A Bethe ansatz approach to the Thirring Quantum Cellular Automaton

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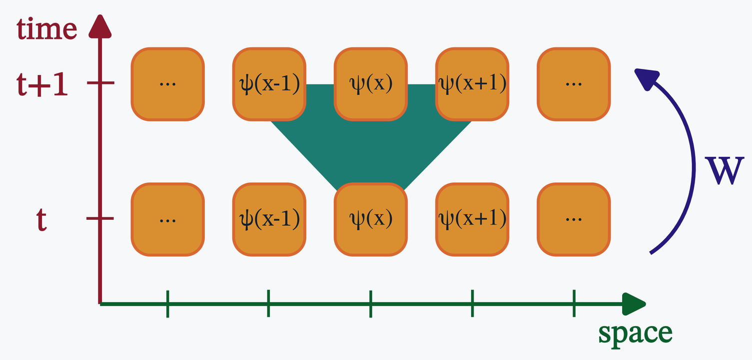
What is a Quantum Cellular Automaton?

QCA: most general
- unitary
- local
- time-discrete

dynamics of a lattice of quantum systems

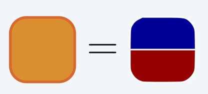


Why?

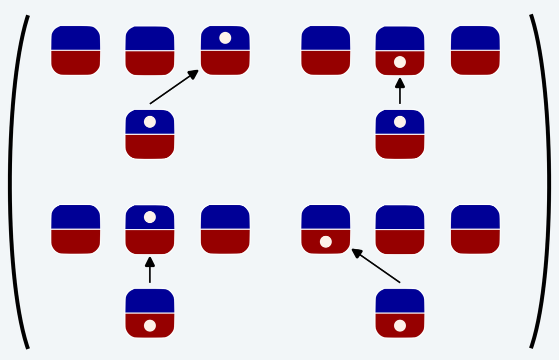
- Algorithms for simulating Quantum Physics;
- Quantum Theory from principles of information processing;
- Discrete Quantum Field Theory model?

Free Theory: Dirac QCA [1]

Assumptions:
- isotropy
- homogeneity
- minimal internal dimension of the cells



Update rule: either flip or shift



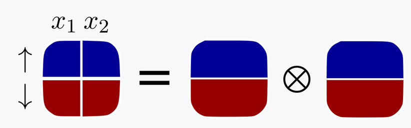
$$W = \begin{pmatrix} \nu T & -i\mu \\ -i\mu & \nu T^\dagger \end{pmatrix} \in \mathrm{Lin}[\mathbb{C}^2 \otimes l^2(\mathbb{Z})] \xrightarrow{\text{Fourier transform}} \tilde{W}(k) = \begin{pmatrix} \nu e^{ik} & -i\mu \\ -i\mu & \nu e^{-ik} \end{pmatrix}$$

In the low-mass (μ) and low-momentum (k) regime, it recovers the dynamics described by the Dirac equation of Quantum Field Theory.

Introducing interactions: Thirring QCA [2,3]

- QCA counterpart of Thirring model;
- most general on-site number preserving interaction.

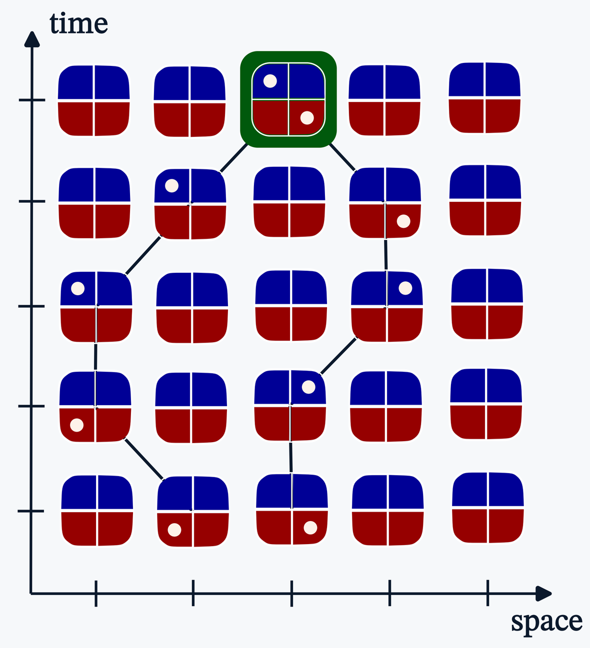
$$U = J(\chi)W$$



$$J(\chi) = \begin{cases} e^{i\chi} & \text{if } x_1 = x_2 \ , a_1 \neq a_2 \\ 1 & \text{else} \end{cases}$$

Whenever two particles occupy
- the same lattice site;
- with opposite internal degrees of freedom;

they interact.

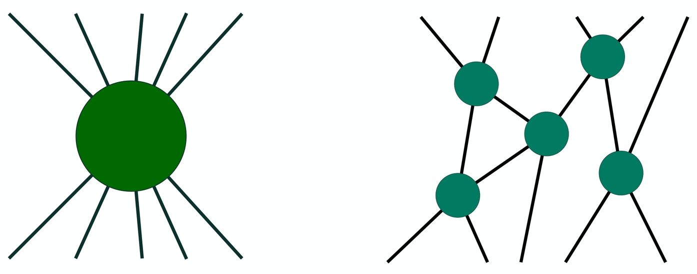


Bethe Ansatz solution

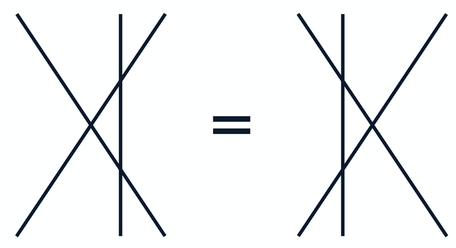
What is quantum integrability?

No formal definition, loosely speaking, we call a quantum system integrable if:

Many-body interactions = ∏ (Pairwise interactions)



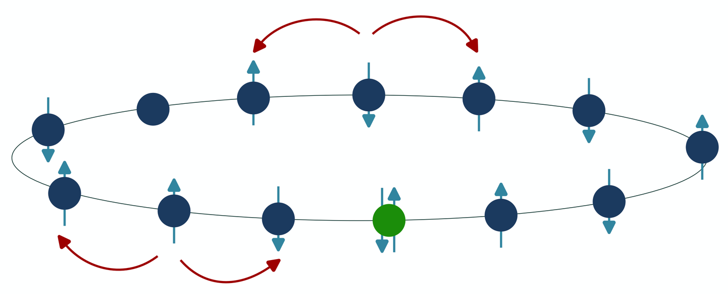
Consistency condition: Yang-Baxter equation



Traditional approach (e.g. Hubbard model)

Closed spin chain with Hamiltonian

H = Hopping term + Local interaction

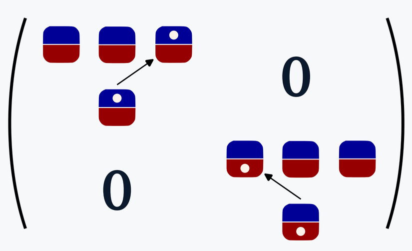


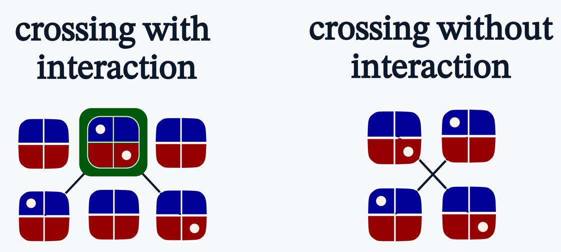
Approach: exploit symmetries to make an educated guess about the form of the wave function; then plug it into the Schrödinger equation to find its parameters.

Main differences of the QCA framework:
- time-discrete evolution;
- (quasi)-energy invariant mod 2π;
- spin-dependent dynamics;
- infinite open chain.

Aim: Find eigenstates of the interacting eigenvalue equation for the massless Thirring QCA.

$$J(\chi)W\,|\psi(\mathbf{k},\mathbf{a})\rangle = e^{-i\omega(\mathbf{k},\mathbf{a})}\,|\psi(\mathbf{k},\mathbf{a})\rangle$$



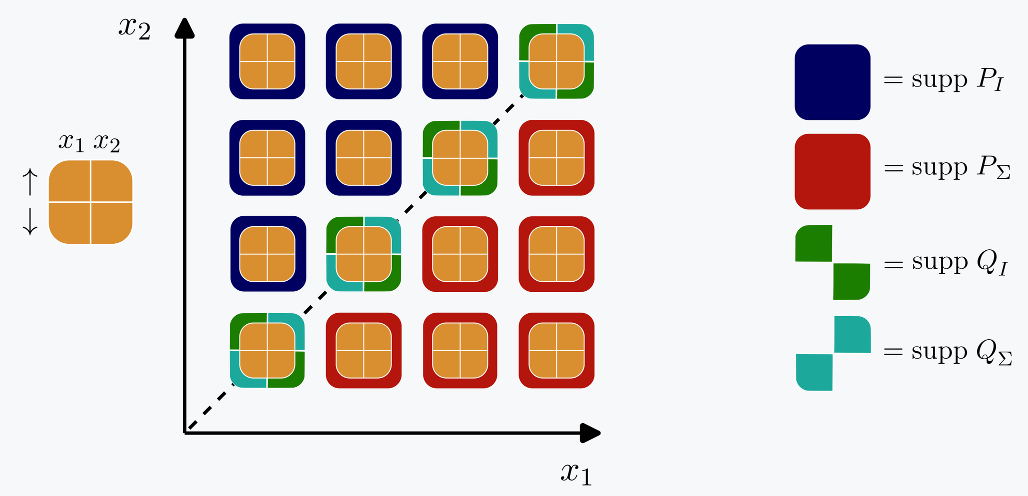


Ansatz: Anti-symmetric superposition of same-energy free eigenstates (i.e. plane waves).

$$|\psi(\mathbf{k},\mathbf{a})\rangle = |\psi\rangle_I + |\psi\rangle_\Sigma + |\psi\rangle_{\partial I} + |\psi\rangle_{\partial\Sigma}$$

$$|\psi(\mathbf{k},\mathbf{a})\rangle = \sum_{\sigma,\tau\in S_N}\left[P_{\sigma,\tau} + \sum_{\mathbf{v}\in B^N} Q_{\sigma,\tau}(\mathbf{v})\right]|\sigma\mathbf{k},\sigma\mathbf{a}\rangle$$

P and Q partition the coordinate space, based on the relative ordering of the particles.



Solutions: We map the solutions on a directed graph $\Gamma_\mathbf{a} := \{\Gamma_\mathbf{a}(\sigma,\tau)\}_{\sigma,\tau\in S_N}$.

Each sub-graph is associated with a particle ordering and a permutation of spins and momenta:

$$\Gamma_\mathbf{a}(\sigma,\tau) = (V,E) \qquad V := \{\text{particle orderings}\} \qquad E := \{\text{orderings connected by a single QCA step}\}$$

Edges encode relations between bulk and boundary coefficients:

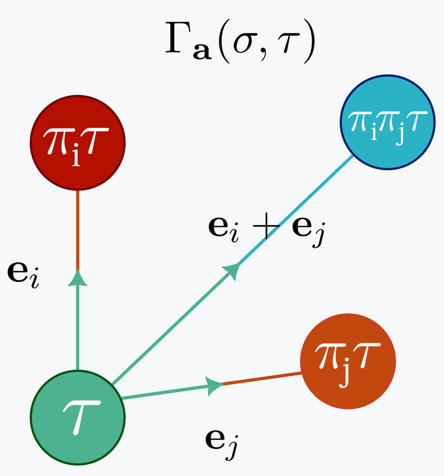
$$(\tau, \pi_{\mathbf{e}_i}\tau) \in E \quad\longleftrightarrow\quad \begin{cases} f_{\sigma,\pi_{\mathbf{e}_i}\tau}(\mathbf{x}) = \Omega_{\mathbf{e}_i}(\mathbf{x})\, f_{\sigma,\tau}(\mathbf{x}), \\ g^{\mathbf{e}_i}_{\sigma,\tau}(\mathbf{x}) = e^{-i|\mathbf{e}_i|^2\chi}\, f_{\sigma,\tau}(\mathbf{x}). \end{cases}$$

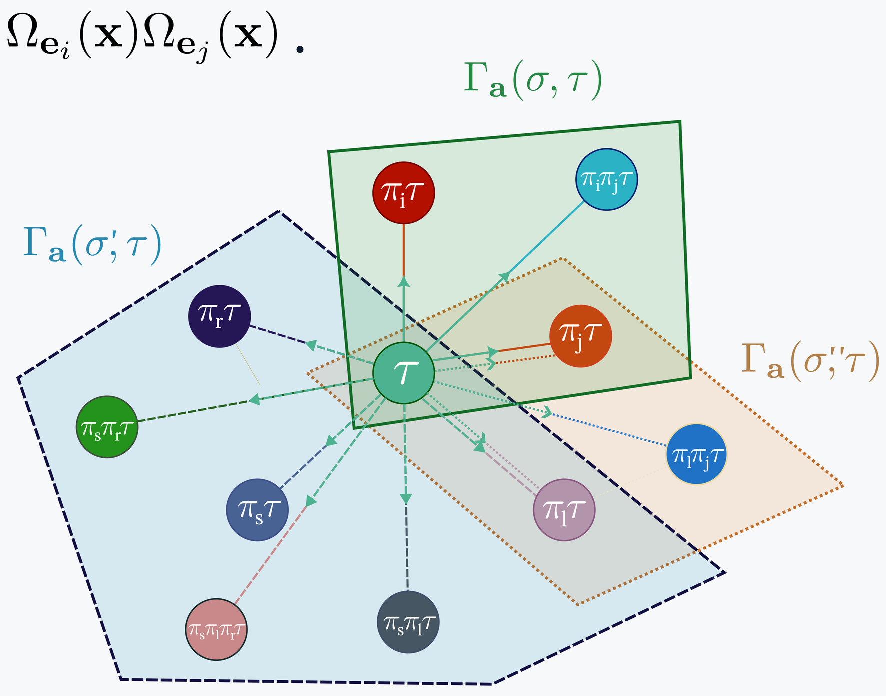
The composition rule

$$\mathbf{v} = \mathbf{e}_i + \mathbf{e}_j \ \longmapsto\ (\tau, \pi_\mathbf{v}\tau) = (\tau, \pi_{\mathbf{e}_i}\pi_{\mathbf{e}_j}\tau) \ ,$$

guarantees that multiple interactions factorise as:

$$\Omega_\mathbf{v}(\mathbf{x}) = \Omega_{\mathbf{e}_i}(\mathbf{x})\Omega_{\mathbf{e}_j}(\mathbf{x}) \ .$$





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[1] [2] [3]