Classification of qubit cellular automata on hypercubic lattices

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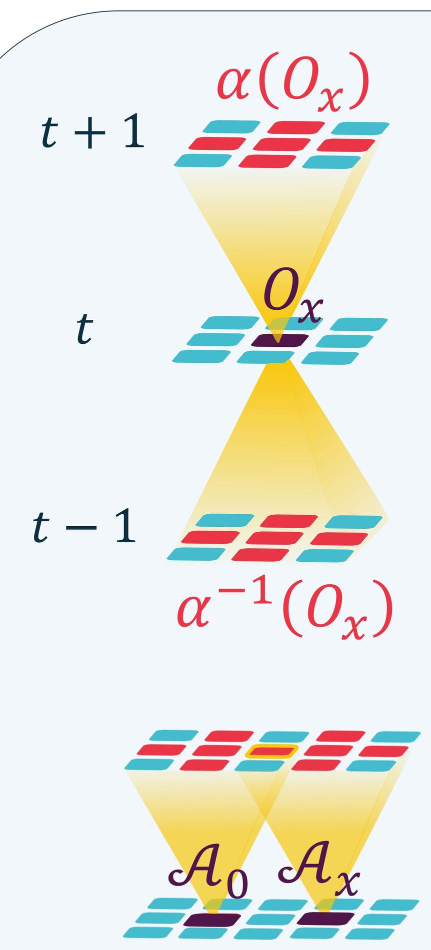
FRAMEWORK: minimal QCA

Discrete-time, local, unitary, translation-invariant dynamics of qubits on $\mathbb{Z}^d$ lattices with nearest neighbor interaction.

Sandbox for condensed matter, QFTs, and simulations.



A Quantum Cellular Automaton (QCA) $\alpha : \mathcal{A}_{\mathbb{Z}^d} \to \mathcal{A}_{\mathbb{Z}^d}$ with (nearest) neighborhood $\mathcal{N}$ is an automorphism of a quasi-local operator algebra $\mathcal{A}_{\mathbb{Z}^d} := \overline{\bigcup_{\Lambda \subset \mathbb{Z}^d} \mathcal{A}_\Lambda}^{\|\cdot\|_\infty}$ of qubits $\mathcal{A}_x = \mathcal{L}(\mathbb{C}^2)$, $\mathcal{A}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{A}_x$, such that:

- $O_x \in \mathcal{A}_x \Rightarrow \alpha(O_x) \in \mathcal{A}_{x+\mathcal{N}}$ (locality)
- $[\alpha, \tau^y] = 0 \;\; \forall$shift τ^y by $y \in \mathbb{Z}^d$ (translation invariance)

The Local Rules of α consist of unitary operators supported on the neighborhood of a cell s.t. $\alpha(O_x) = U^\dagger_{x+\mathcal{N}}(O_x \otimes I)\, U_{x+\mathcal{N}}$.

U provides a local rule of α iff $[\alpha(\mathcal{A}_0), \alpha(\mathcal{A}_x)] = 0$ for $0 \neq x \in \mathbb{Z}^d$.

Overlaps constrain the rules.

RESULTS: classification beyond 1d

✓ Classification of Local Rules.

✓ Classification via local invariants (Index): equivalence classes of QCA modulo Finite-Depth Quantum Circuits (FDQCs).

✓ Study of entanglement production via (classical) simulation.

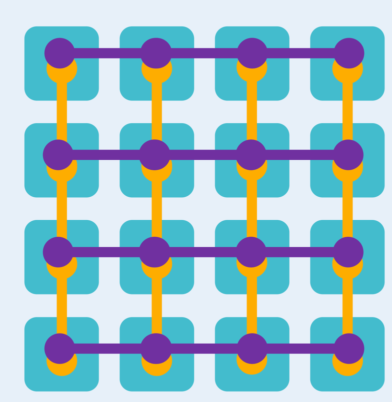
Results hold in any dimension $\mathbb{Z}^d$.

Interest

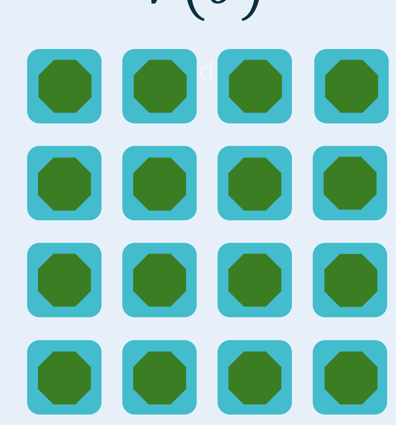
- Dynamics with no Hamiltonian
- Topological phases of matter
- Cluster(graph) states and one-way quantum computing

Local Rules

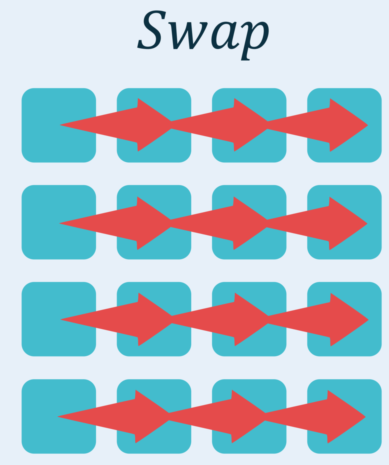
Multiply Controlled-Phase $CP(\vec{\varphi})$



Site-wise unitary $V(\vec{\theta})$

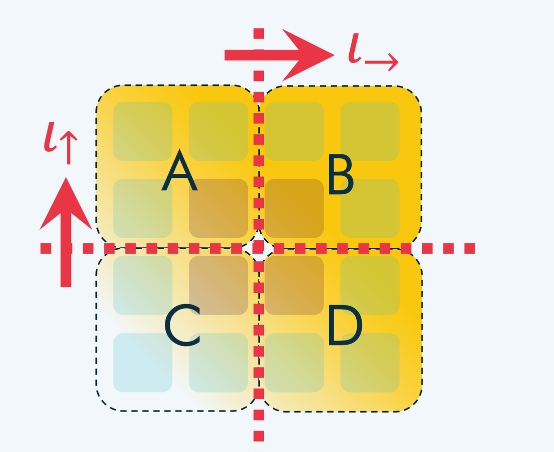


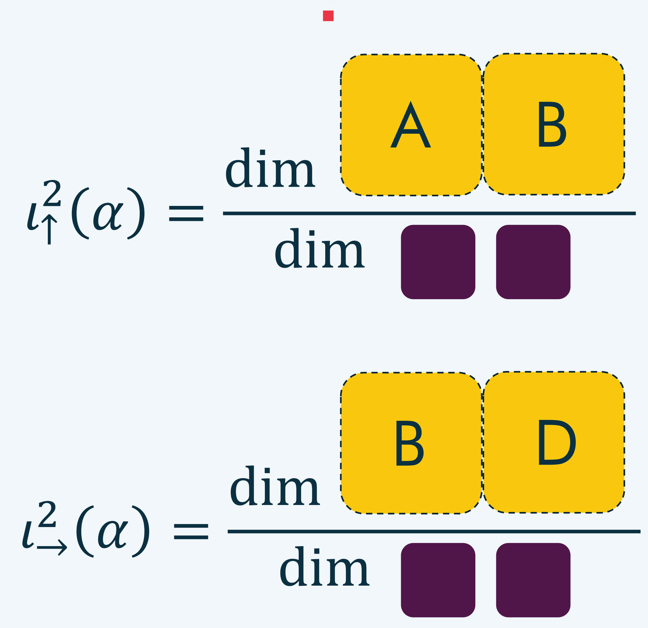
Shift *Swap*



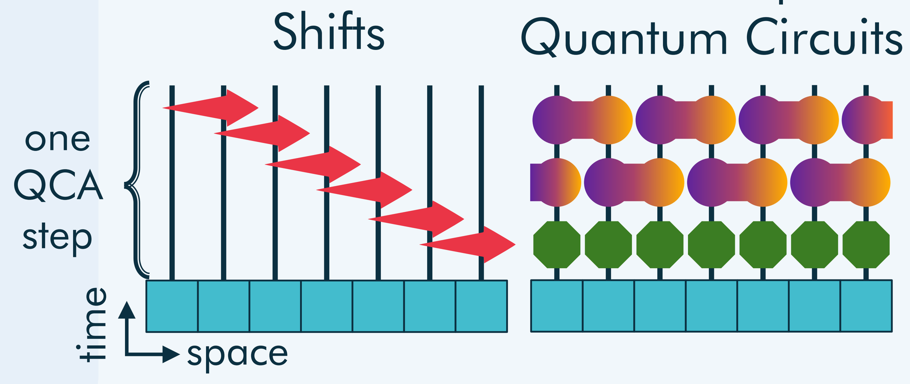
Index

Information Flux



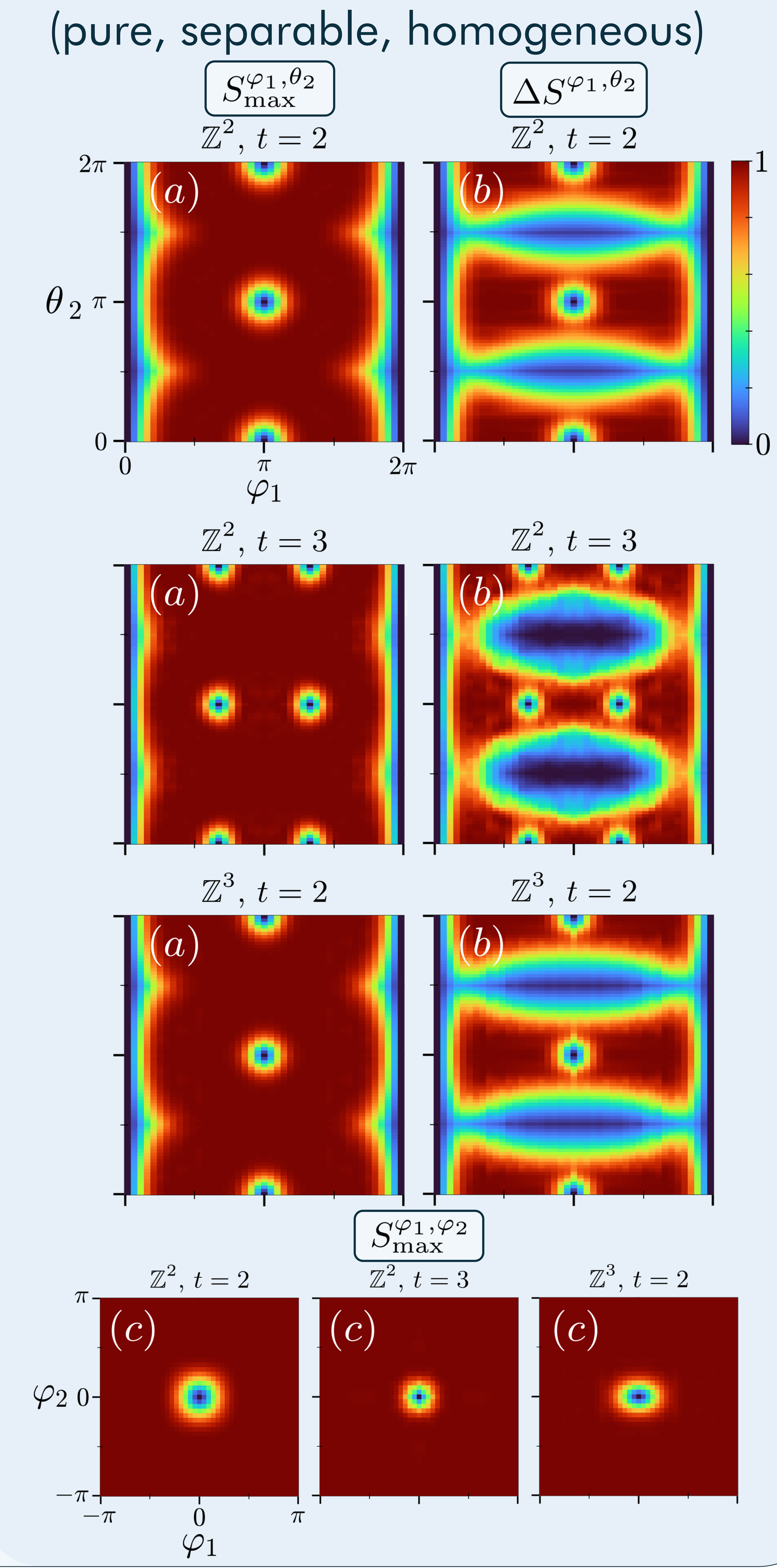


Unique Equivalence Classes
$\alpha \sim FDQC \Leftrightarrow \vec{\iota}(\alpha) = \vec{1}$
$\alpha \sim$ Shift $\Leftrightarrow \vec{\iota}(\alpha) \neq \vec{1}$



Entanglement

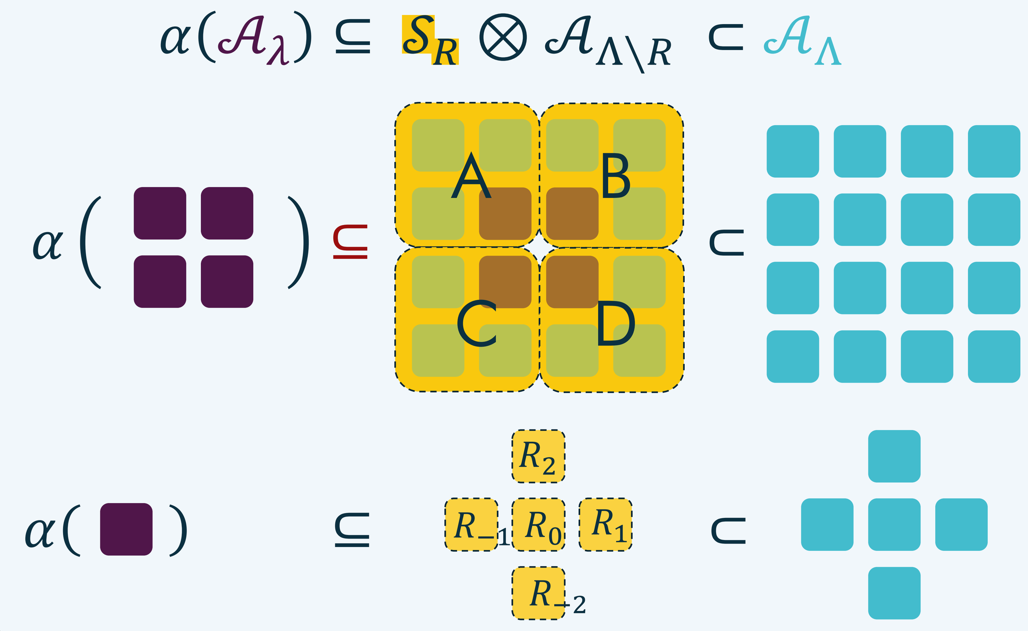
Entanglement entropy S of a qubit vs QCA parameters $(\vec{\varphi}, \vec{\theta})$, maximized over input states (pure, separable, homogeneous)



METHODS: support algebras

Support algebras allow to study how an algebra of operators distributes among neighboring algebras.

Let $\lambda \subset \Lambda, \{R\} \subset \Lambda$. The support algebras $\mathcal{S}_R$ are the smallest (C*) algebras such that $\alpha(\mathcal{A}_\lambda) \subseteq \mathcal{S}_R \otimes \mathcal{A}_{\Lambda\setminus R} \subset \mathcal{A}_\Lambda$



Theorem: the inclusion becomes an equality for our qubit QCA.

[This is true for *any* 1d QCA: « GNVW index theory » D. Gross, V. Nesme, H. Vogts, R.F. Werner, CMP 310, 419 (2012)]

Theorem: all configurations of $\{R_j\}$ can be listed.

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