UNIVERSITÀ
DI PAVIA

"Repeated interactions as a thermodynamic microscope: spin rings and clustering collision models"

End of year seminars A.A 2025/2026

17/09/2026

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Supervisors: Giacomo Guarnieri
Dario Gerace

Open vs closed quantum systems

Closed quantum systems

Open vs closed quantum systems

Closed quantum systems

Von Neumann equation: $\frac{d\rho_S(t)}{dt} = -i[H(t), \rho_S(t)]$

Unitary dynamics: $\rho_S(t) = U(t)\rho_S(0)U(t)^\dagger$

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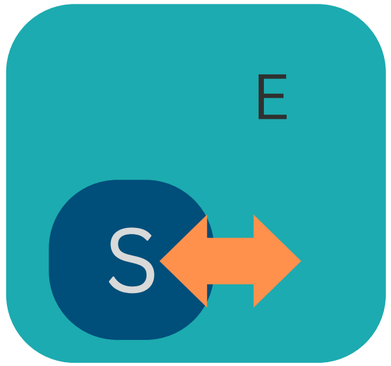
Open quantum systems



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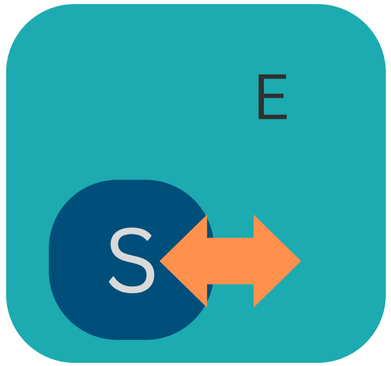
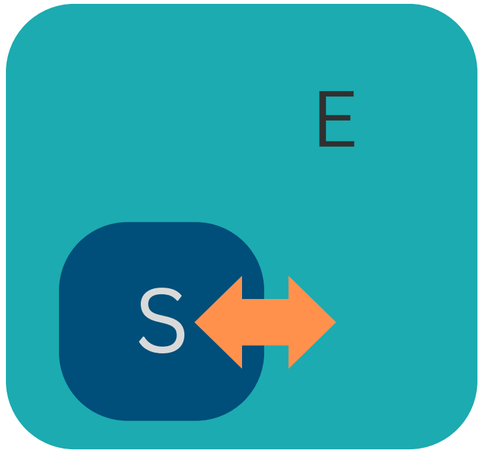


More degrees of freedom to describe
→ the formalism is not immediate

Lindblad master equations



reduced dynamics:

$$\frac{d\rho_S(t)}{dt} = -i[H_S(t) + H_{LS}(t), \rho_S(t)] + \mathcal{D}_t[\rho_S(t)]$$

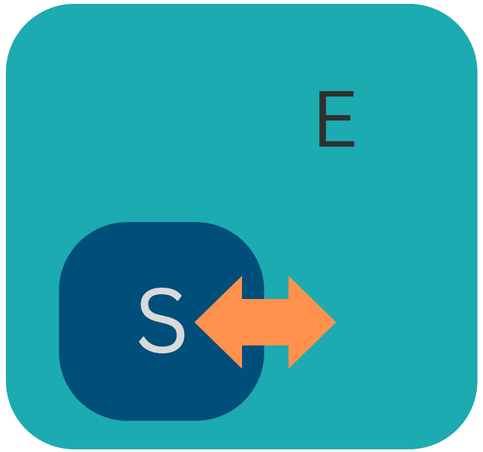
UNITARY CONTRIBUTION

DISSIPATOR

$$\mathcal{D}_t[\rho_S(t)] = \sum_k \gamma_k(t) \left(S_k(t)\rho_S(t)S_k^\dagger(t) - \frac{1}{2}\{S_k^\dagger(t)S_k(t), \rho_S(t)\} \right)$$

$\gamma_k(t) \in \mathbb{R}$

Lindblad master equations



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Needed approximations:

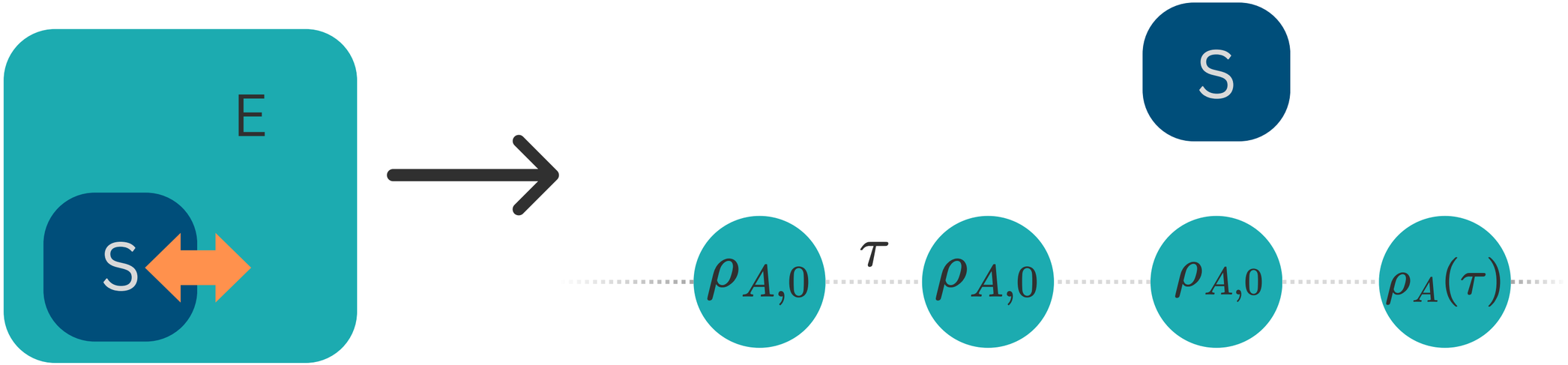
Weak coupling

Born approximation

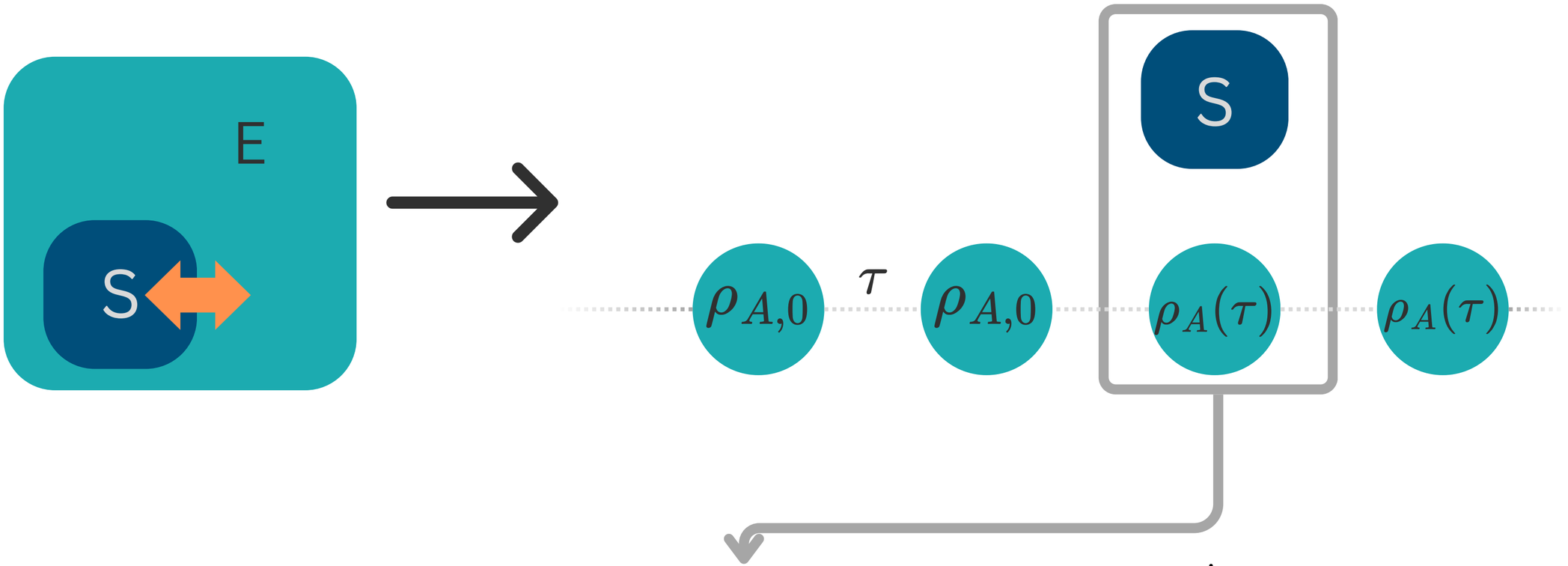
Markov approximation

Secular approximation

Collision models

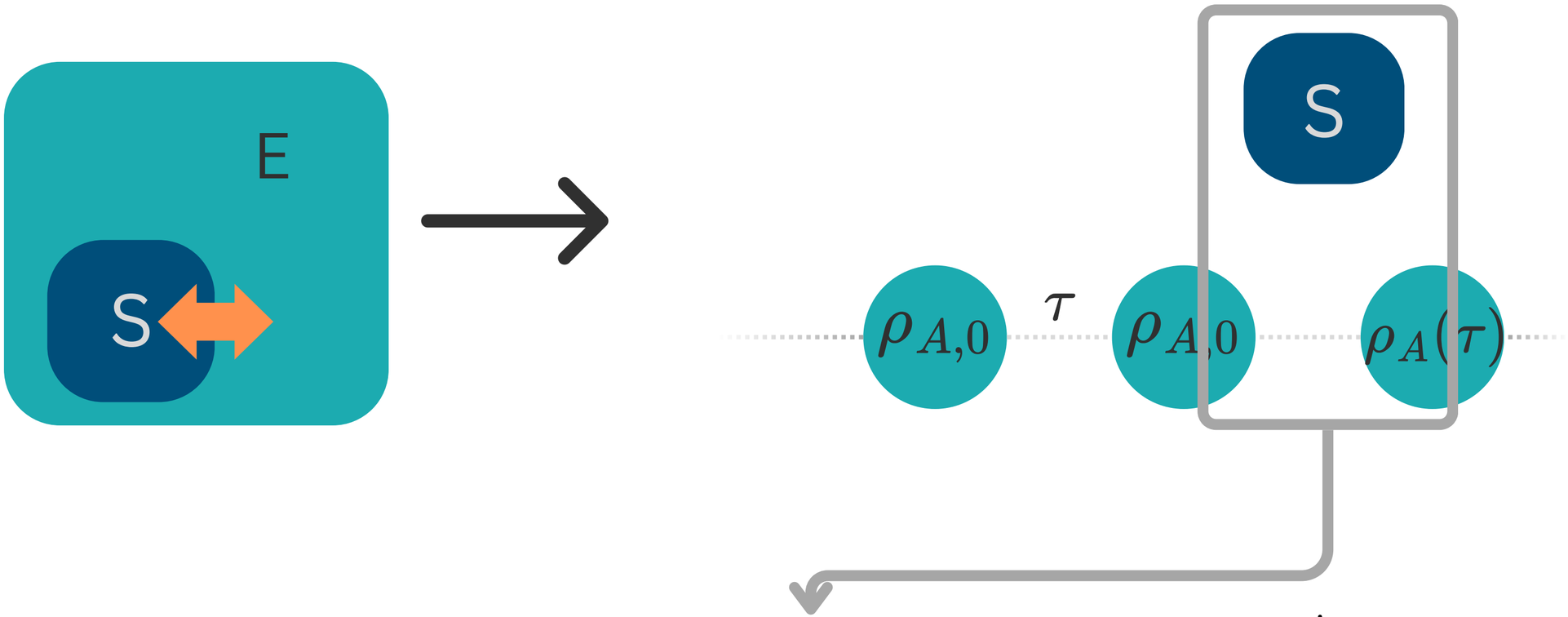


Collision models



$$\rho_S(\tau) = Tr_A[U(\tau)(\rho_S(0) \otimes \rho_A(0))U(\tau)^\dagger)]$$

Collision models



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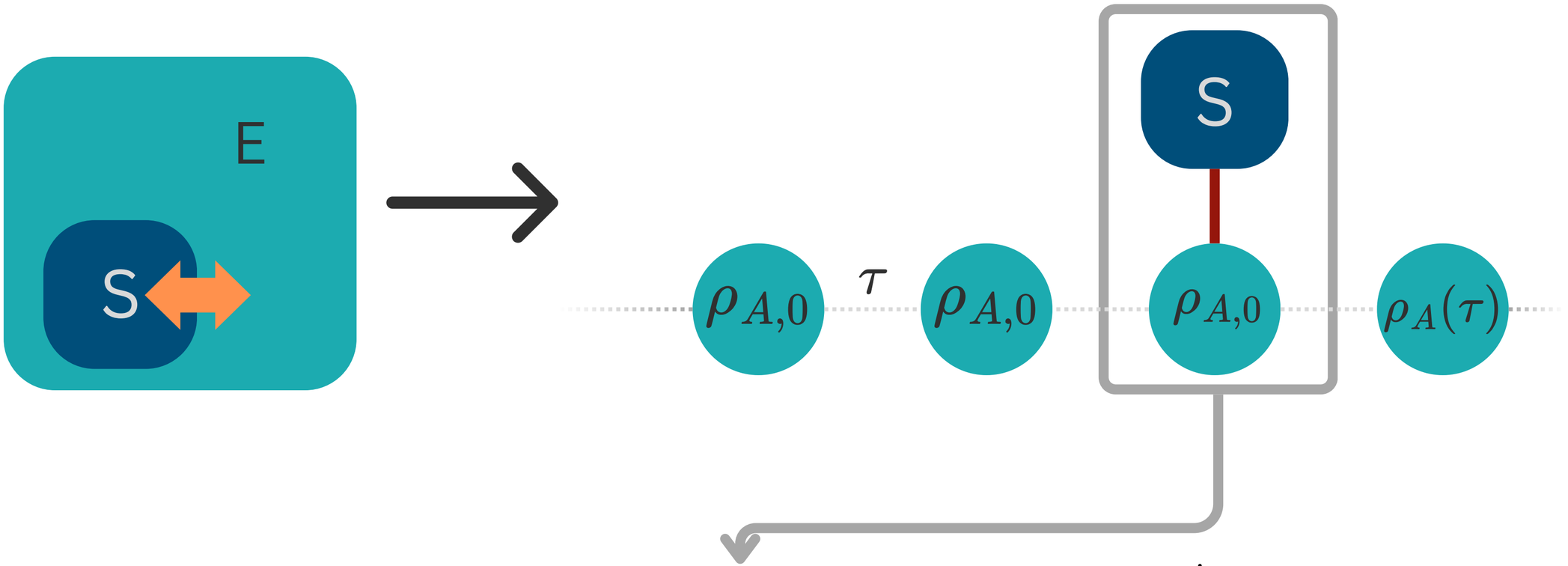
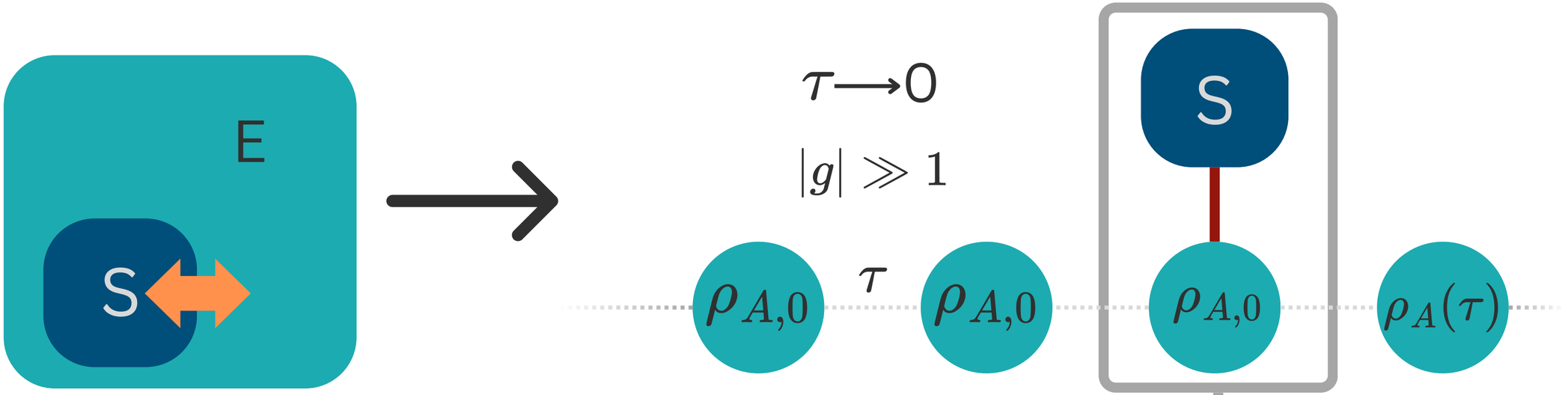


$$\rho_{SA}(2\tau) = U(\tau)(\rho_S(0) \otimes \rho_A(0))U(\tau)^\dagger$$

Collision models



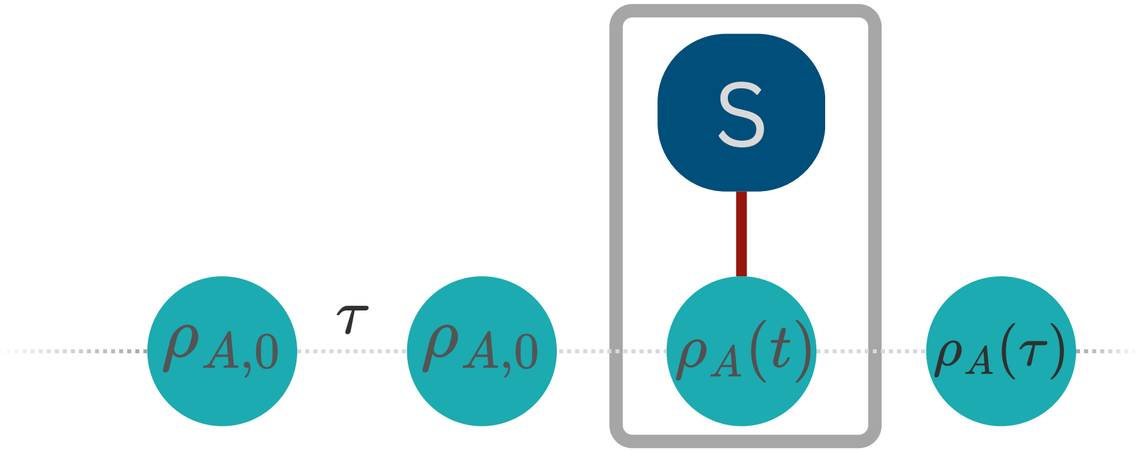
Taylor expansion at 2nd order

$$\frac{d\rho_S(t)}{dt} = -i[H_S, \rho_S(t)] + \mathcal{D}[\rho_S(t)]$$

Collision models

1) Simple structure

2) No weak coupling, secular approximation



Ciccarello, Francesco, et al. Physics Reports 954 (2022): 1-70.

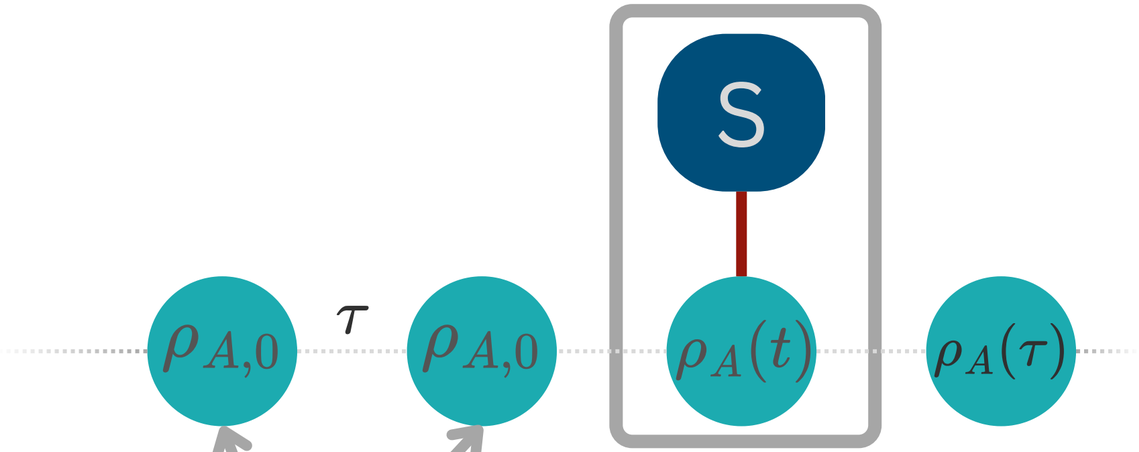
Collision models

1) Simple structure

2) No weak coupling, secular approximation

3) Versatility:

- adding coherence



$$\rho_A(0) = \rho_A^d + \varepsilon \begin{pmatrix} 0 & \alpha \\ \alpha^* & 0 \end{pmatrix}$$

Ciccarello, Francesco, et al. Physics Reports 954 (2022): 1-70.

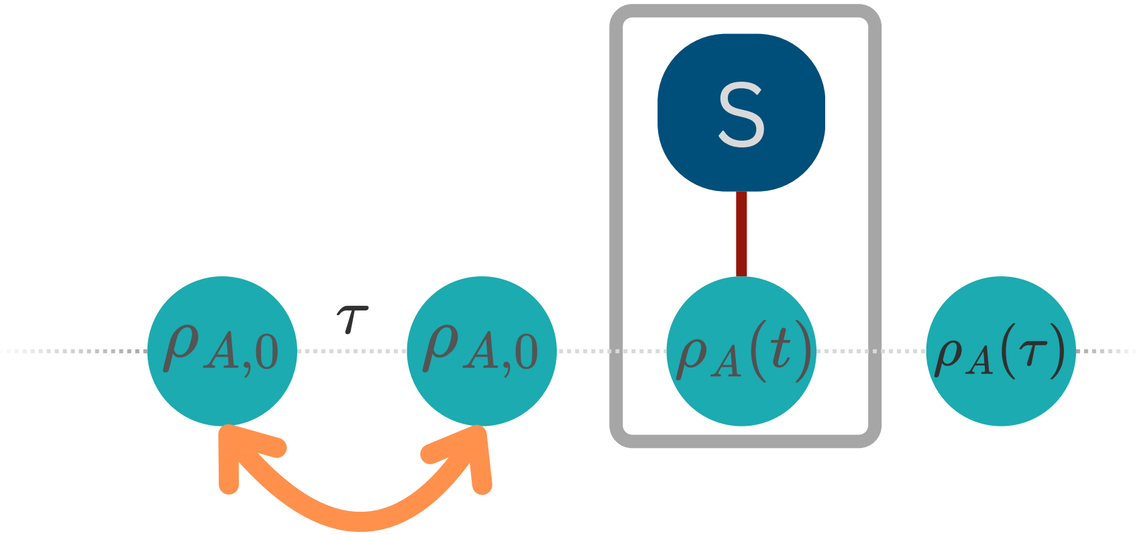
Collision models

1) Simple structure

2) No weak coupling, secular approximation

3) Versatility:

- adding coherence
- memory effects



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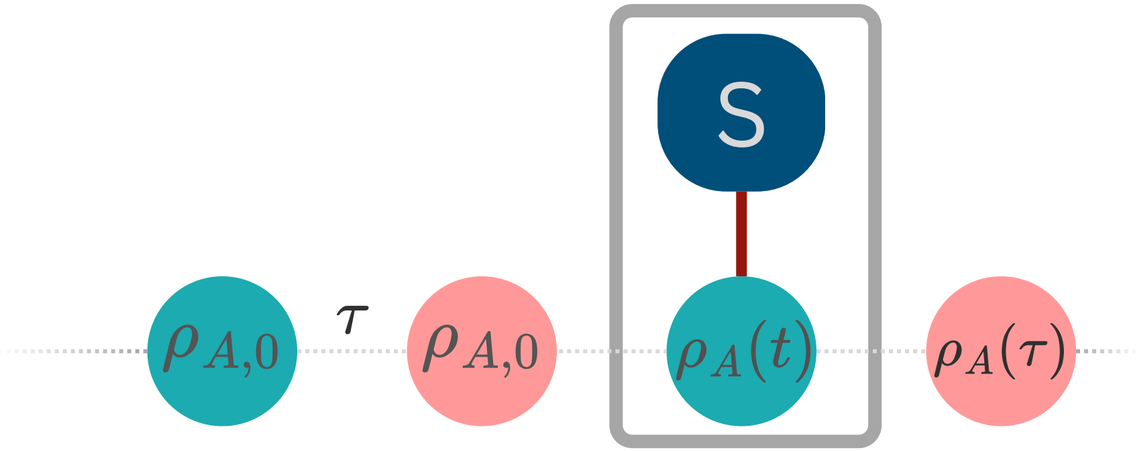
- more that one bath



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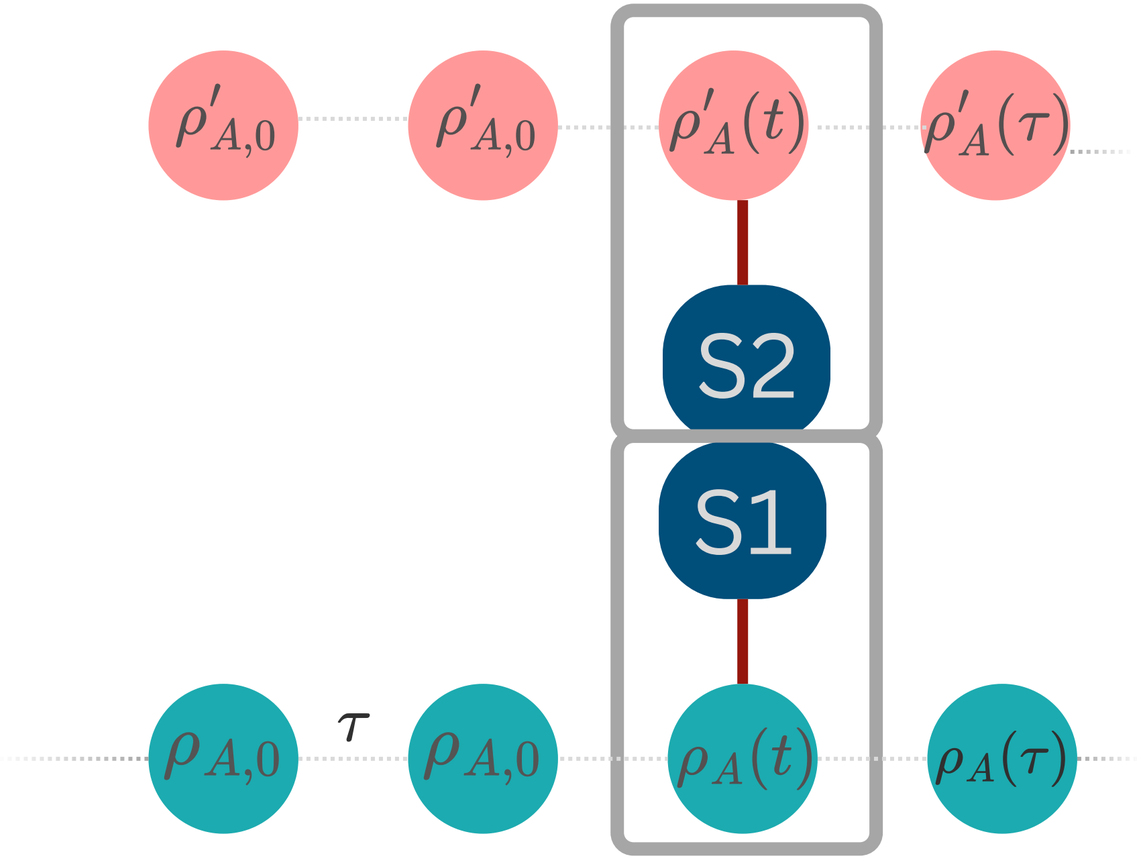
Collision models

1) Simple structure

2) No weak coupling, secular approximation

3) Versatility:

- adding coherence
- memory effects
- more that one bath
- multi-partite system



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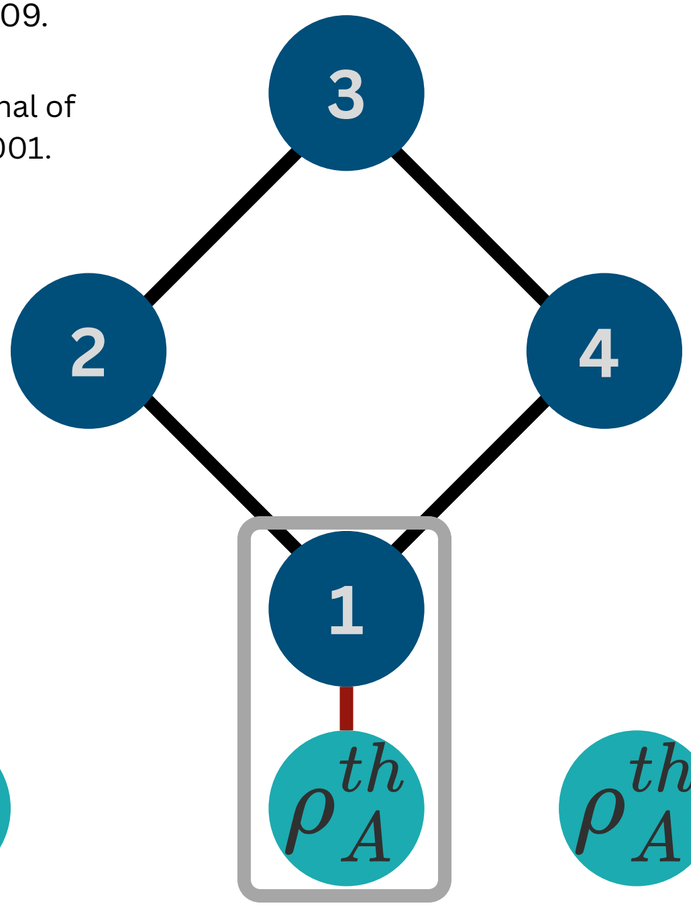
Heisenberg spin ring

Guarnieri, et al. Physical Review A 106.2 (2022): 022209.

Campbell et al. New Journal of Physics 27.2 (2025): 023001.



Random arrival time θ_i

ρ_A^{th} ρ_A^{th} ρ_A^{th} ρ_A^{th}

Hamiltonian:

$$H_{tot} = H_S + H_{SA}(t) + H_A$$

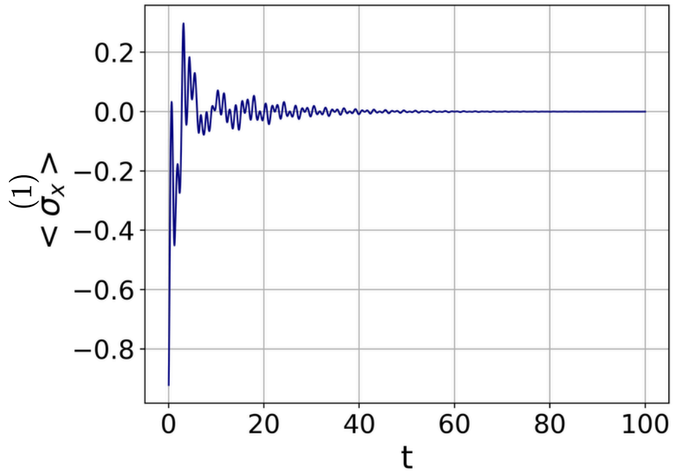
On-site energy + interaction between sites

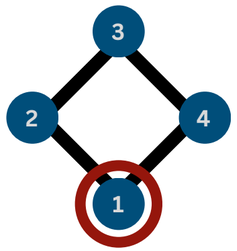
Interaction Hamiltonian

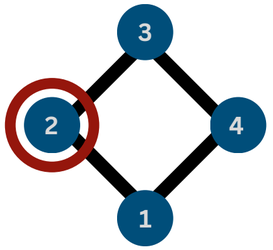
On-site energy (environment)

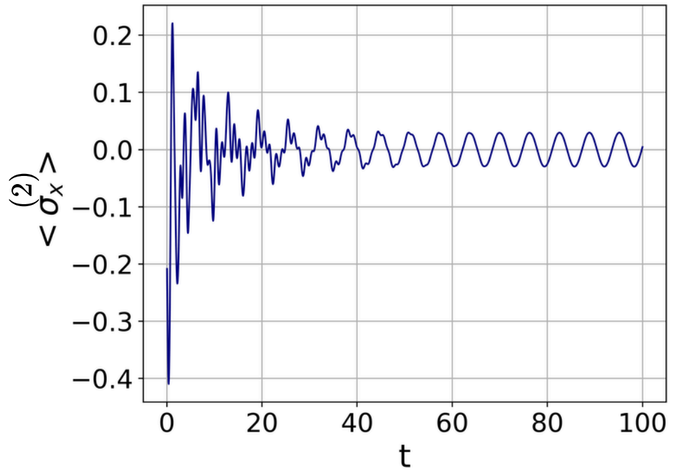
$$H_{SA}(t) = \sum_n g(\sigma_x^{(1)}\sigma_x^{(A)} + \sigma_y^{(1)}\sigma_y^{(A)})\Theta(t-\theta_n)\Theta(\theta_n+\tau-t)$$

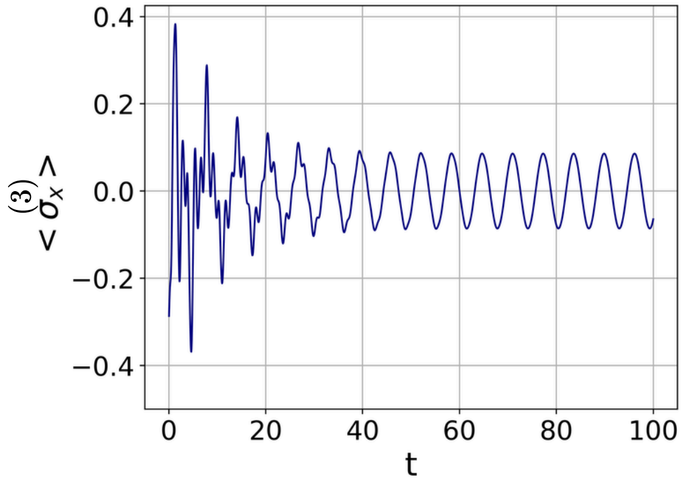
Dynamics of the N=4 ring

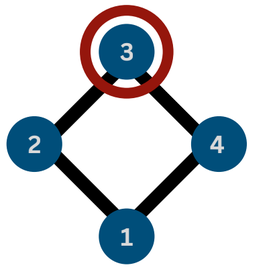


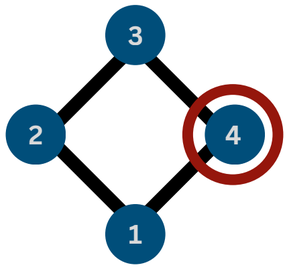


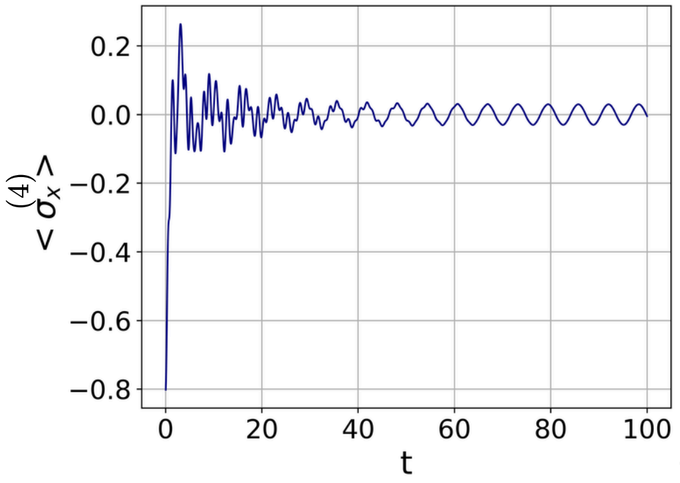




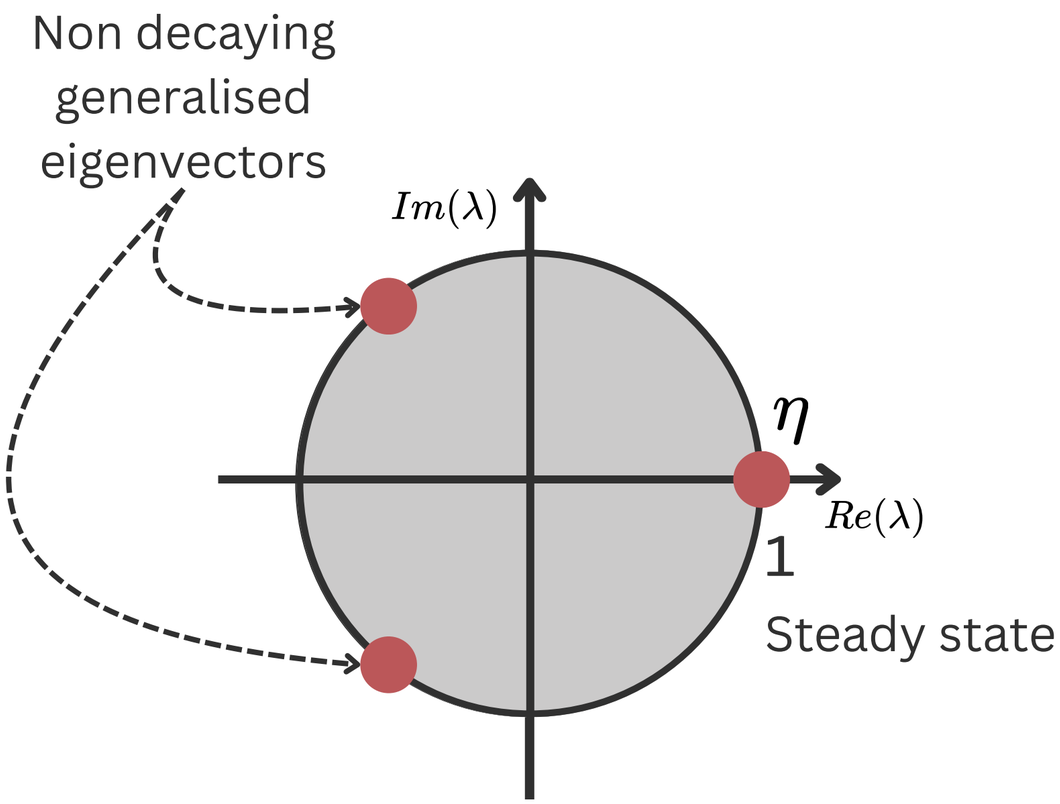








Dynamical symmetries



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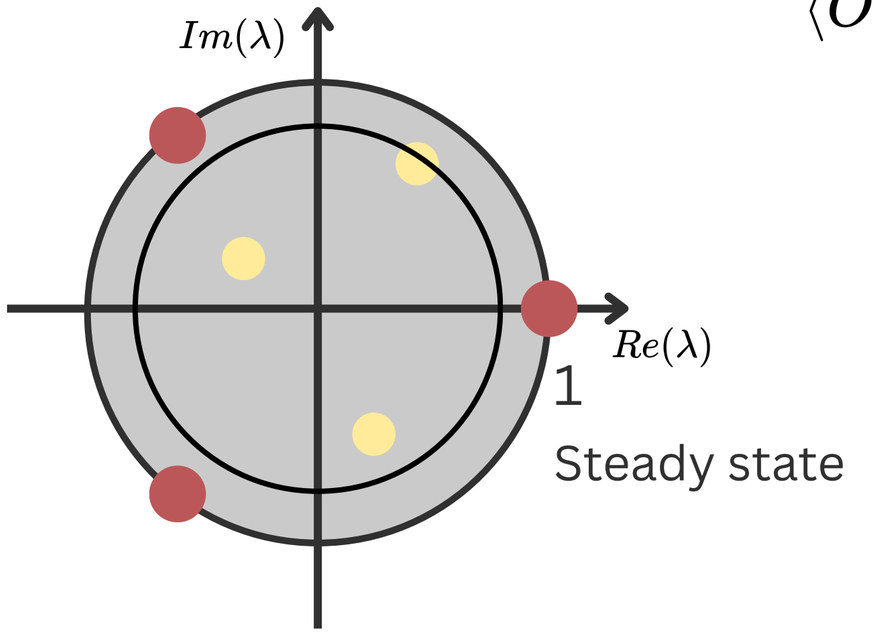

Steady state

$$\langle O(nt)\rangle \xrightarrow{n\to\infty} r_0 Tr[\eta O] + \sum_m e^{i\omega_\alpha t} r_\alpha Tr[O\Xi_\alpha \eta] + h.c.$$

$$\rho_S(t) = \Lambda_t[\rho_S(0)]$$

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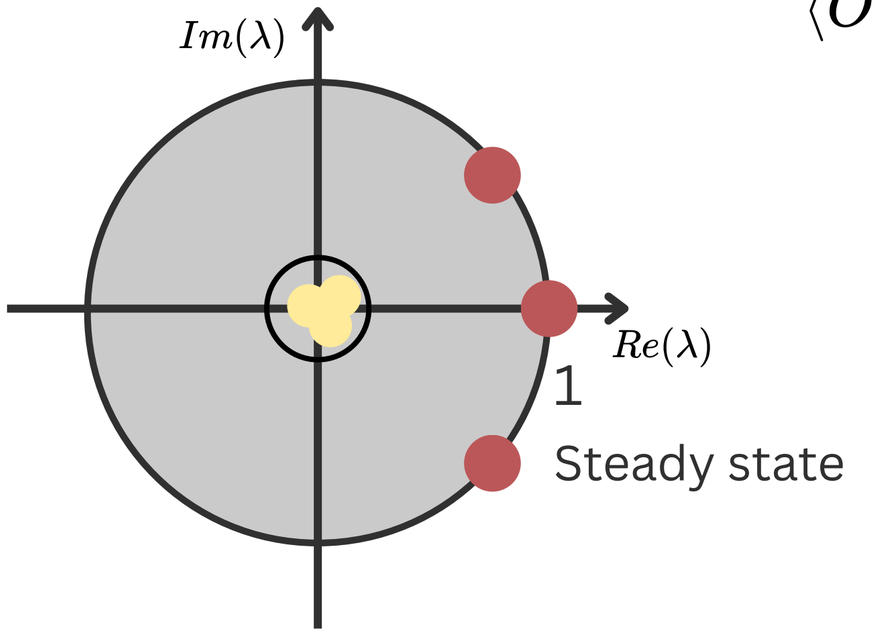

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$$\rho_S(t) = \Lambda_t[\rho_S(0)]$$

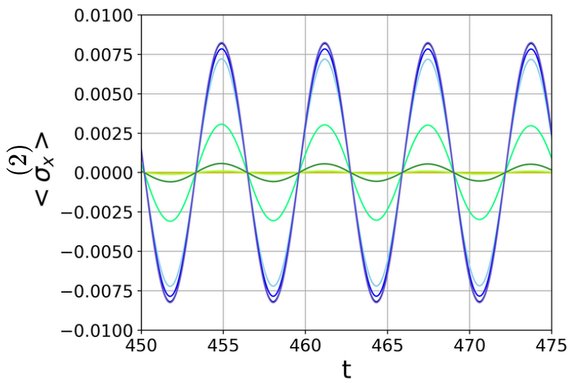
n repetitions

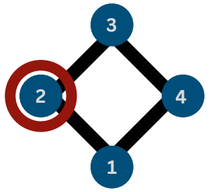
$$\rho_S(nt) = [\Lambda_t]^n[\rho_S(0)]$$

Only the generalised eigenvectors in the peripheral spectrum survive

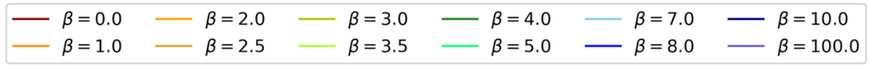
Summary of preliminary results

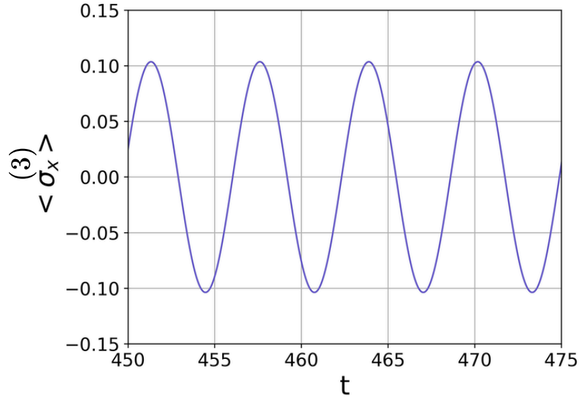
1) Robustness to temperature

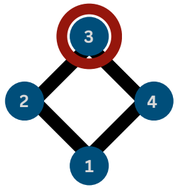




Information about the **TEMPERATURE**







Can be used as a **CLOCK**

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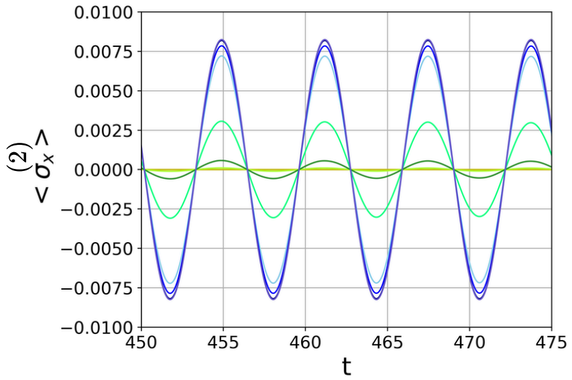

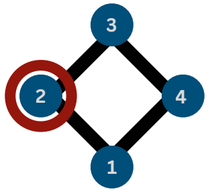

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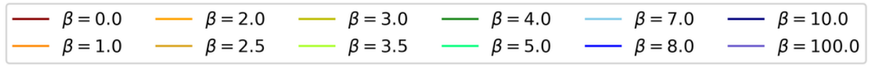
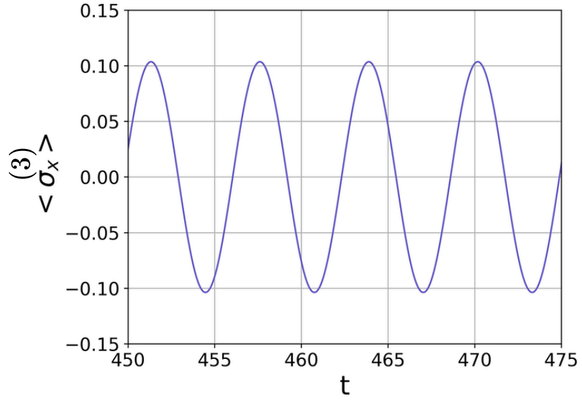
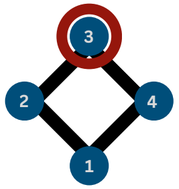
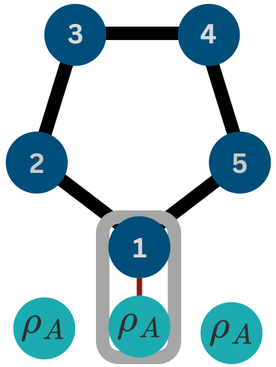


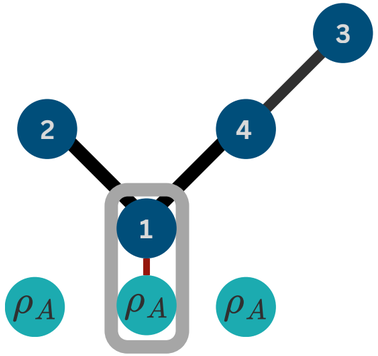




Can be used as a **CLOCK**

2) Different dynamical configurations





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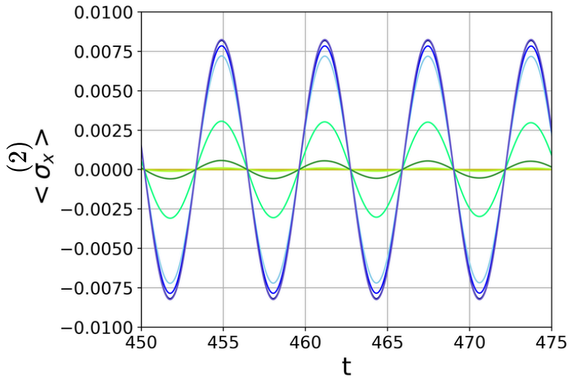

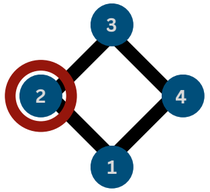

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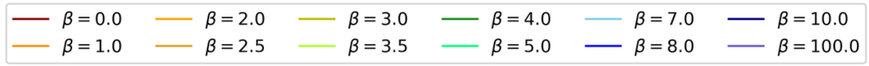

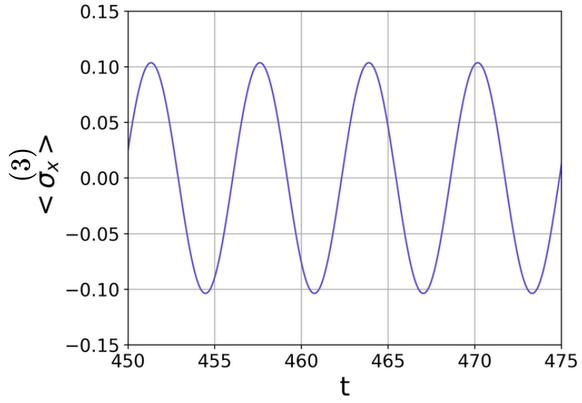

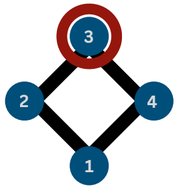

Can be used as a **CLOCK**

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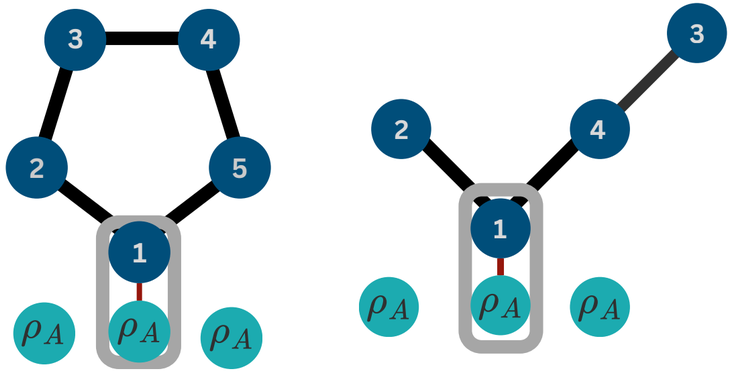
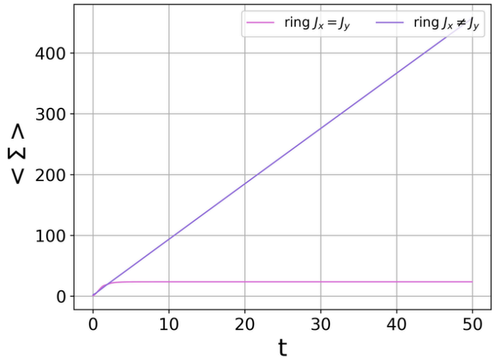
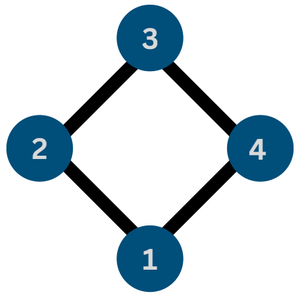


3) Characterisation of the thermodynamic cost



$$\Sigma = \Delta S_S + \Delta S_E \geq 0$$

New questions



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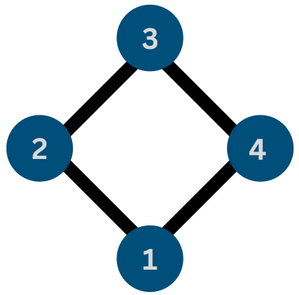
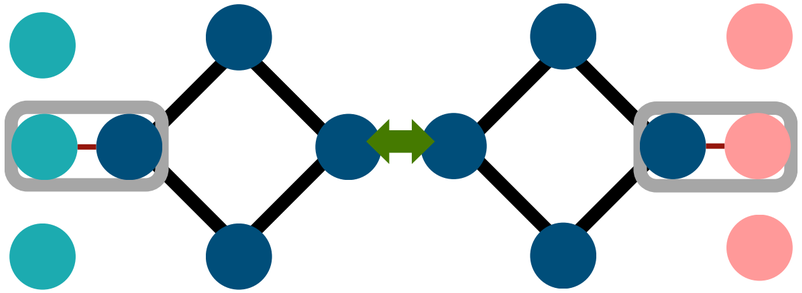


What about **TRANSPORT PROPERTIES** between two thermal baths?



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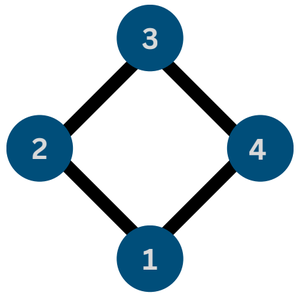

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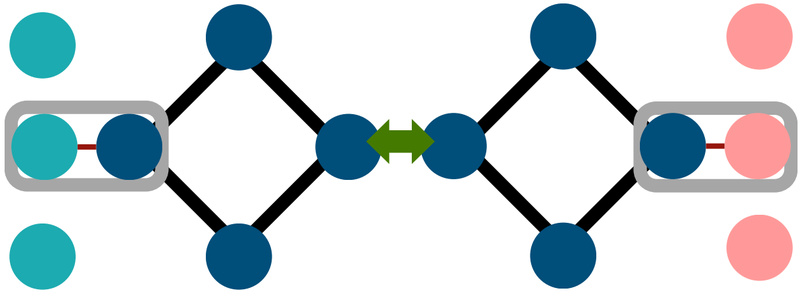

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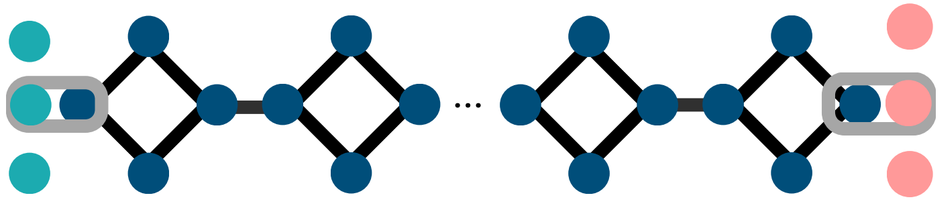

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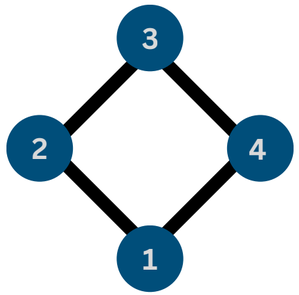

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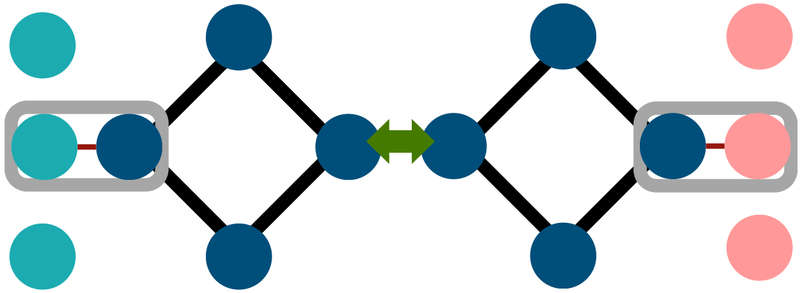

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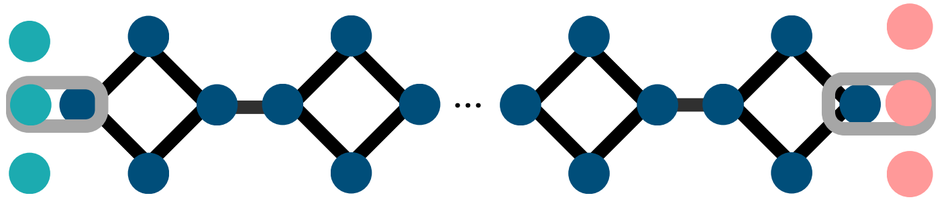

PROBLEM: Computational complexity is too high

Tensor networks - graphical representation

Basic elements

Number

a

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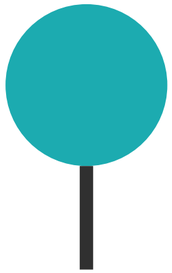

Vector

v_i

Tensor networks - graphical representation

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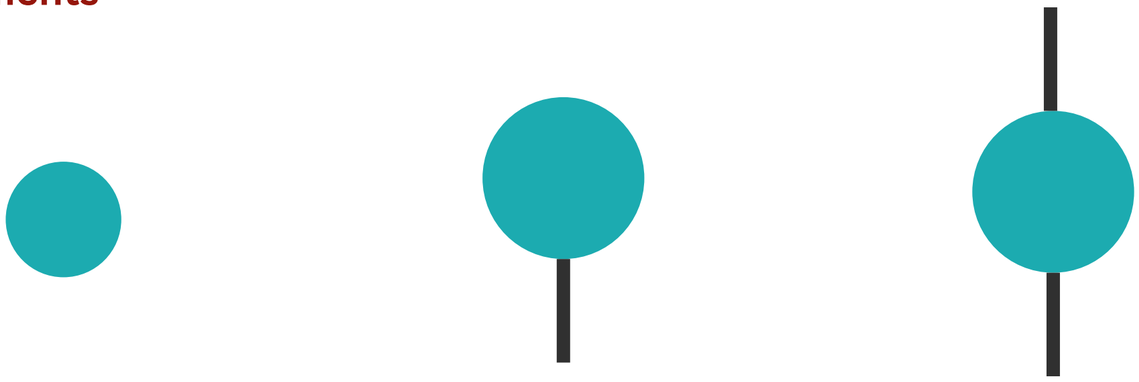

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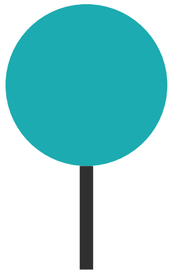

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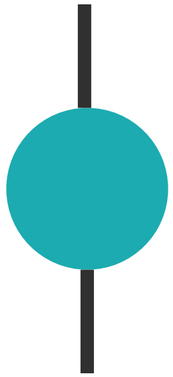

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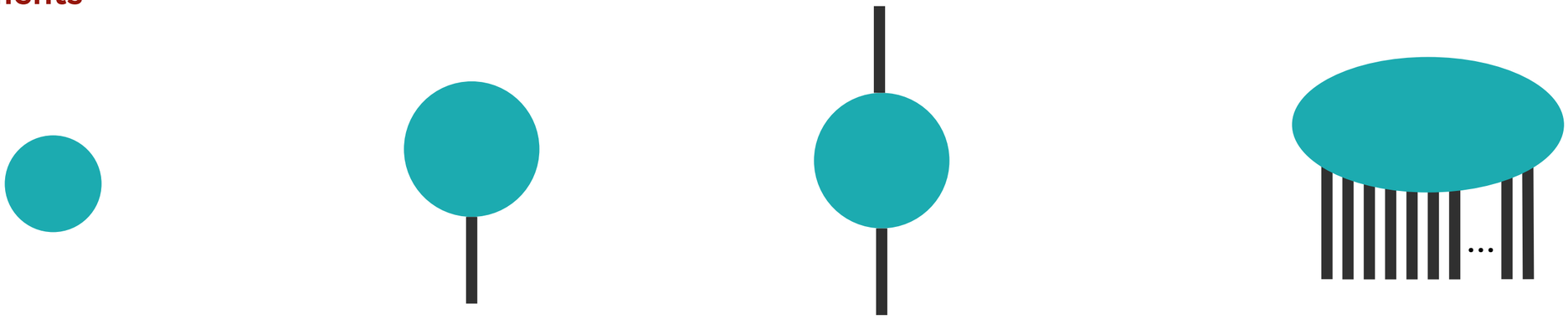

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Basic operation: contraction

$$w_j = \sum_j M_{i,j} v_i$$

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Generic quantum state: $|\psi\rangle = \sum_{\alpha_1,\alpha_2,\ldots,\alpha_N} \psi_{\alpha_1,\alpha_2,\ldots,\alpha_N} |\alpha_1, \alpha_2, \ldots, \alpha_N\rangle$

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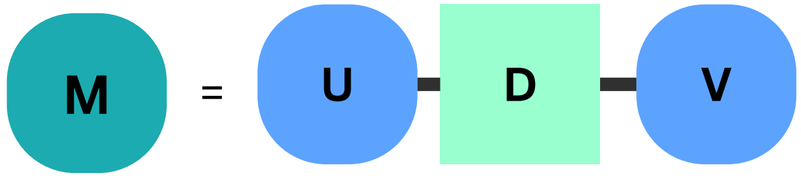

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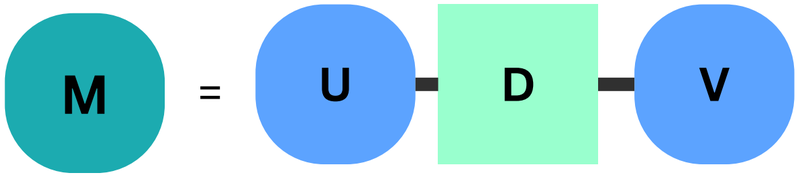

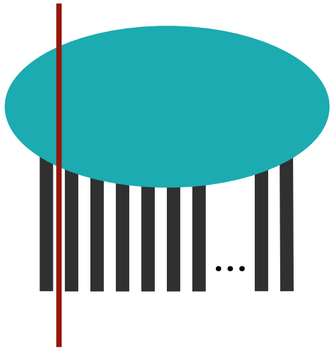

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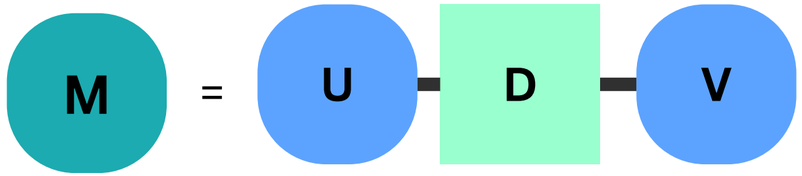

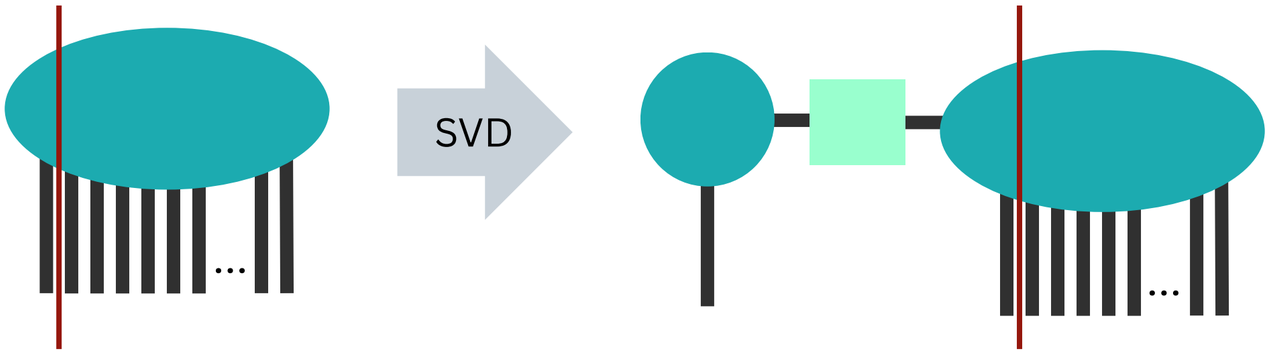

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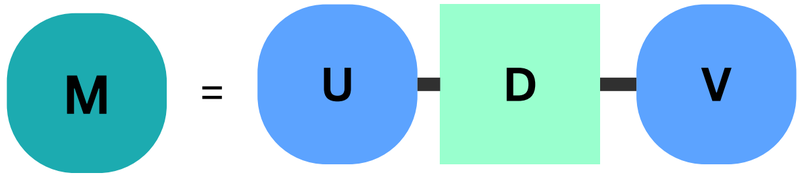

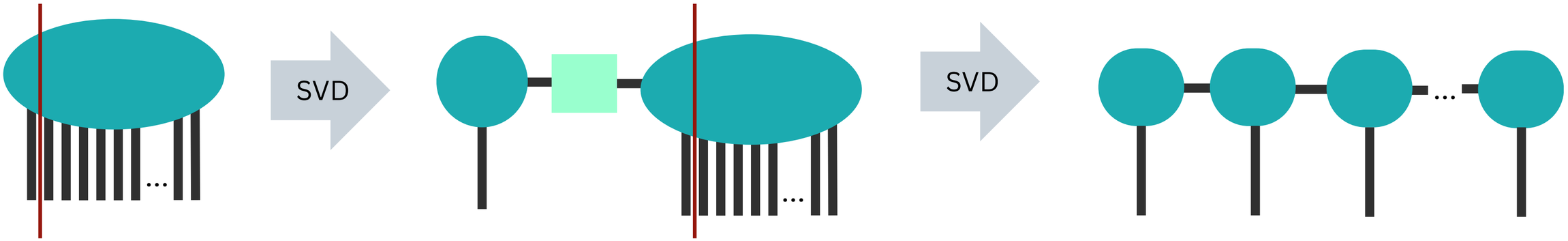

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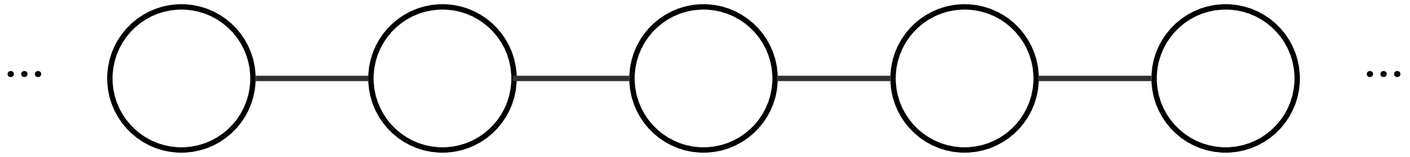

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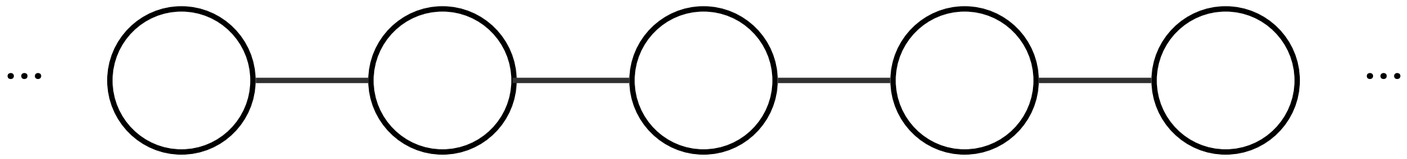

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Schollwöck Annals of physics 326.1 (2011): 96-192.

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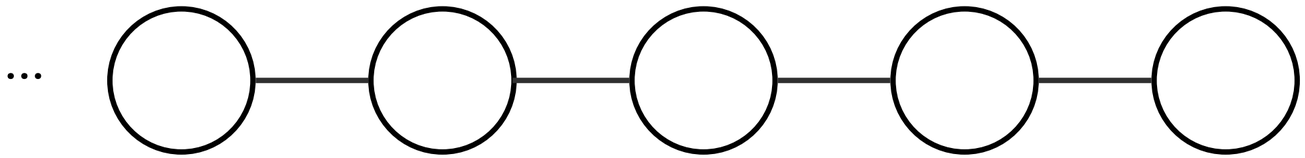

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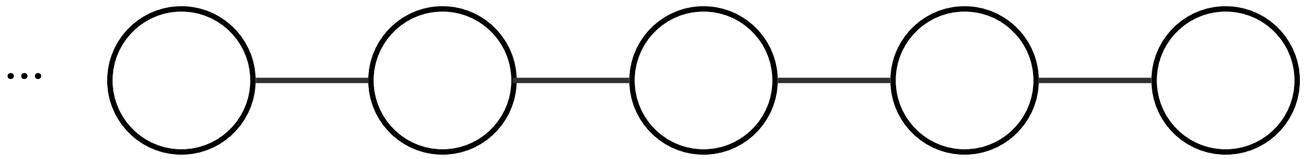

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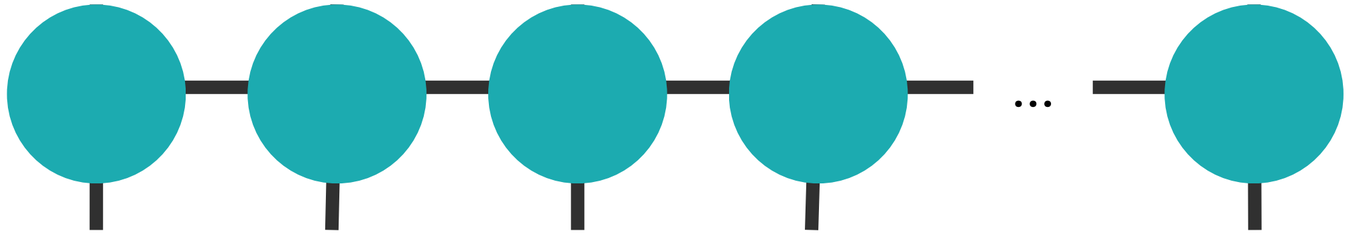
TOTAL Hilbert space

RELEVANT PORTION of the Hilbert space

Tensor networks

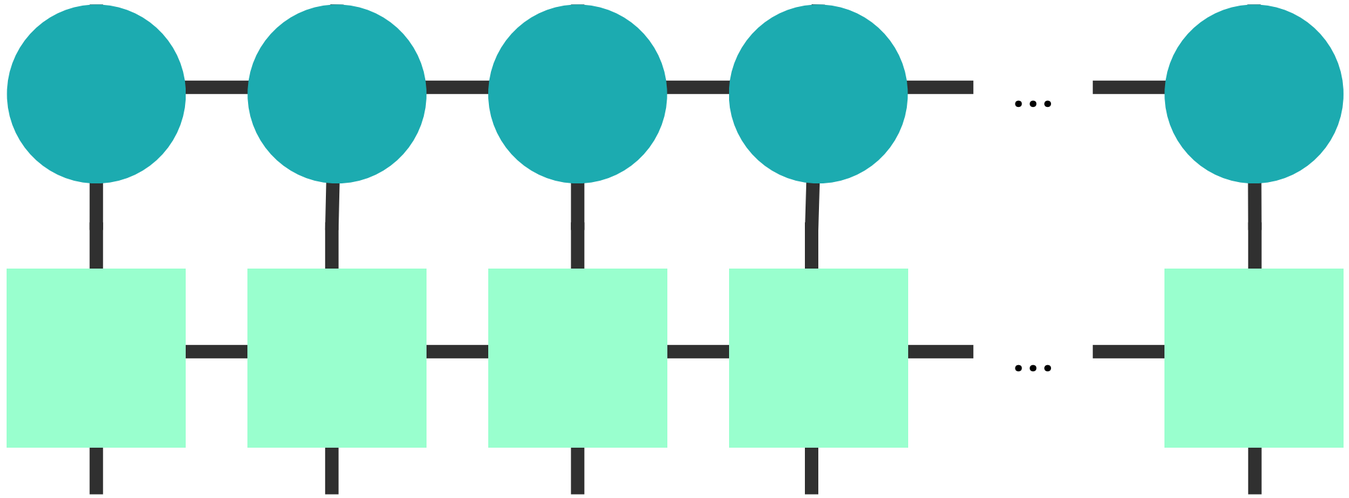
MPS



Tensor networks

MPS

MPO



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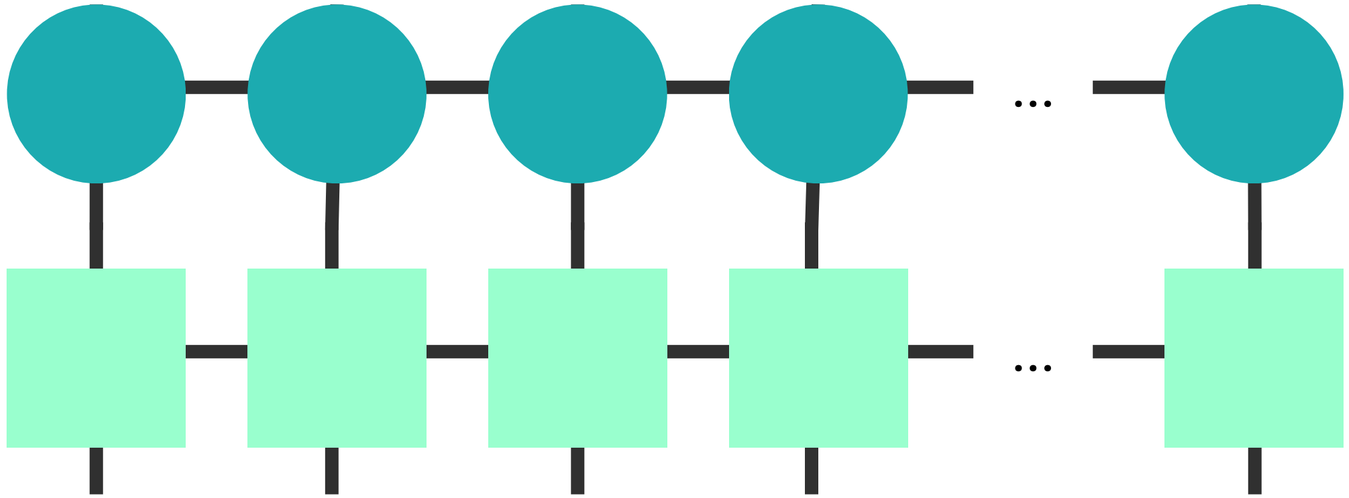

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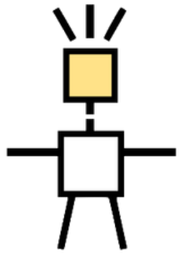

Matthew Fishman, et al. SciPost Phys. Codebases 4 (2022)

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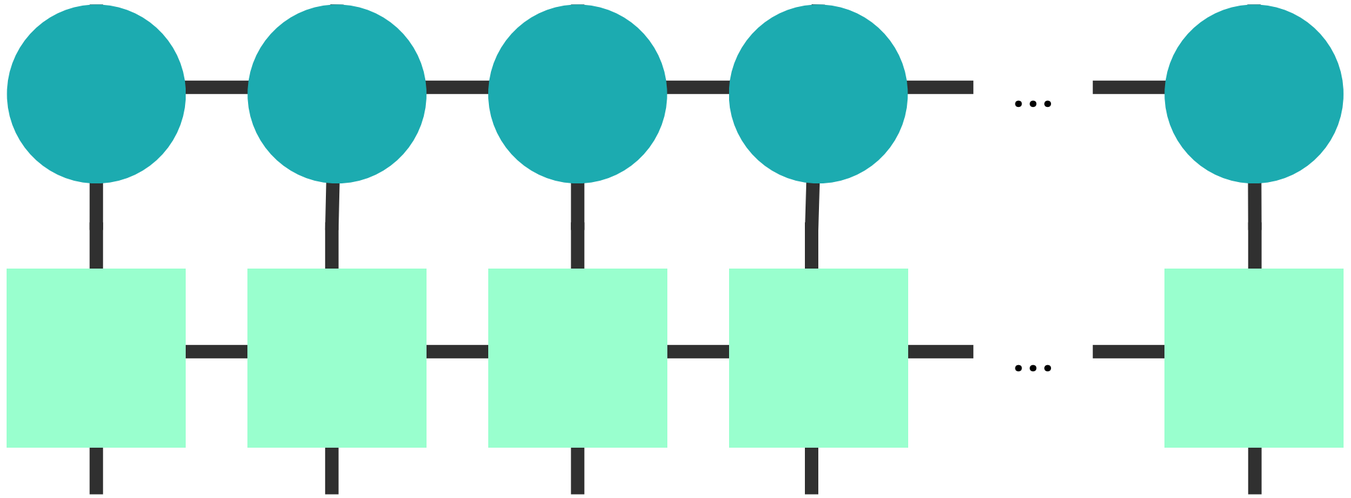

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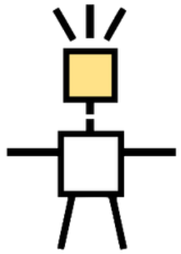

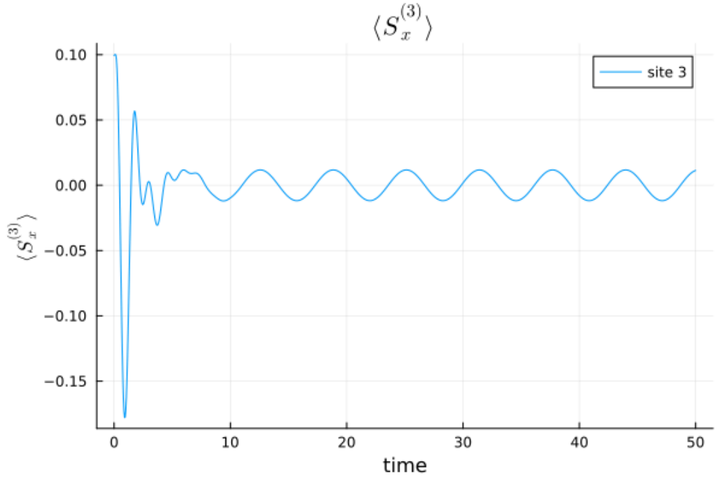

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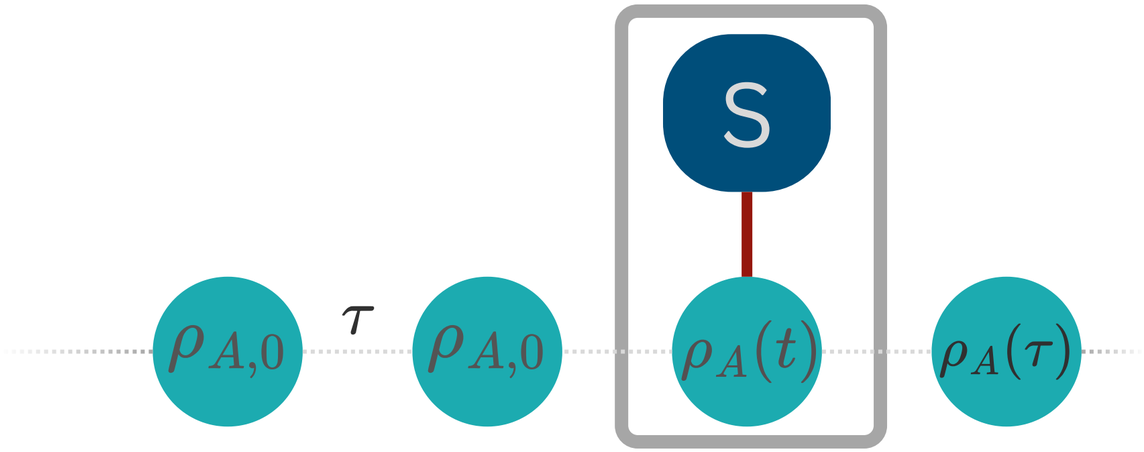
Problem: matter exchange



Collision models:

- ✓ good description of the dynamics
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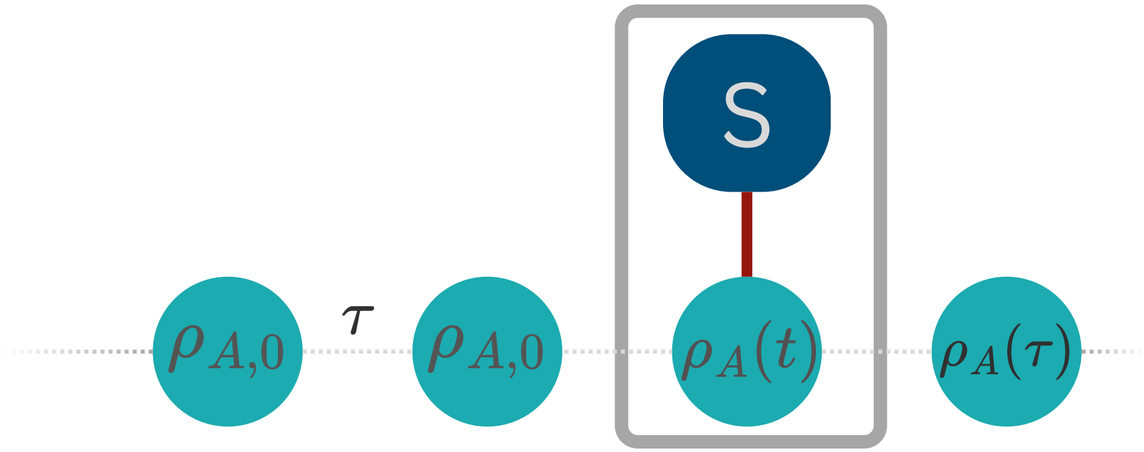

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- ✓ good description of the dynamics
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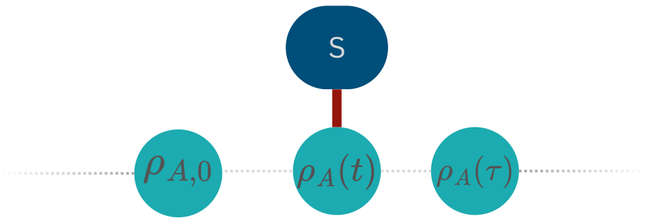
Problem: can systems that exchange particles be characterised?

Clustering collision models

Attachment $\rho_S(\tau) = \mathcal{A}_p \circ \xi\left[\rho_S(0)\right]$



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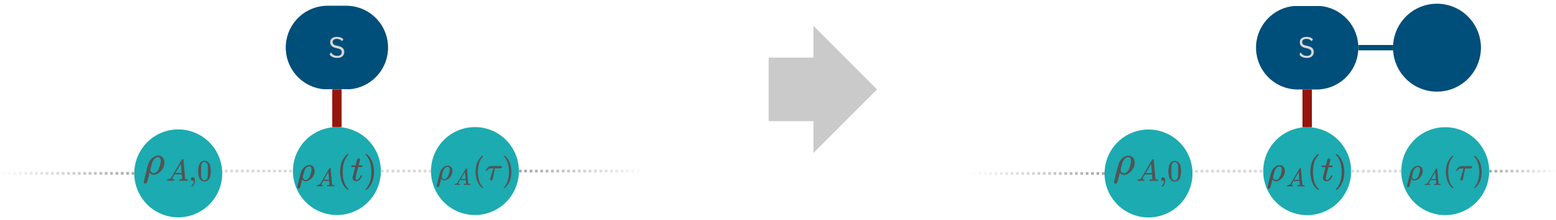

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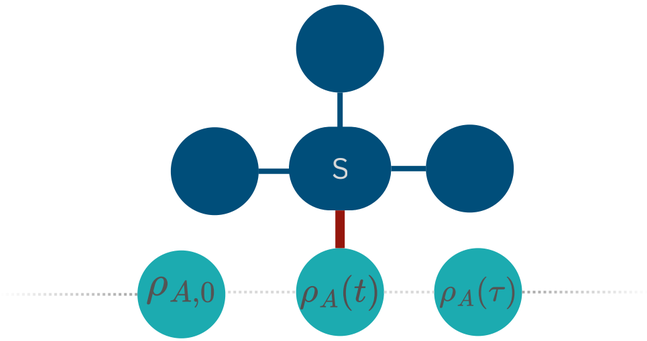

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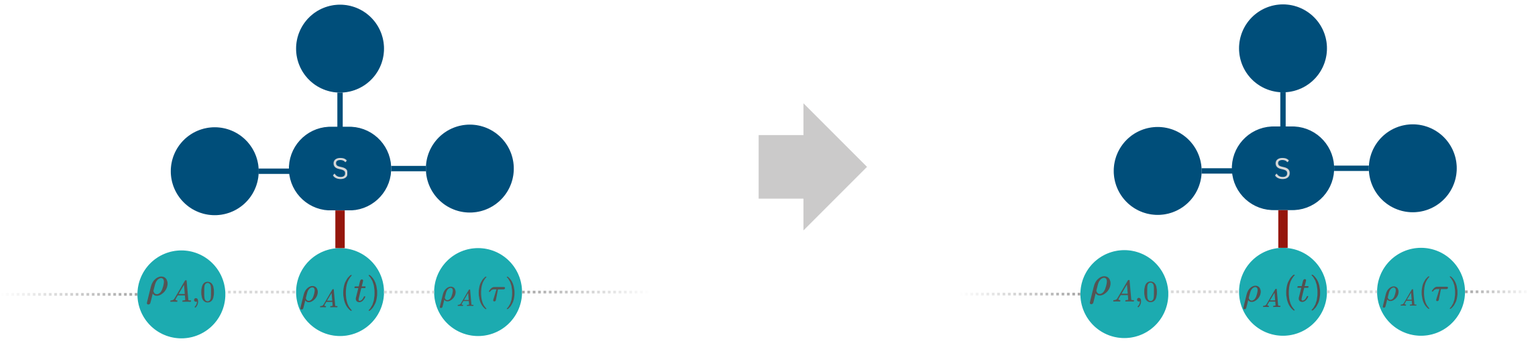

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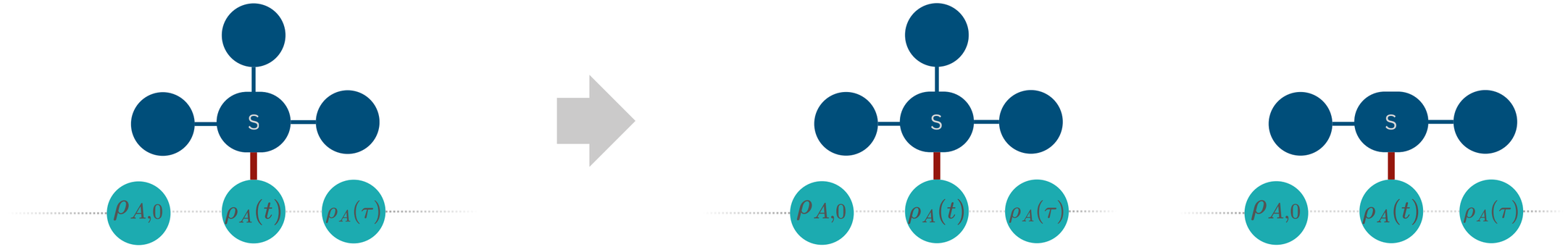
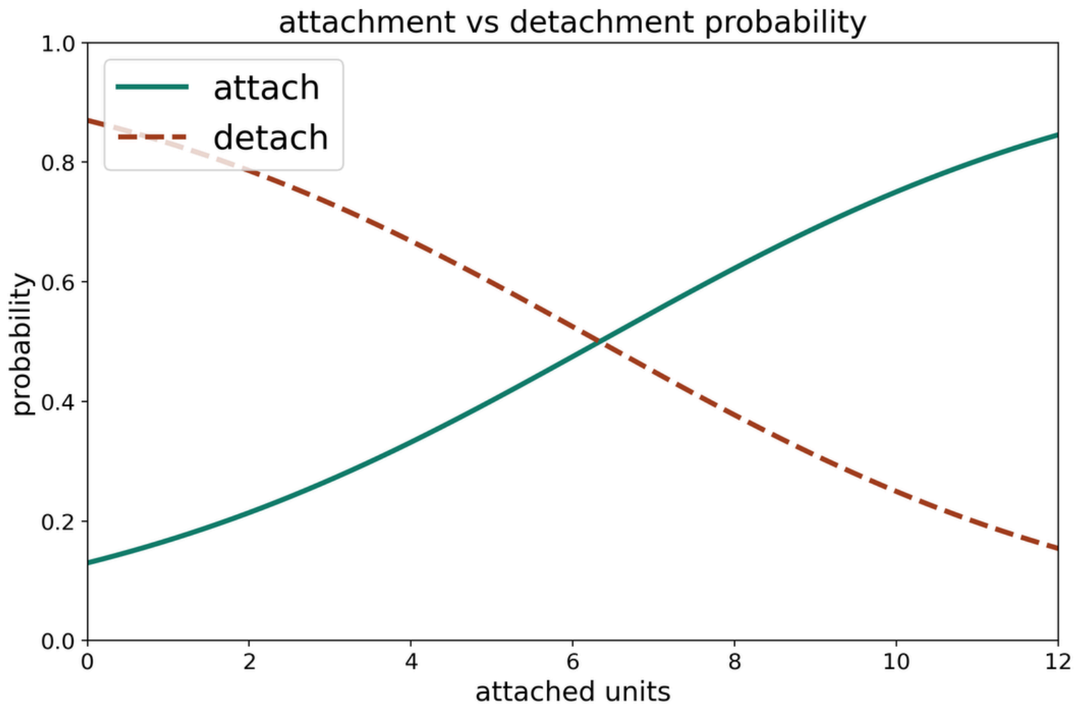


Stabilising number of auxiliary units

Question: how do we define the probabilities in the stochastic operators?



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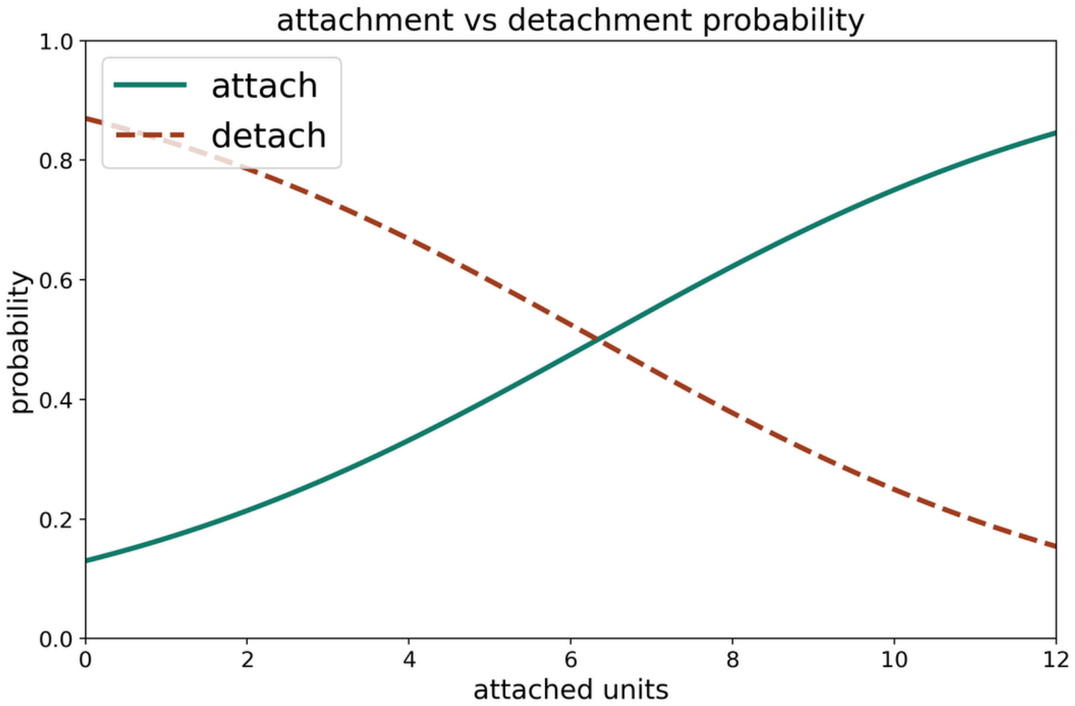

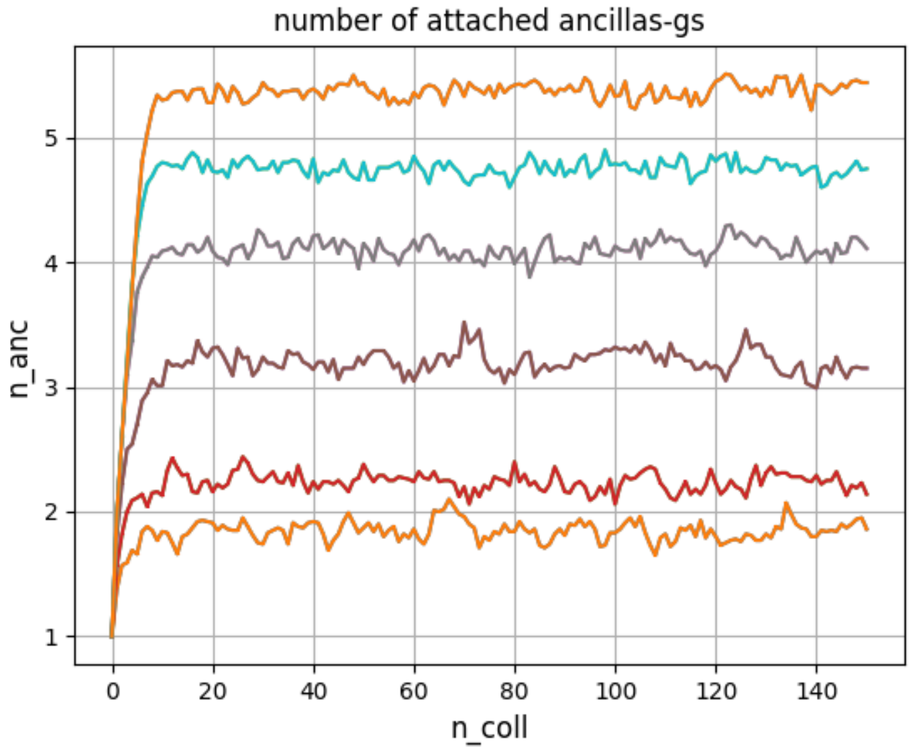

Non-Markovianity in CluCMs

How do we distinguish two quantum states?

One answer is **TRACE DISTANCE**

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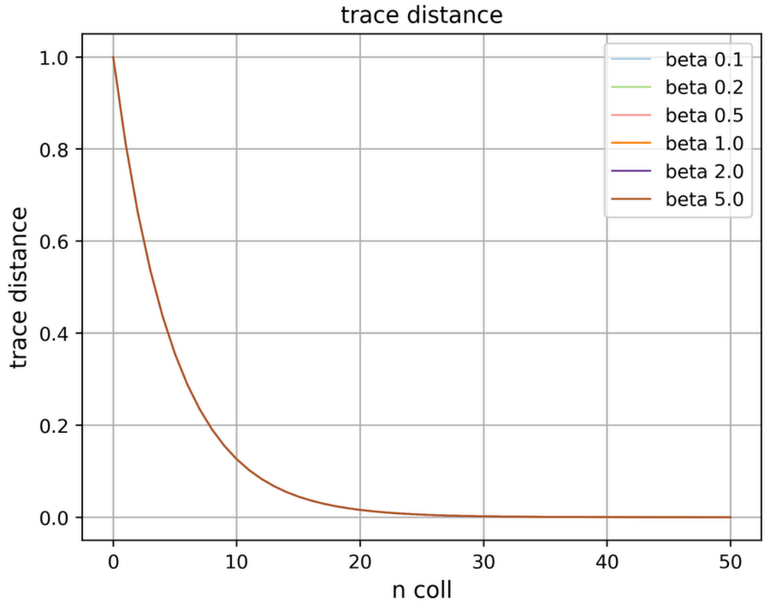

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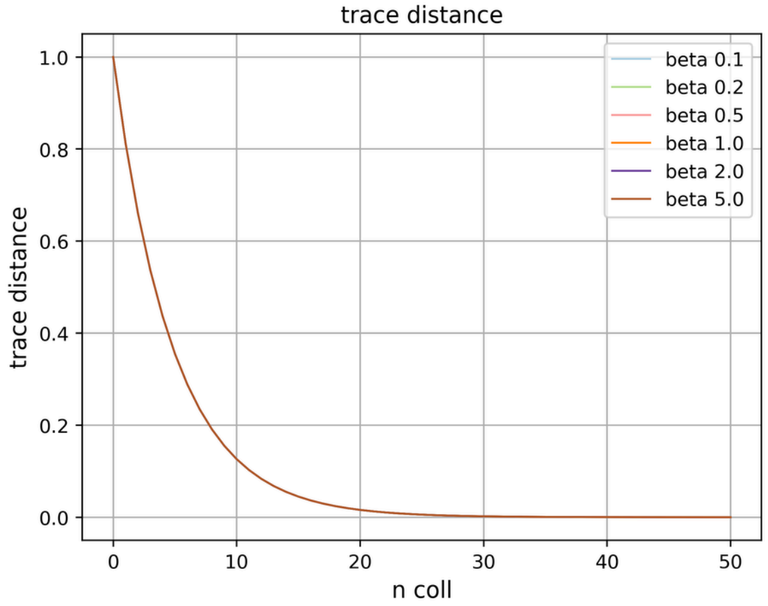

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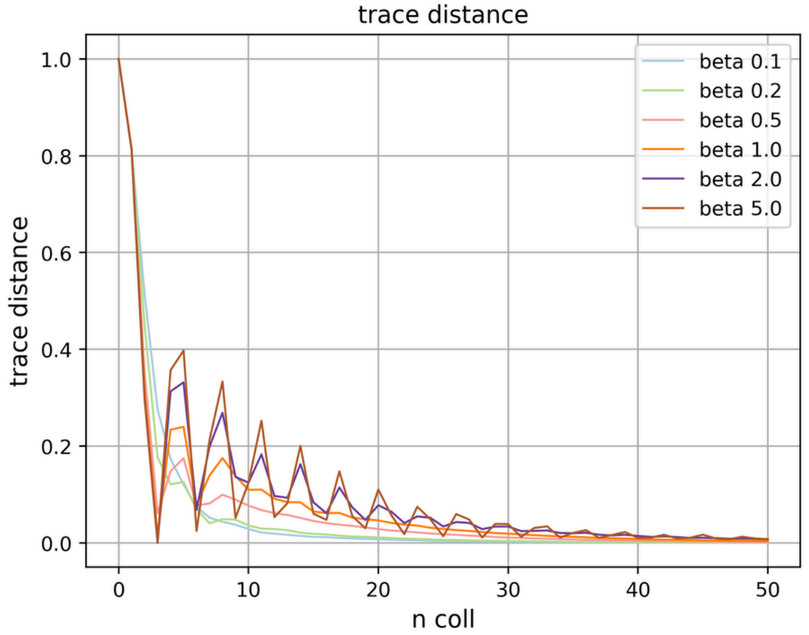

A second detachment mechanism: natural decay

Introducing an average lifetime for the auxiliary units attached to the cluster:

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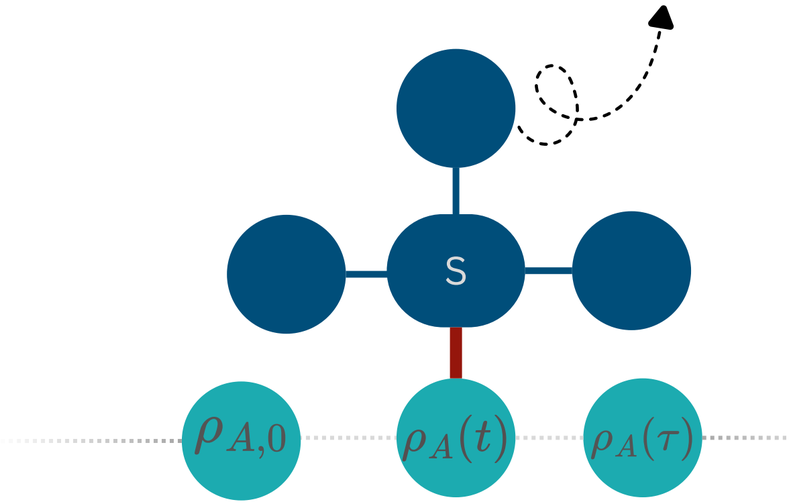
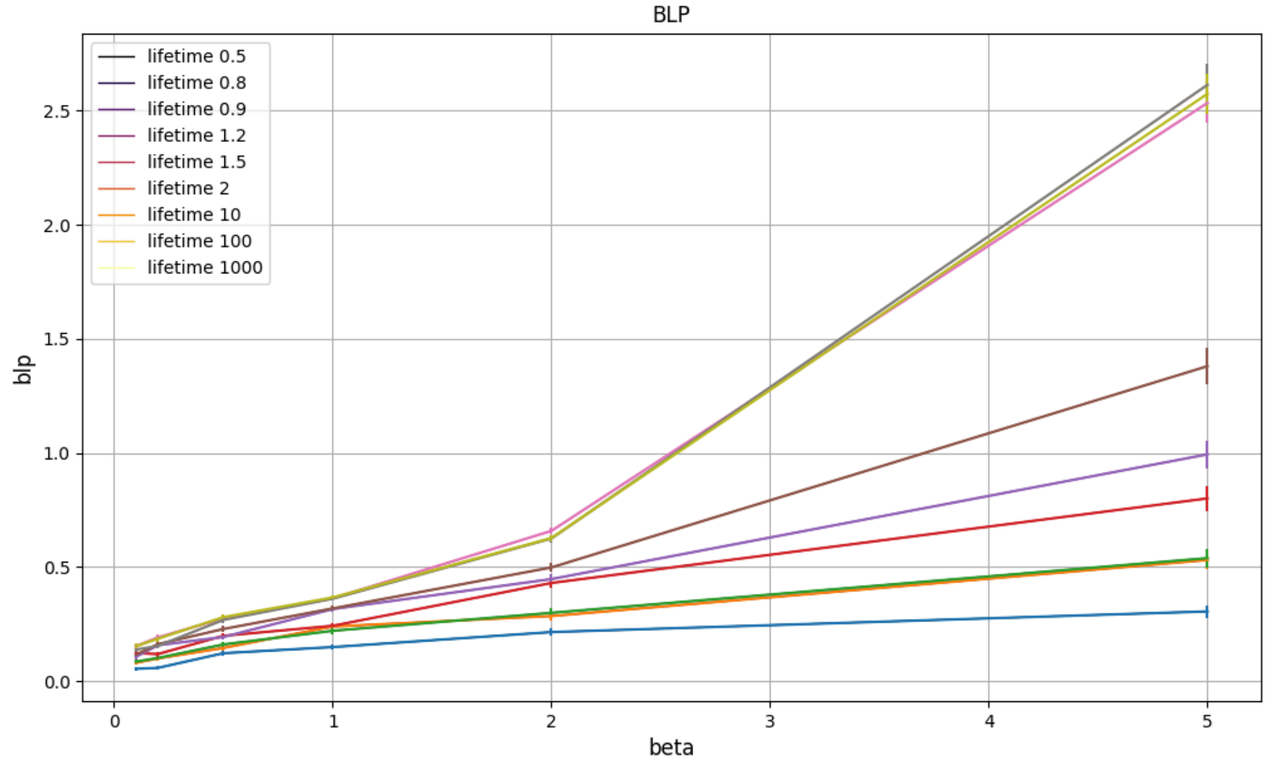


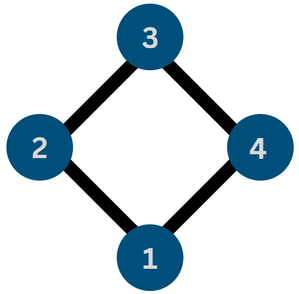
Laineet al. Physical Review A—Atomic, Molecular, and Optical Physics 81.6 (2010): 062115.

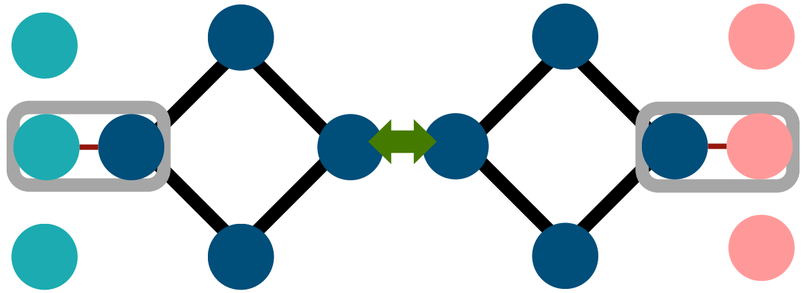


Summary and conclusions

Heisenberg spin ring

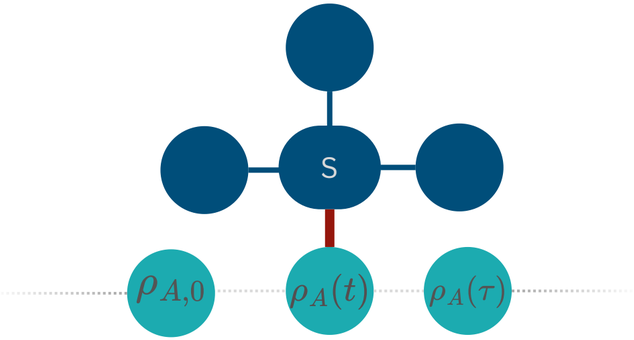
- ✓ Dynamics and thermodynamics
- ✓ TN version of the system





Clustering collision models

- ✓ Dynamics of the cluster



Goal: fully characterising energy exchanges