantify computational resources.

Entanglement Entropy
Nonclassical Correlations

Magicness
Stabilizer Nonclassicality

Stabilizer Rényi Entropy, Lorenzo Leone, Salvatore F. E. Oliviero, Alioscia Hamma (2022)



Magicness quantifies the deviation of a quantum state from the set of *stabilizer states*, which can be efficiently simulable on classical computer.