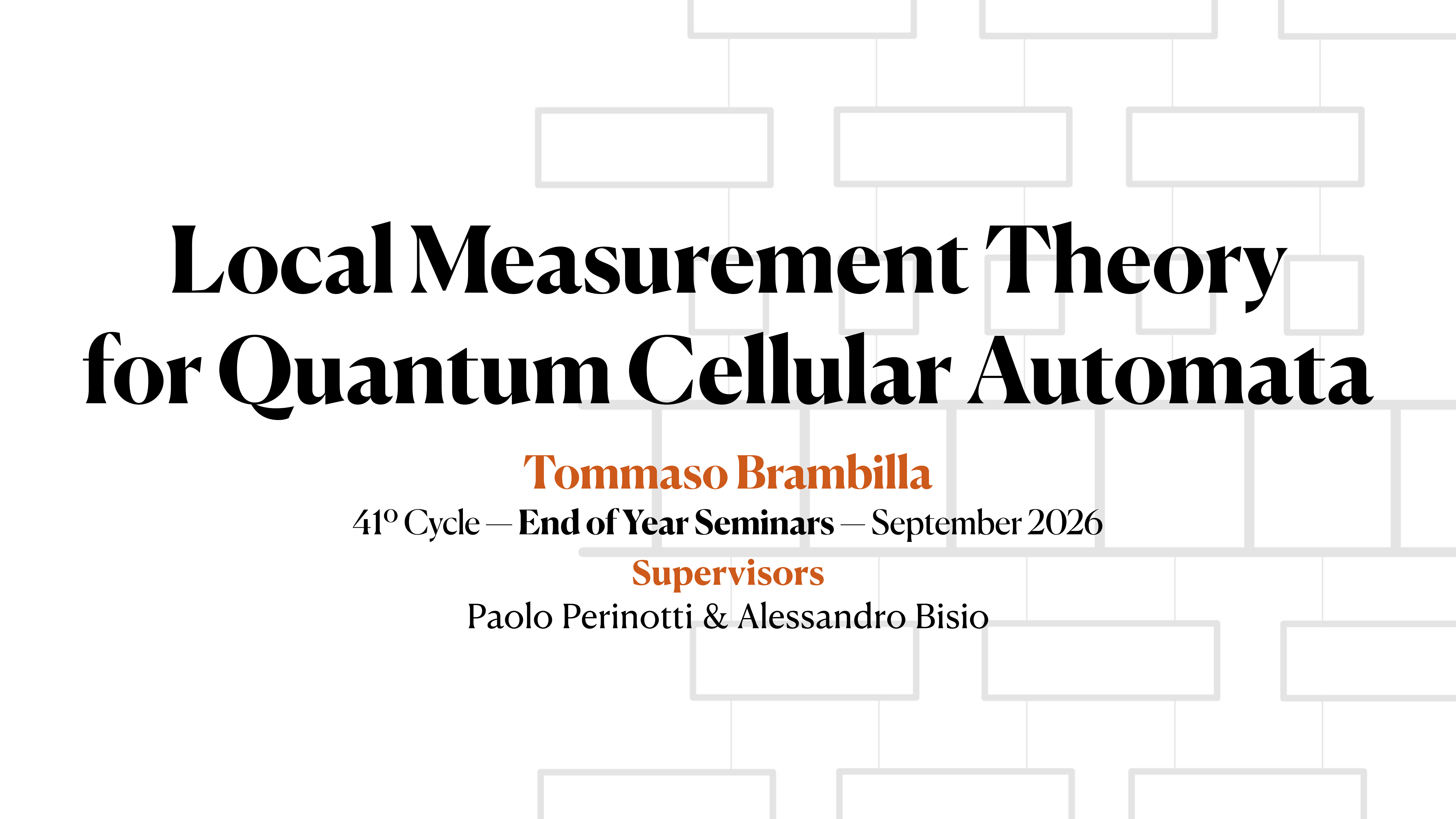




Local Measurement Theory for Quantum Cellular Automata

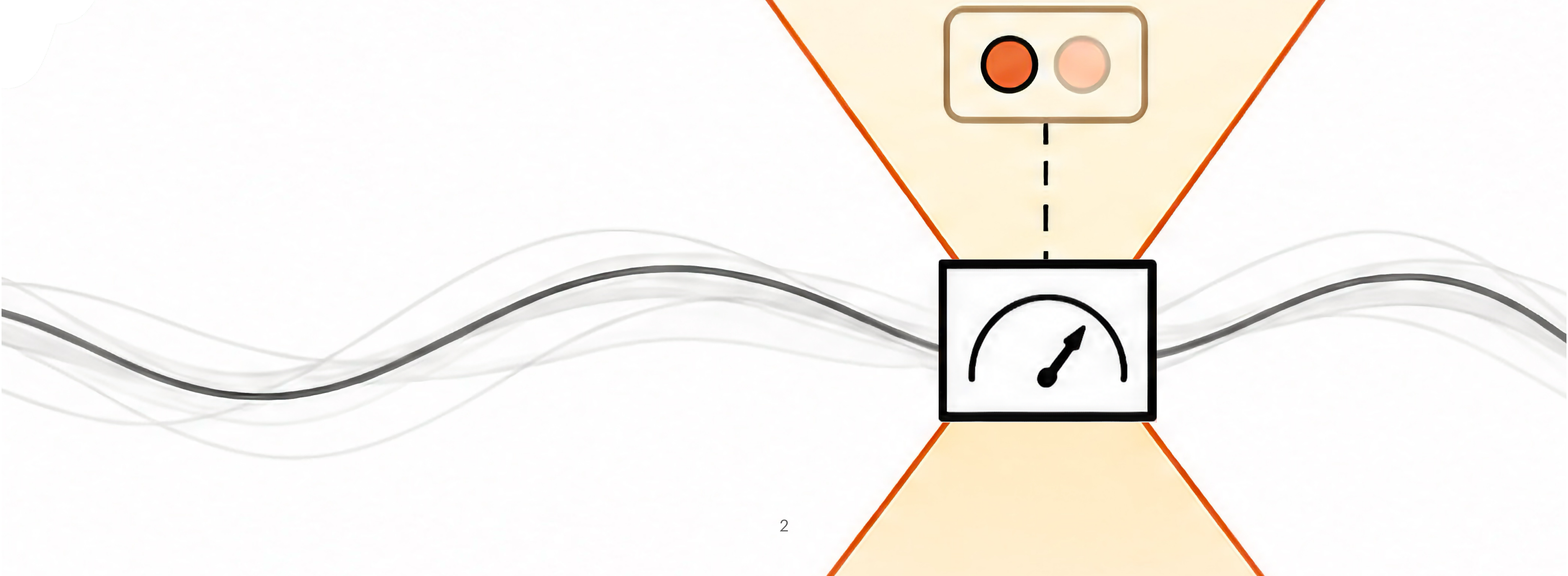
Tommaso Brambilla

41^o Cycle — End of Year Seminars — September 2026

Supervisors

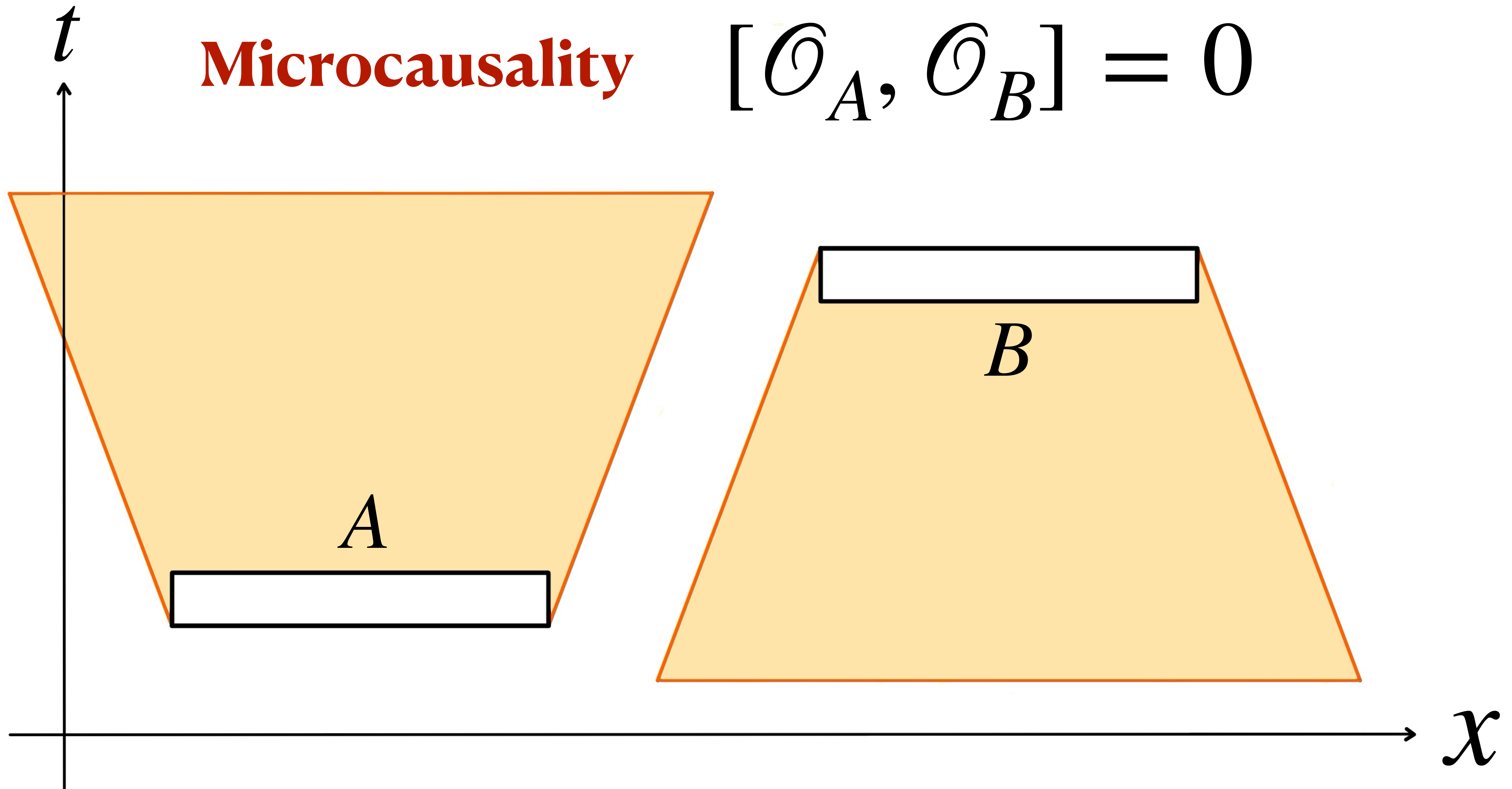
Paolo Perinotti & Alessandro Bisio

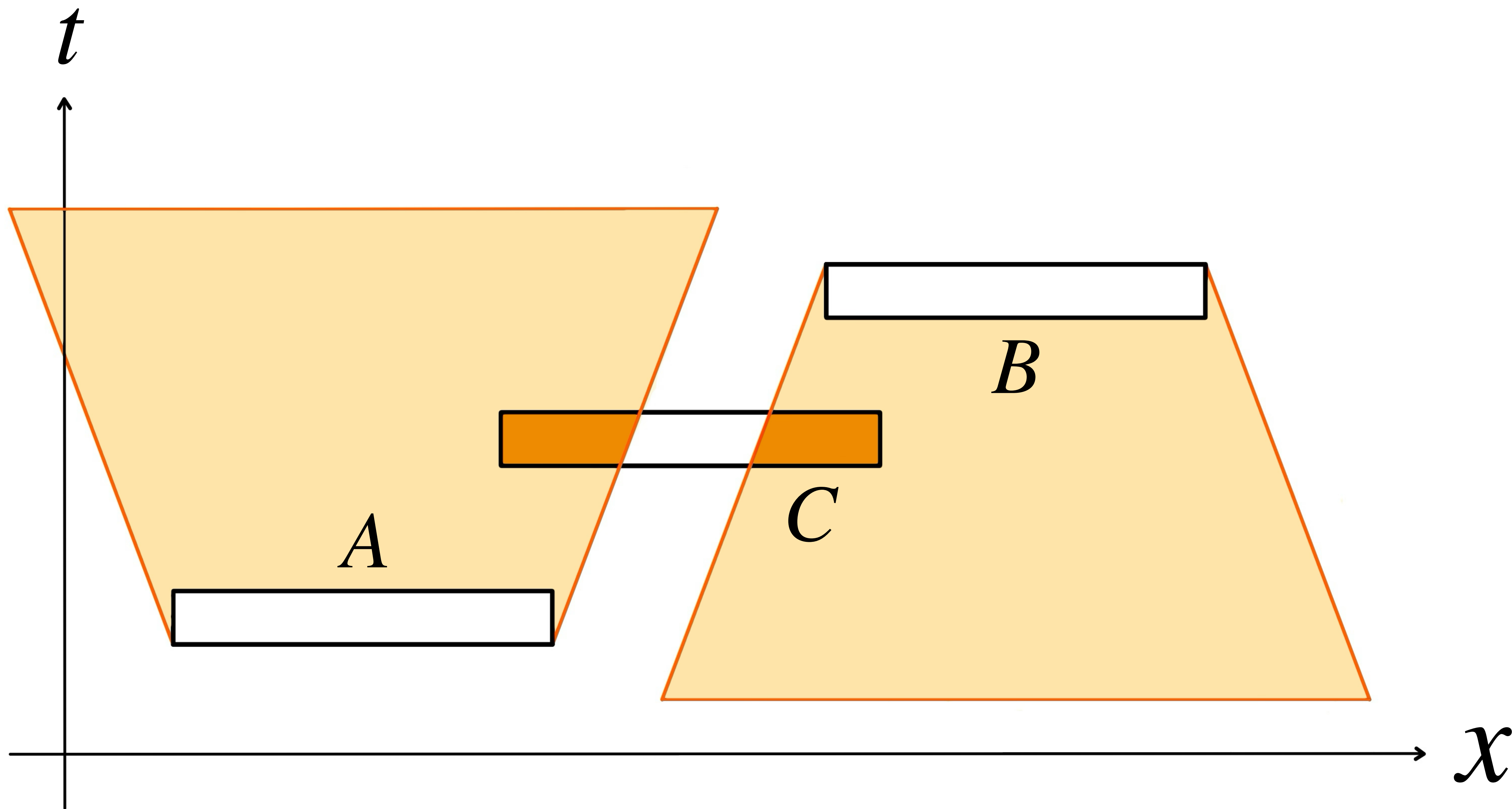
The Problem of Local Measurements in Quantum Field Theory

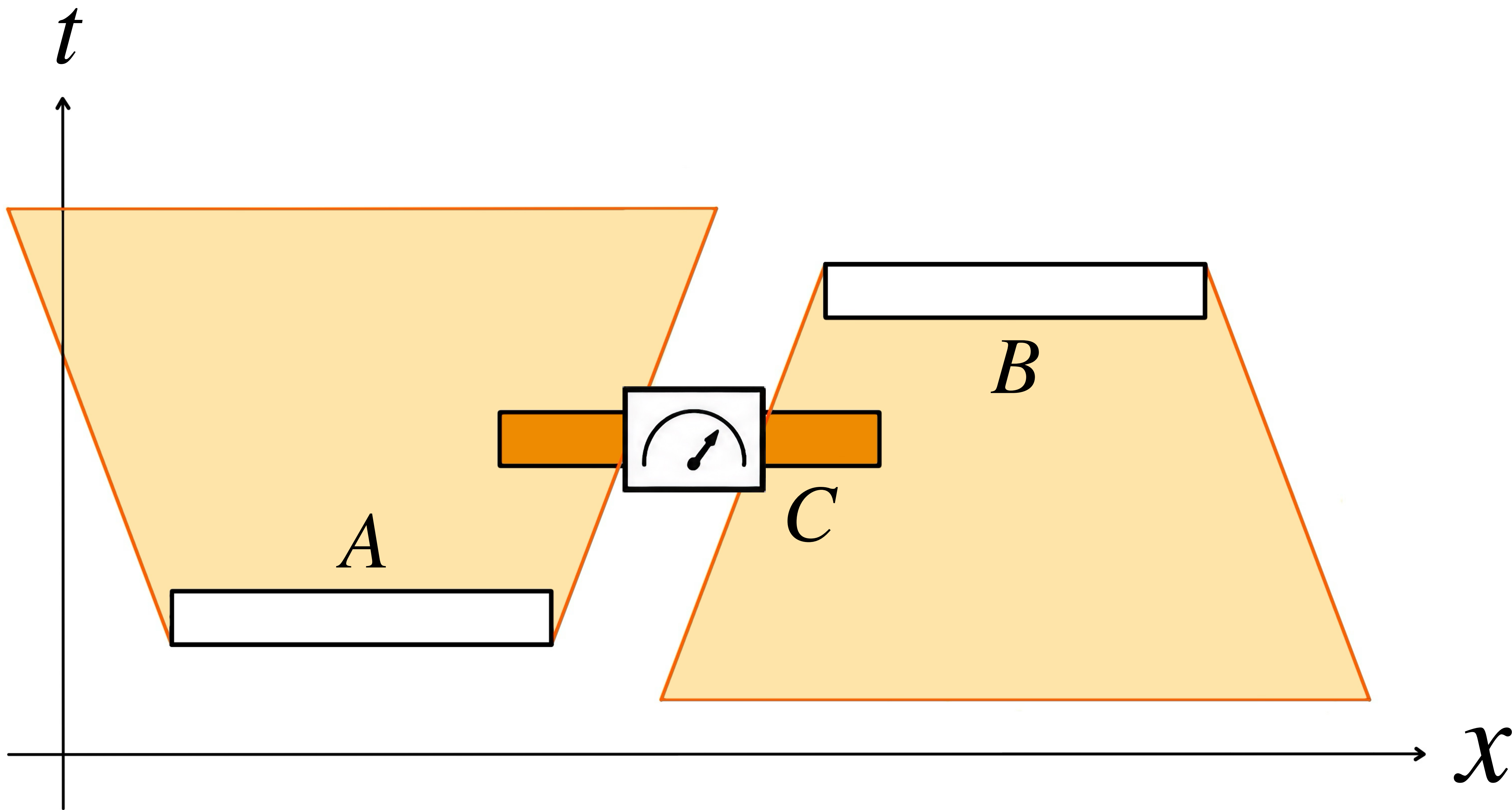


Microcausality

$$[\mathcal{O}_A, \mathcal{O}_B] = 0$$







t

Impossible Measurements on Quantum Fields*

RAFAEL D. SORKIN

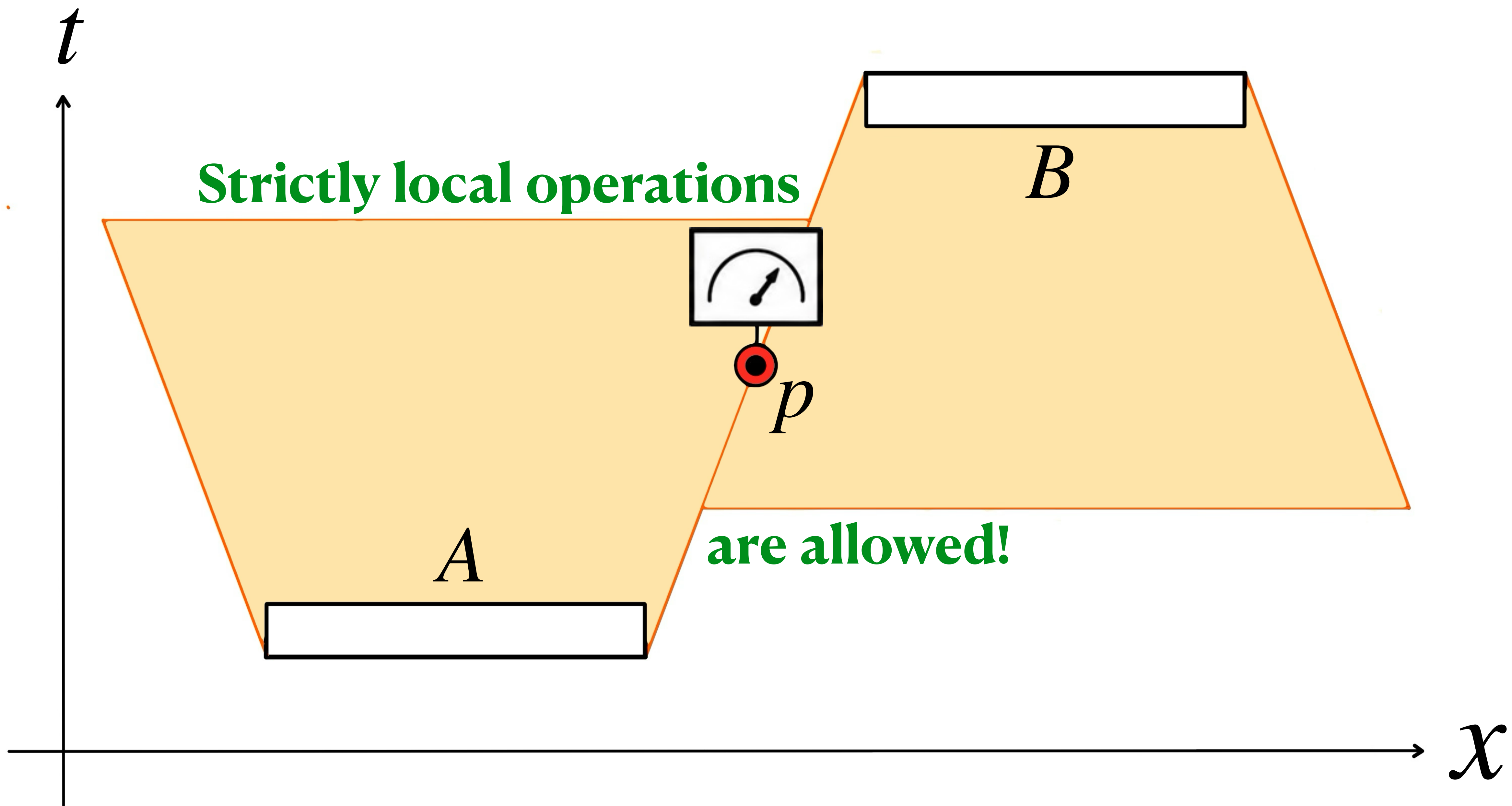
Department of Physics, Syracuse University, Syracuse NY 13244-1130

Abstract

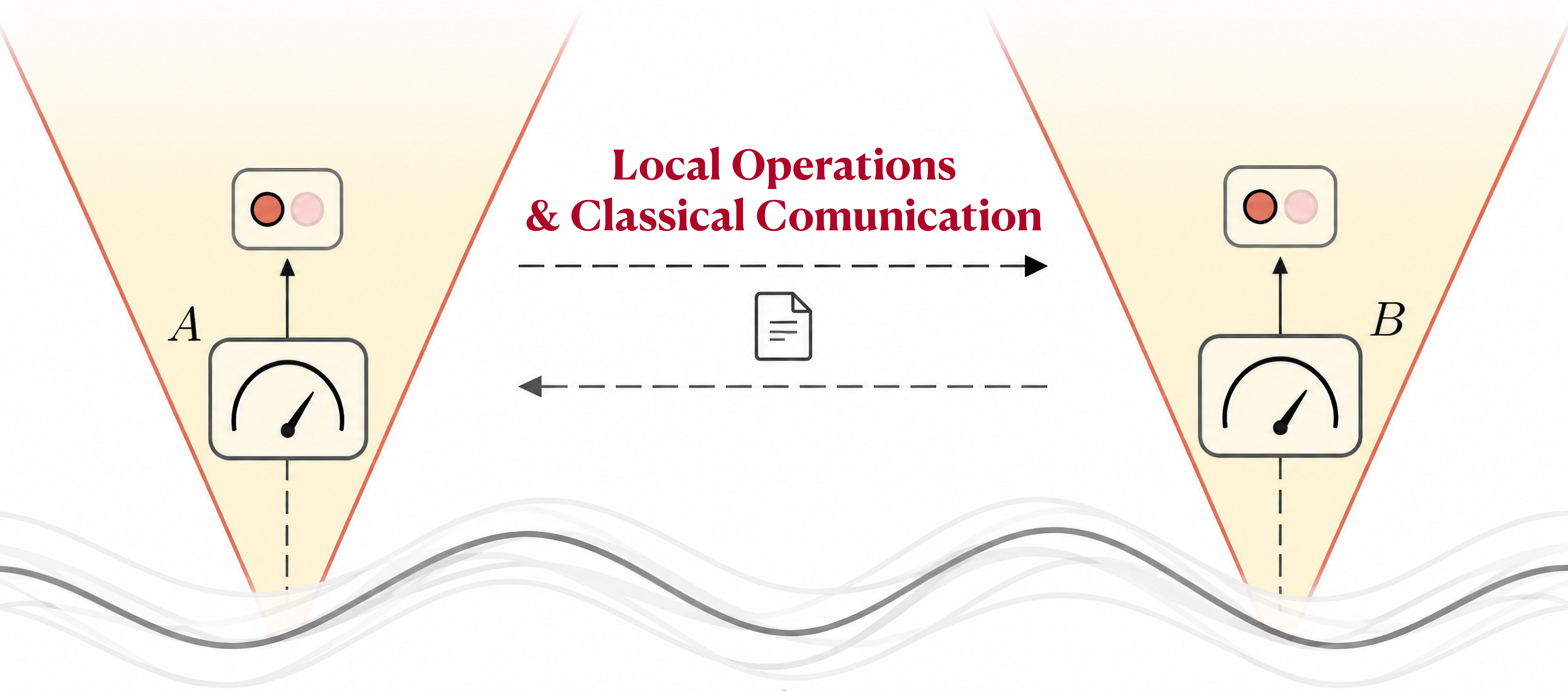
It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality,

with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for the free scalar field in flat spacetime. It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity.

 x



Existing Approches



Existing Approches

Quantum fields and local measurements

Christopher J. Fewster^{*1} and Rainer Verch^{†2}

¹*Department of Mathematics, University of York, Heslington, York YO10 5DD, United Kingdom.*

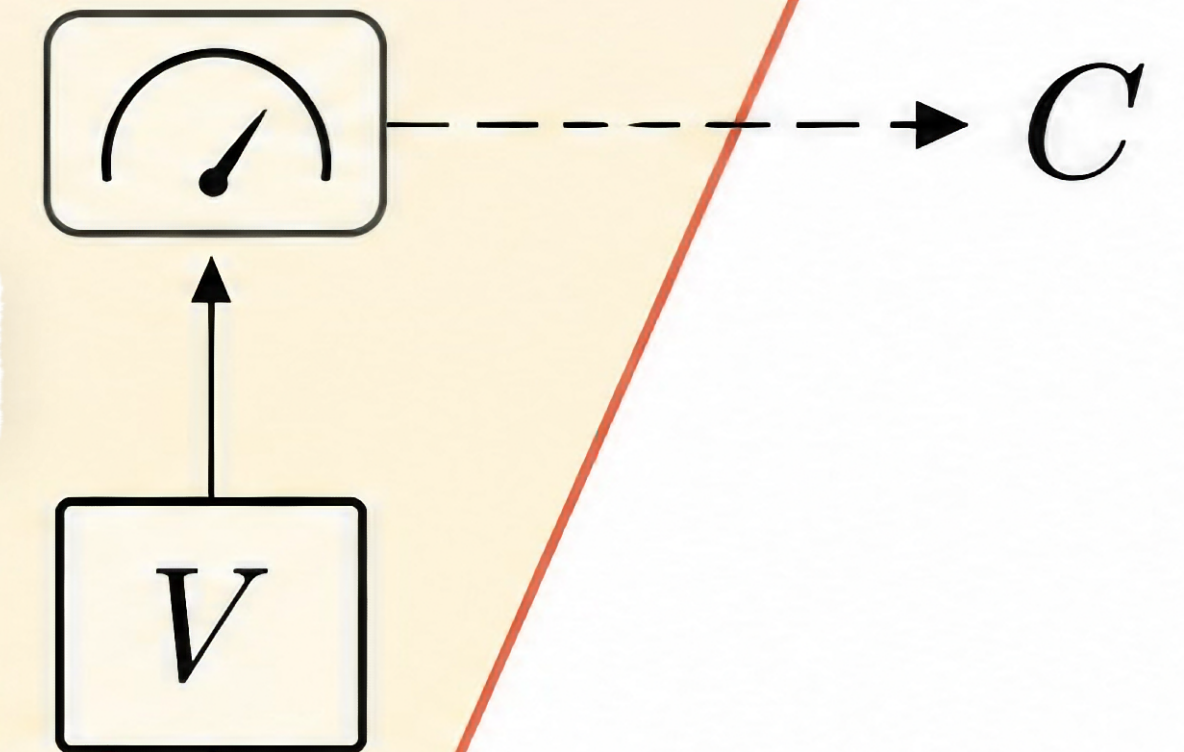
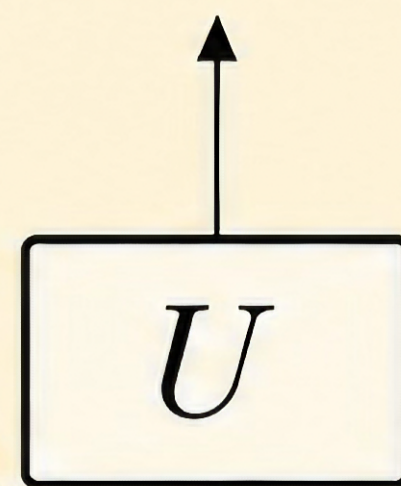
²*Institute for Theoretical Physics, University of Leipzig, 04009 Leipzig, Germany.*

July 27, 2020

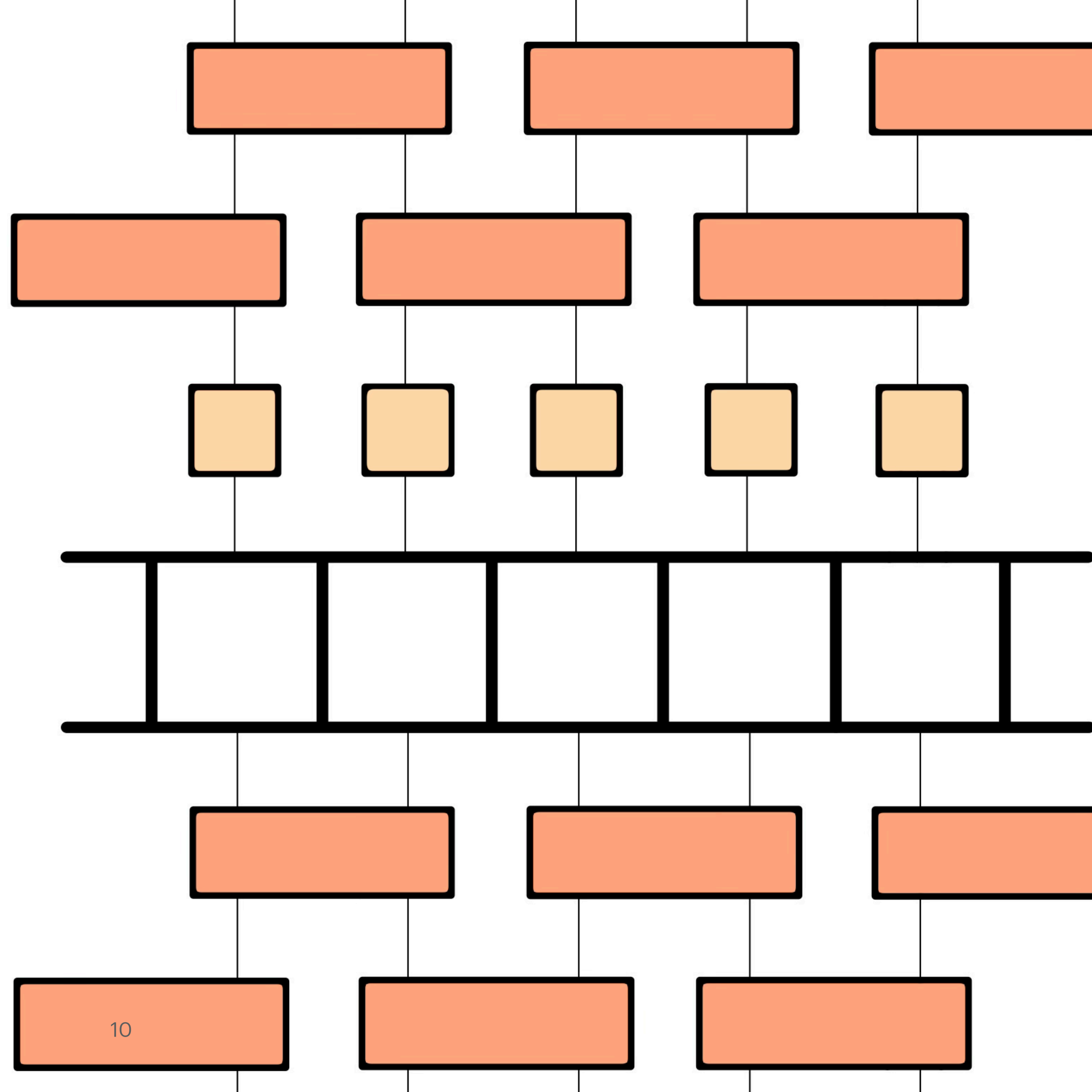
Abstract

field theory on curved spacetimes. Measurements are carried out on one quantum field theory, the “system”, using another, the “probe”. The measurement process involves a dynamical

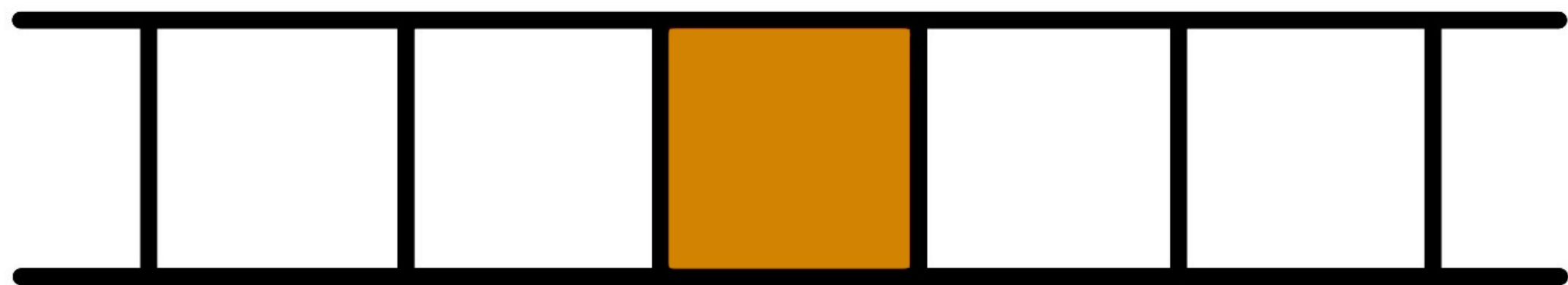
coupling of “system” and “probe” within a bounded spacetime region. The resulting “coupled theory” determines a scattering map on the uncoupled combination of the “system” and “probe” by reference to natural “in” and “out” spacetime regions. No specific interaction is assumed and all constructions are local and covariant.



Quantum Cellular Automata: An Operational Approach



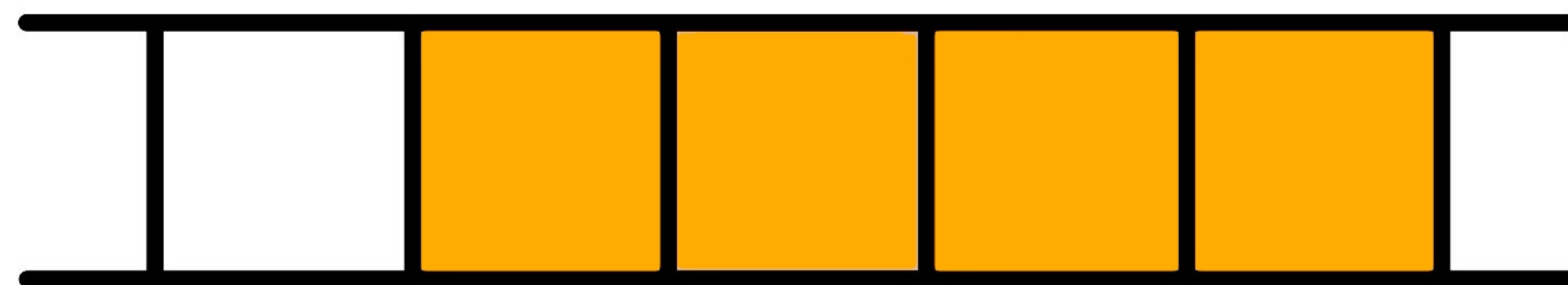
$$x \in \mathcal{L}$$



$$A_x \simeq M_d(\mathbb{C})$$

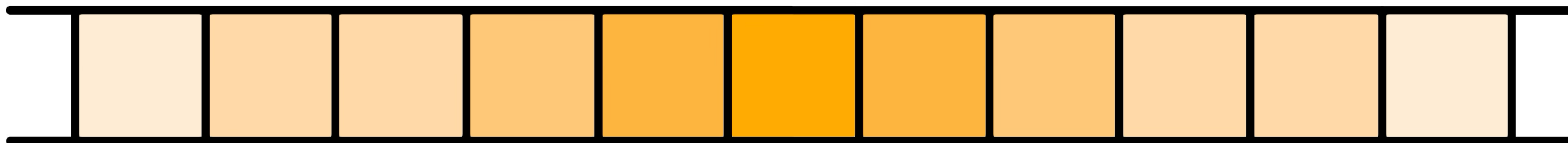
$$d < \infty$$

$$\Omega \subset \mathcal{L}$$

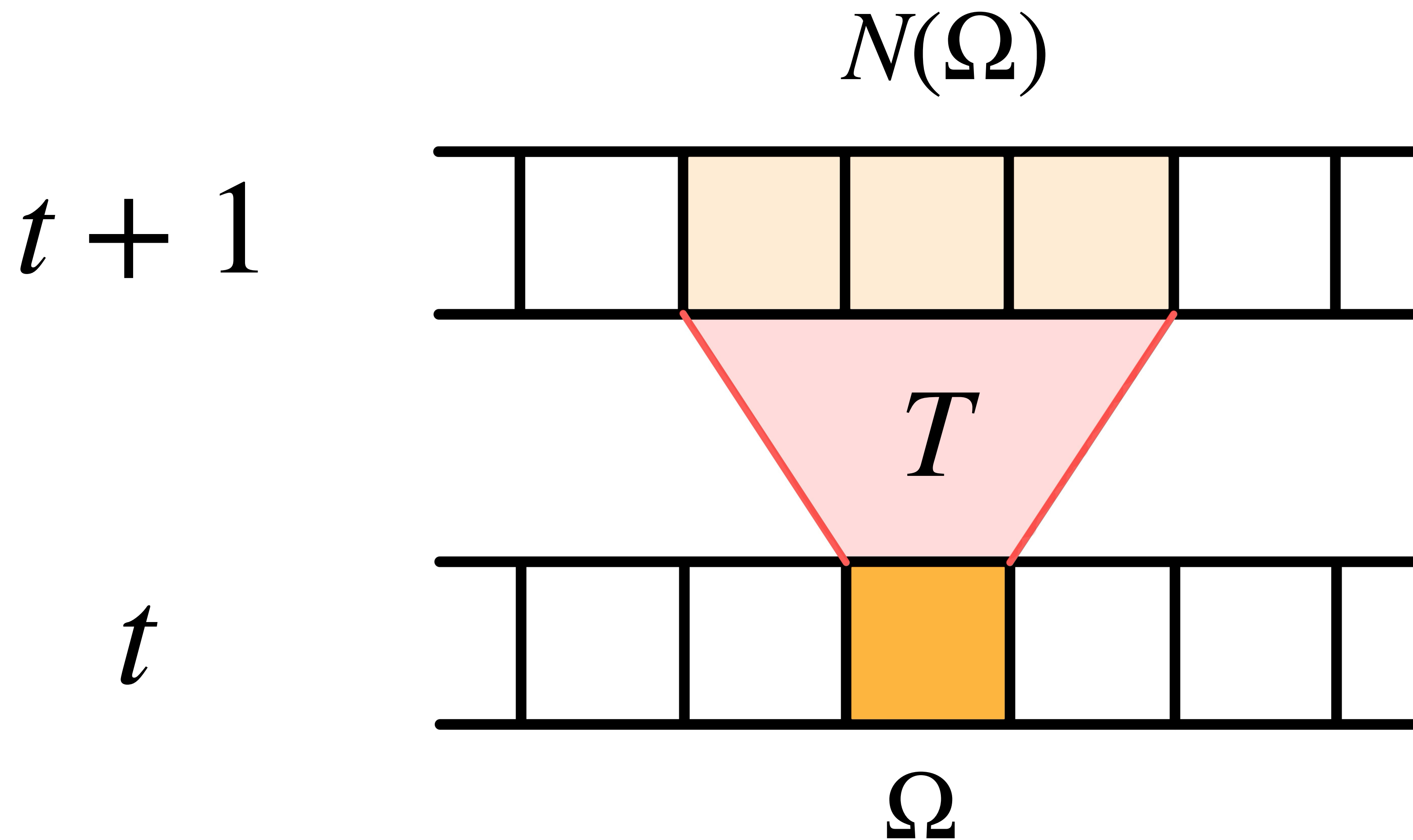


$$A_\Omega = \bigotimes_{x \in \Omega} A_x$$

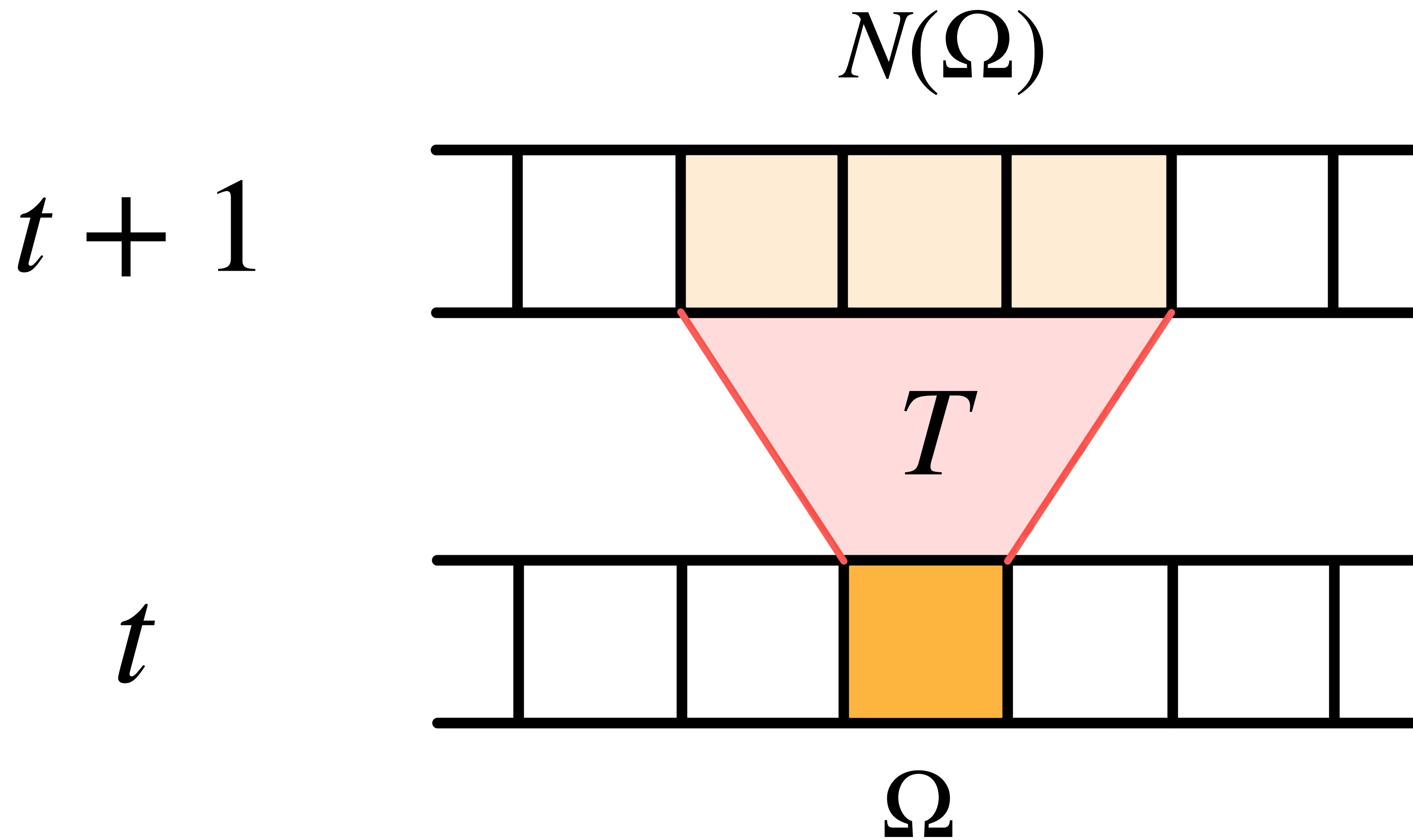
$\mathcal{L}$



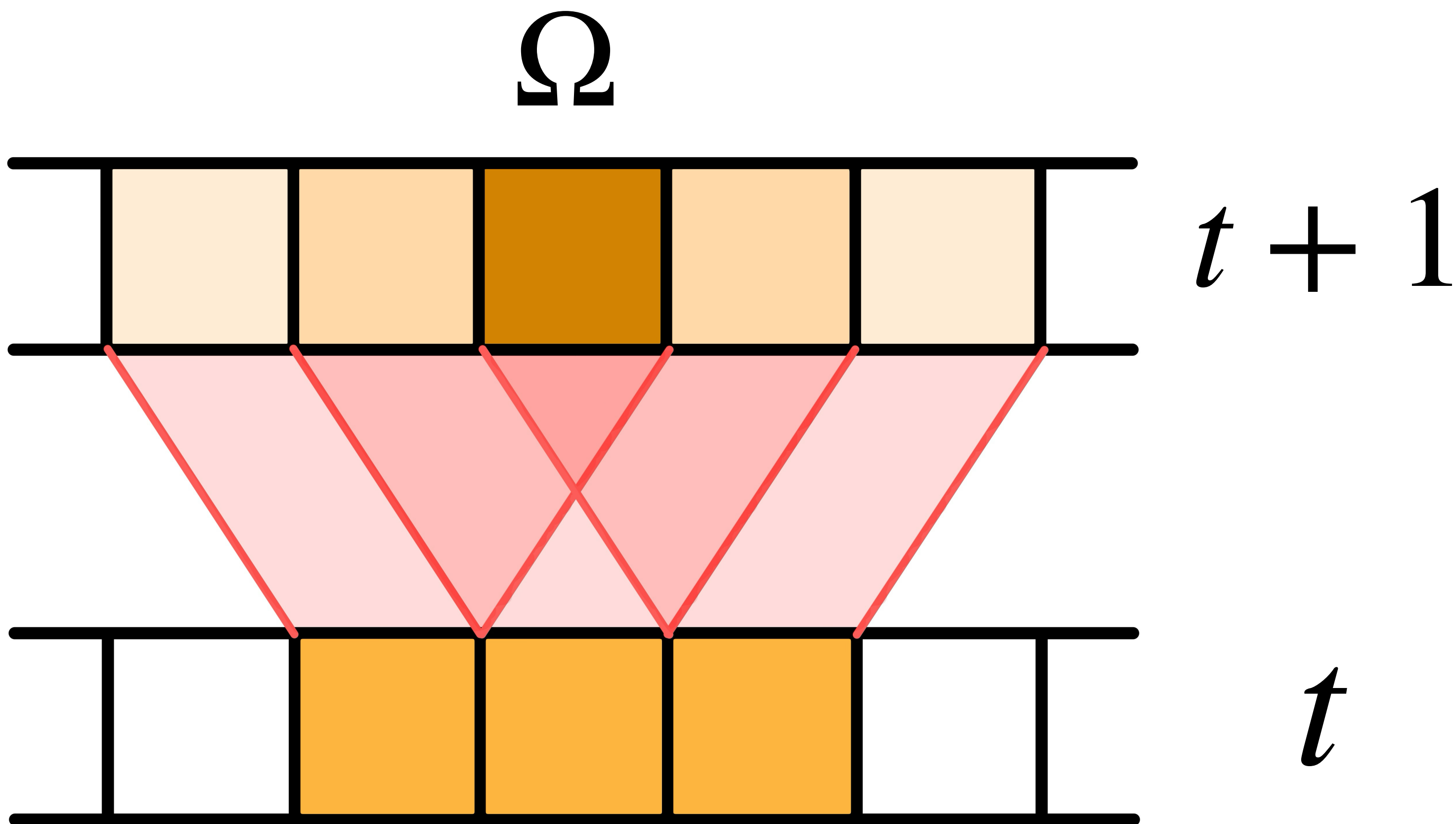
$$A_{\mathcal{L}} := \overline{\bigcup_{|\Omega| < \infty} A_{\Omega}}$$



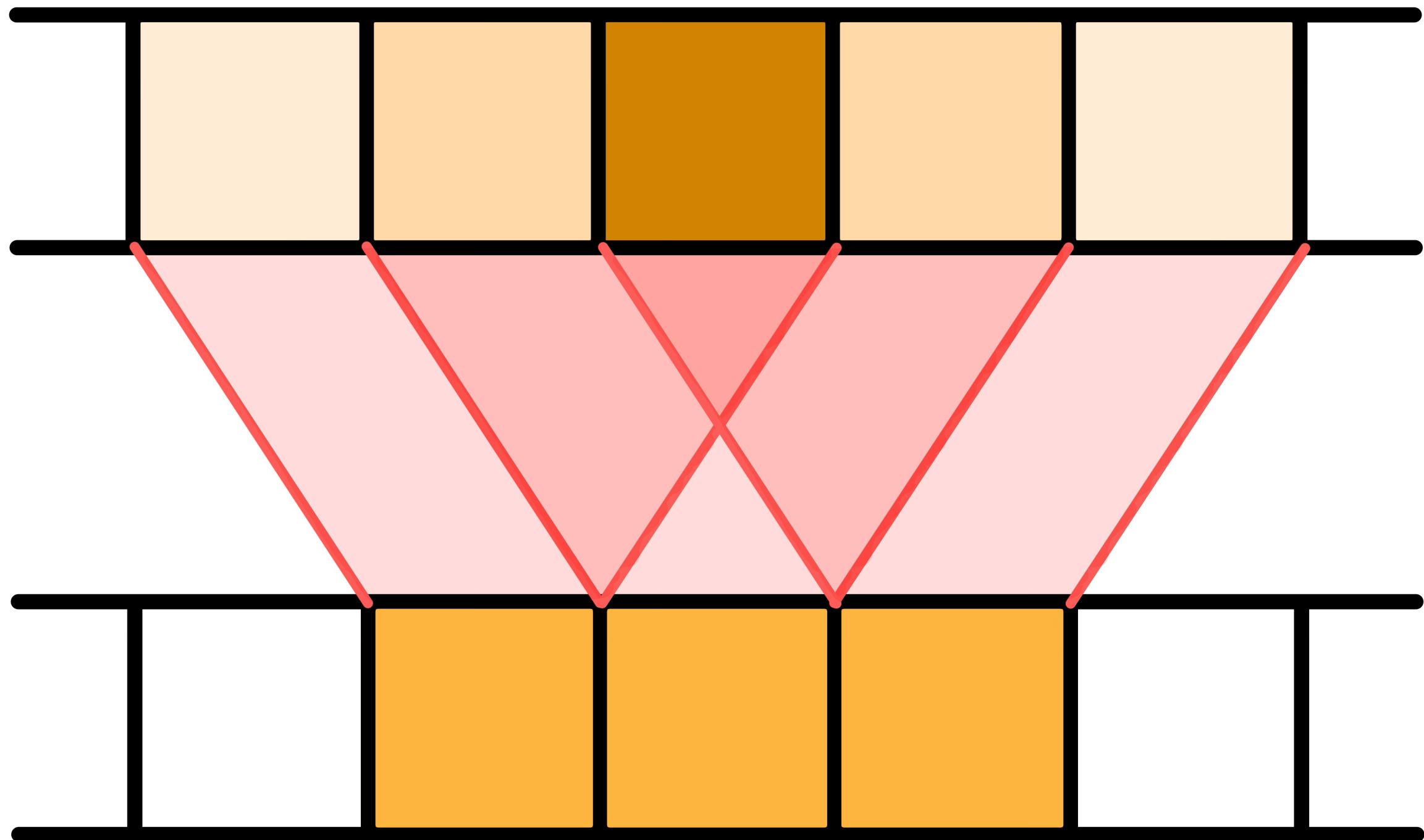
$$T \in \text{Aut}(A_{\mathcal{L}}) \quad T(A_{\Omega}) \subset A_{N(\Omega)} \quad \text{Locality}$$



$$T(A_{\Omega}) = U_{N(\Omega)}^{\dagger} \left(A_{\Omega} \otimes \mathbf{I}_{N(\Omega) \setminus \Omega} \right) U_{N(\Omega)} \subset A_{N(\Omega)}$$

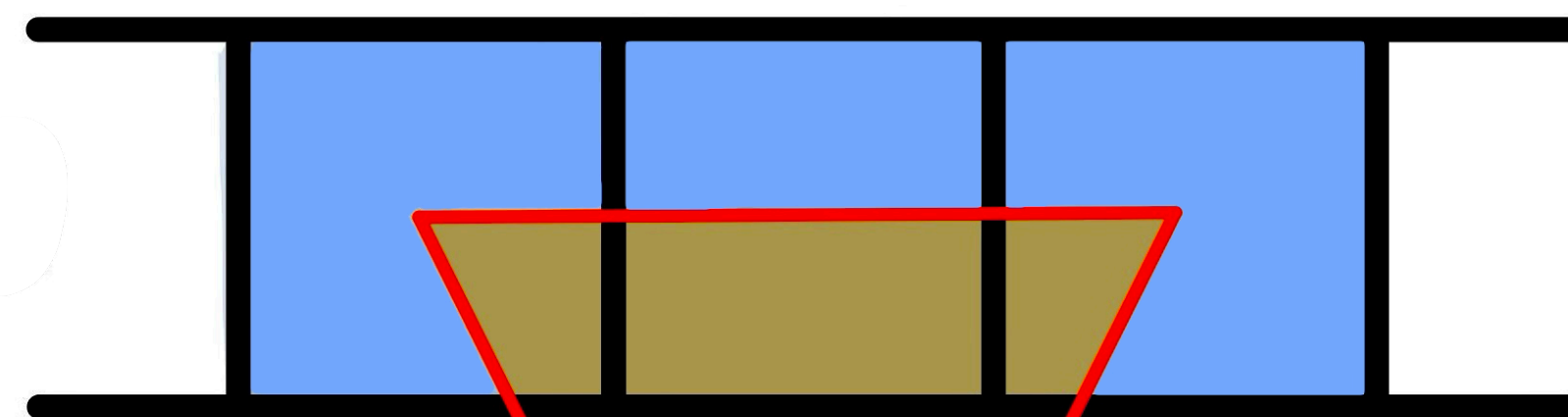
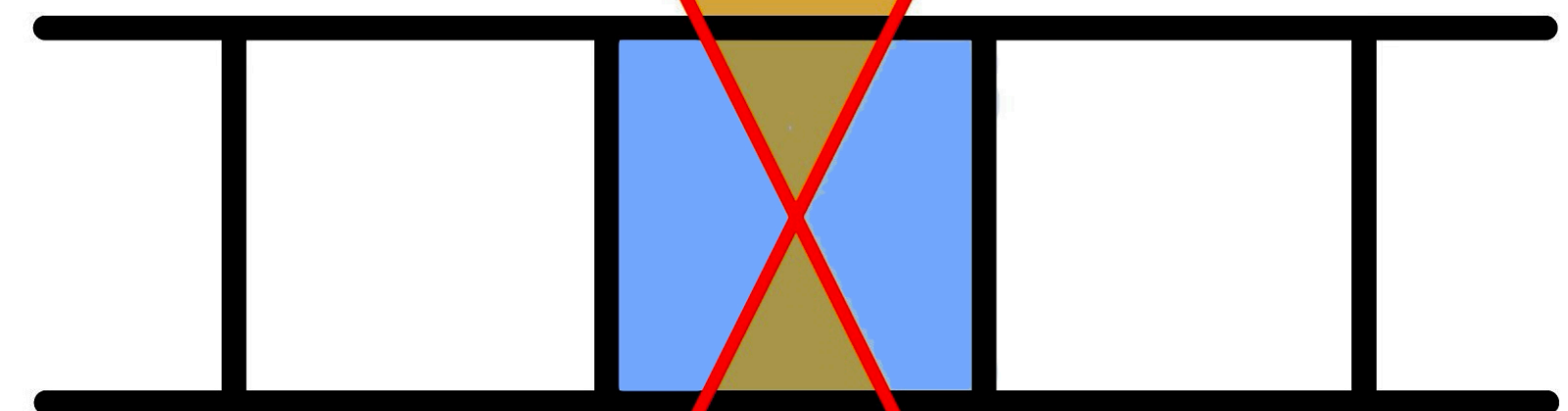
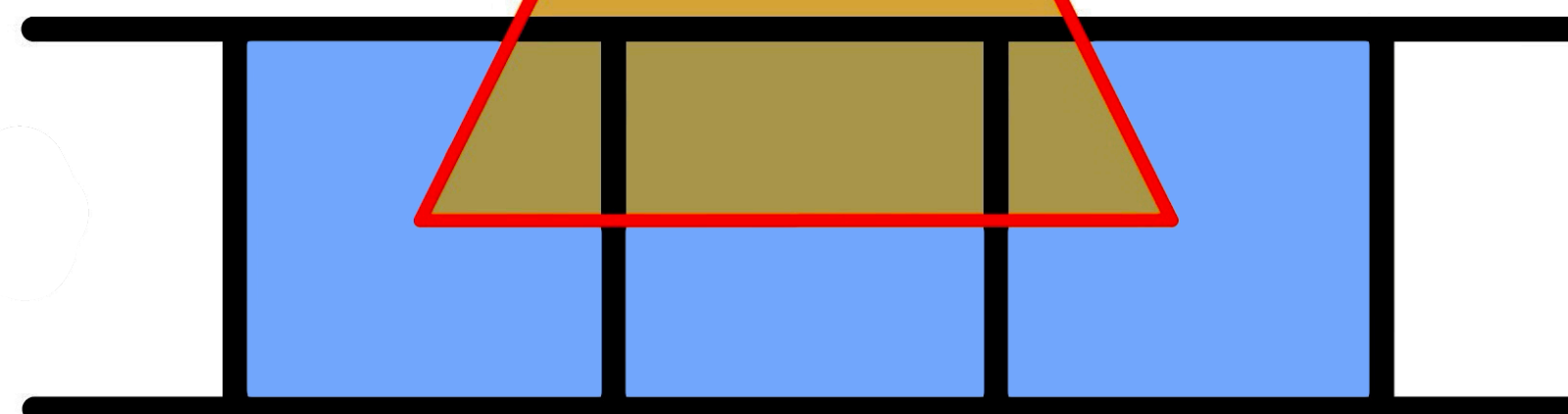


$$T^{-1}(A_{\Omega}) \subset A_{N(\Omega)}$$

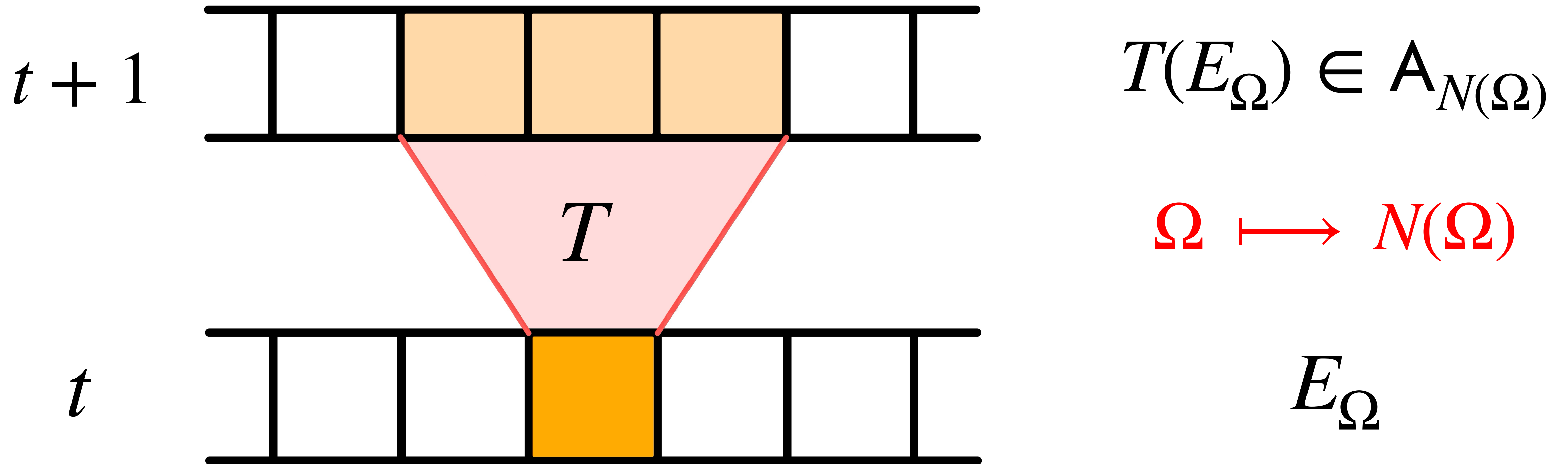
Ω

 $N(\Omega)$

$$T^{-1}(A_{\Omega}) \subset A_{N(\Omega)}$$

Discrete Causal Structure

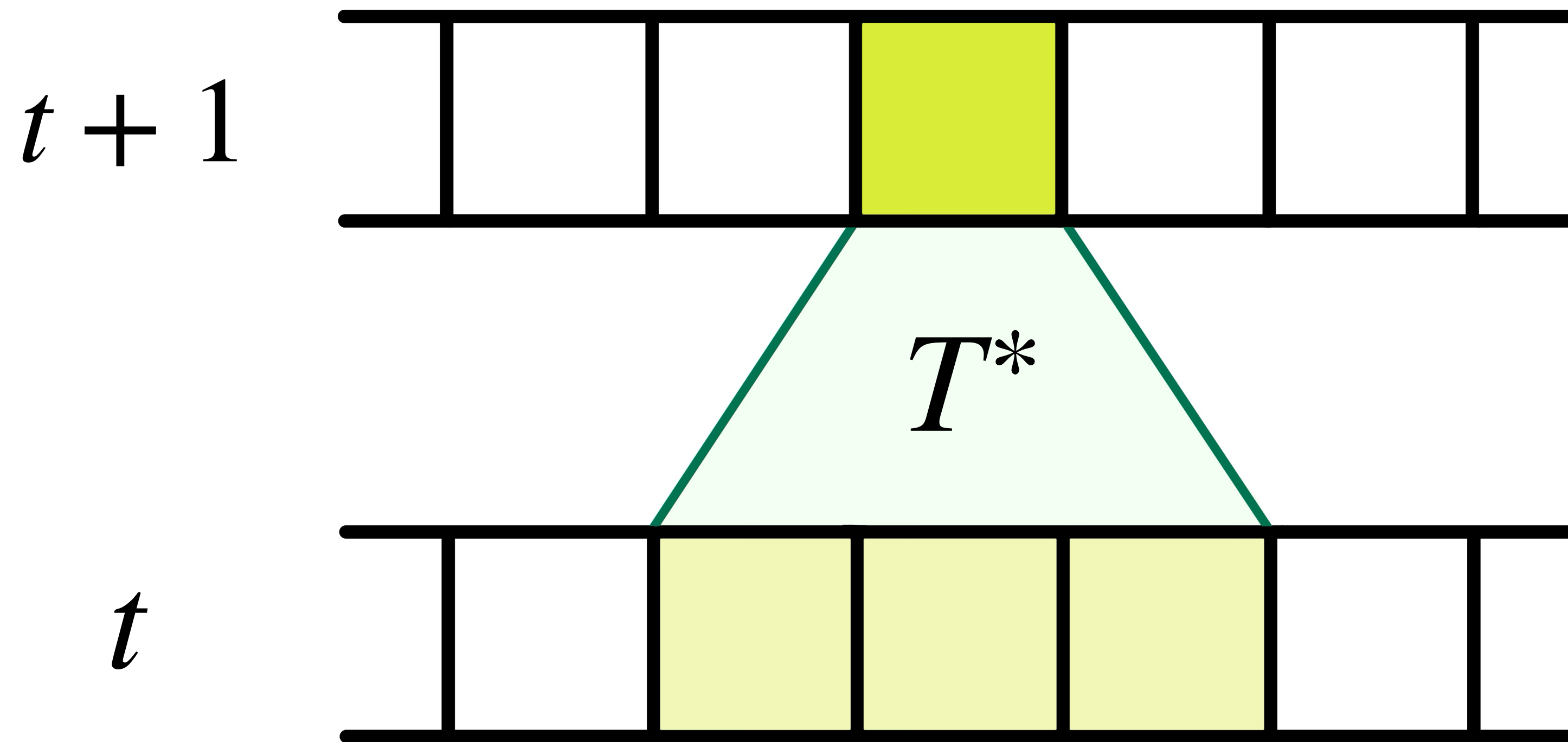
 $t + 1$

 t

 $t - 1$


Heisenberg Picture



$$\mathbf{p}_\rho[E_\Omega \mid t+1] = \text{tr}[\rho_{N(\Omega)} T(E_\Omega)]$$

Schrödinger Picture



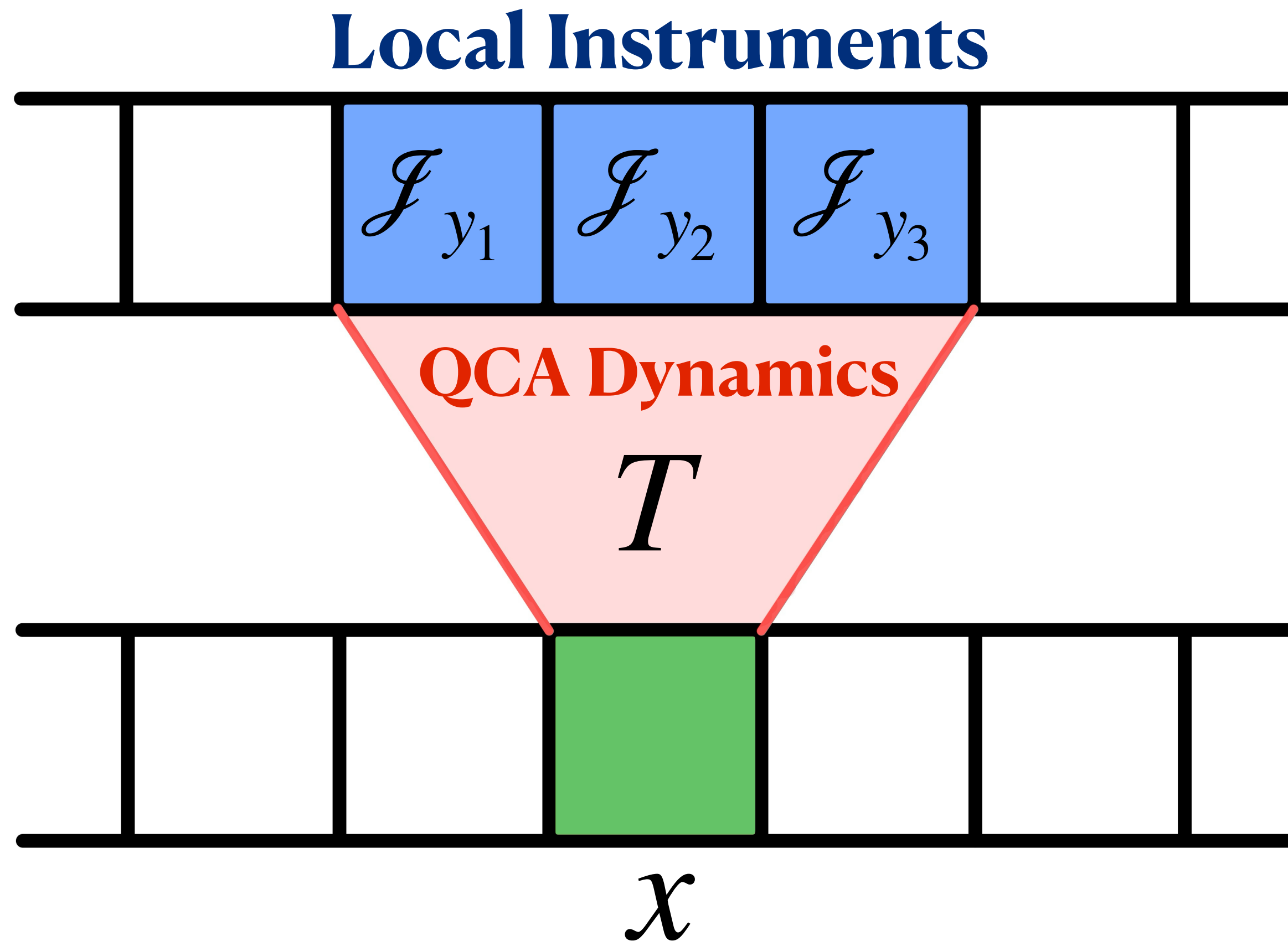
$$T^*(\rho_{N(\Omega)}) \in A_\Omega$$

$$N(\Omega) \mapsto \Omega$$

$$\rho_{N(\Omega)}$$

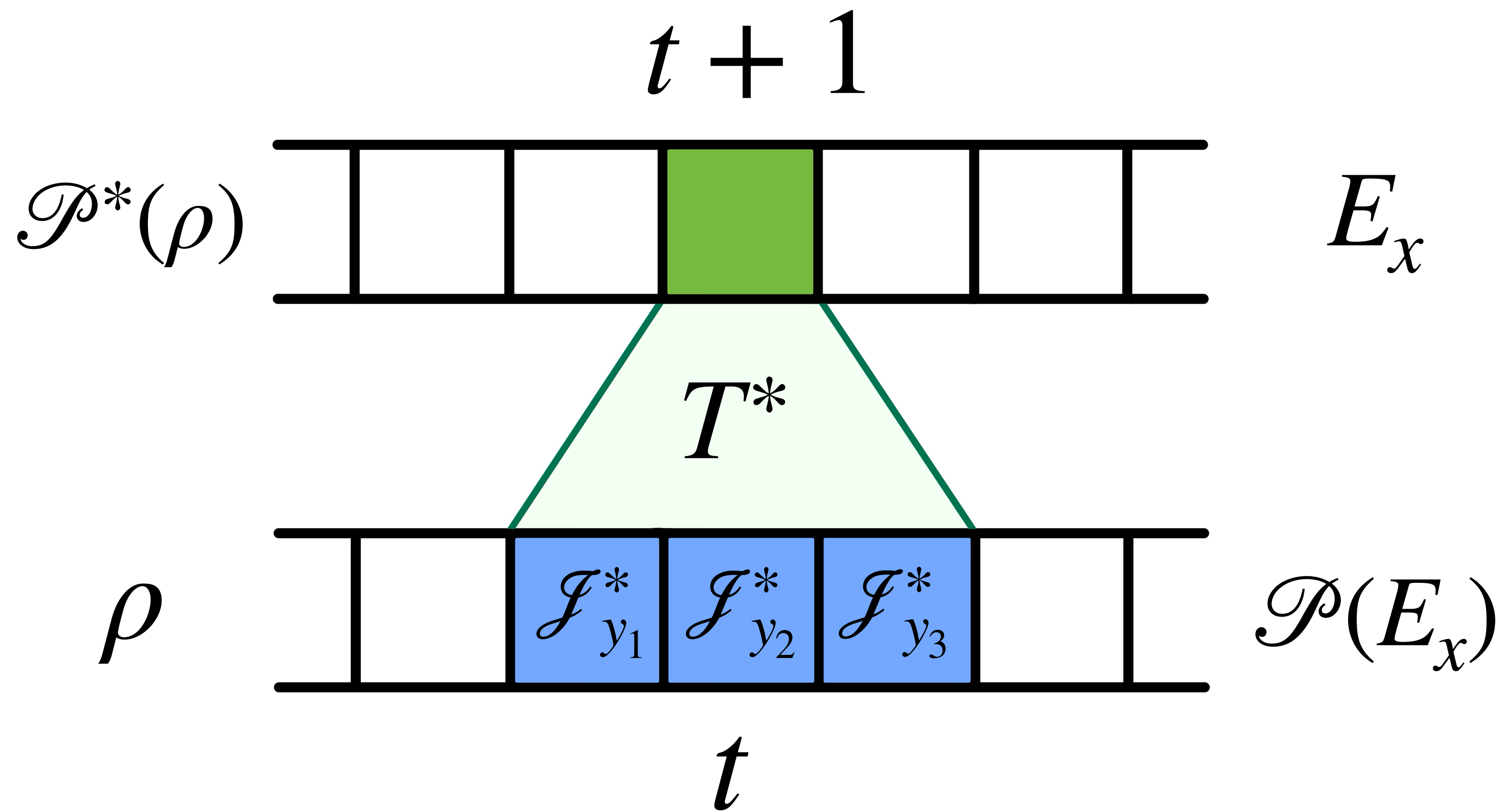
$$\mathbf{p}_\rho[E_\Omega \mid t+1] = \text{tr}[\rho_{N(\Omega)} T(E_\Omega)] = \text{tr}[T^*(\rho_{N(\Omega)}) E_\Omega]$$

Operational Measurement Protocols



$$\mathcal{P} = \bigotimes_{y \in N(x)} \mathcal{J}_y \circ T$$

$$E_x \mapsto \mathcal{P}(E_x) \in A_{N(x)}$$

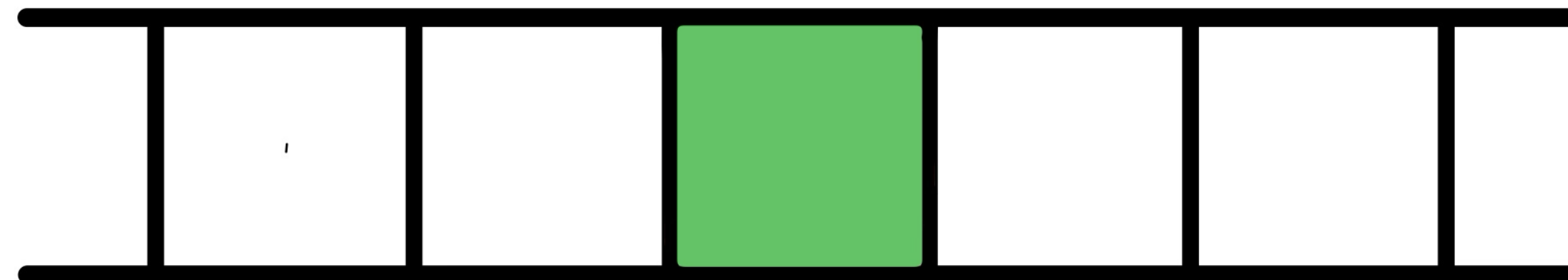


LOCAL

NONLOCAL

$$\mathcal{P}^*(\rho)(E_x) = \rho(\mathcal{P}(E_R))$$

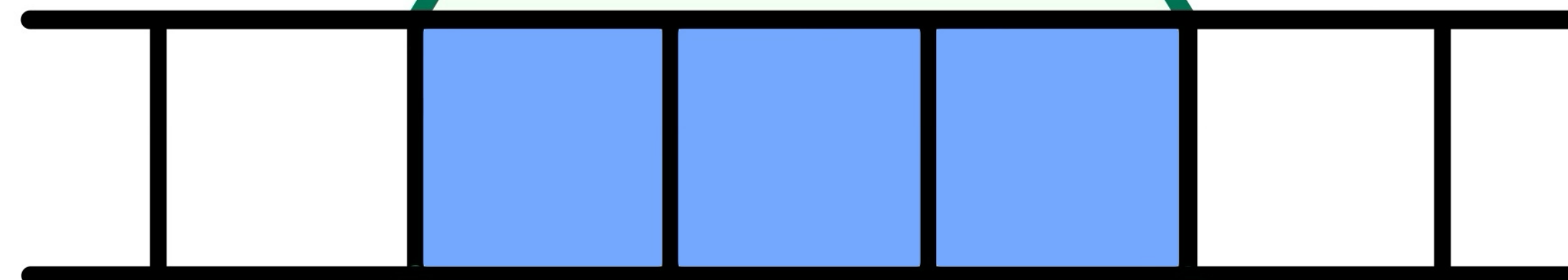
$t + 2$



Ω

T^*

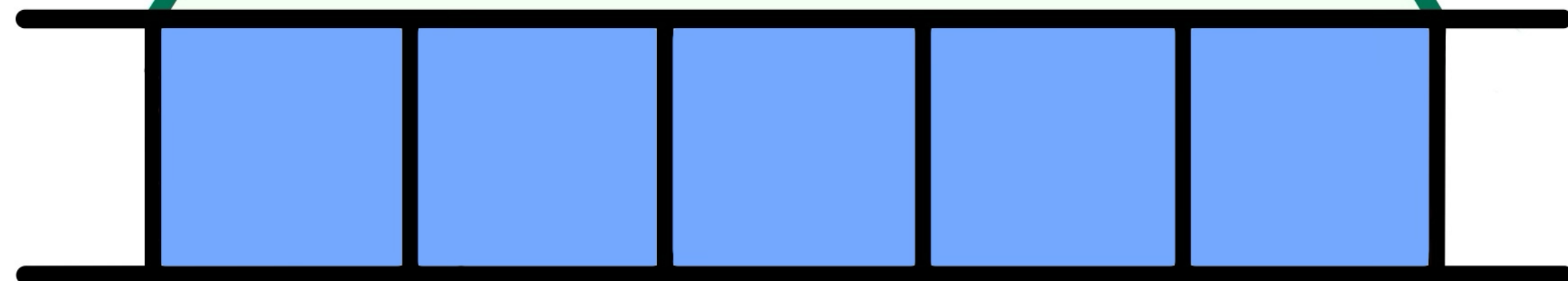
$t + 1$



$N(\Omega)$

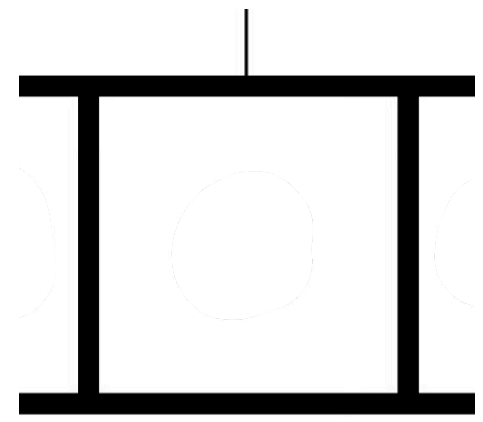
T^*

t

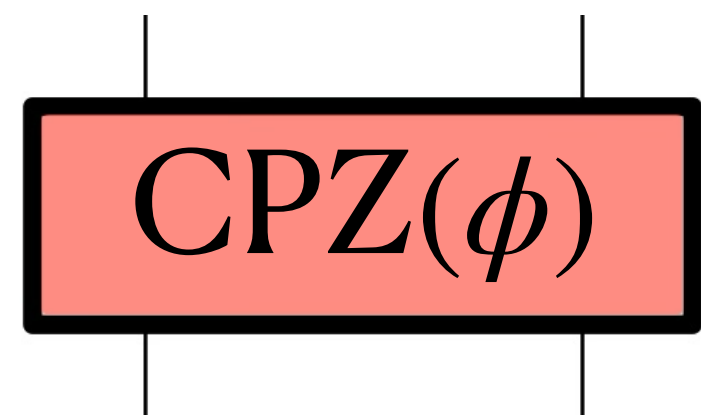
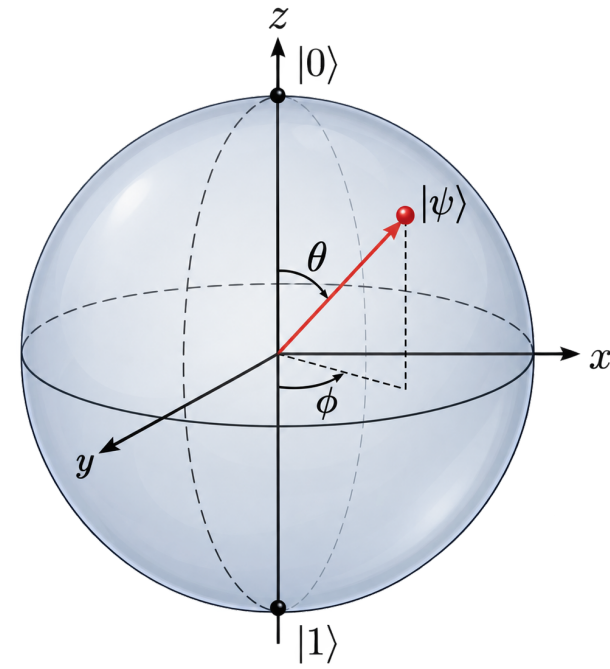


$N^2(\Omega)$

1D Nearest-Neighbour Qubit QCAs



$$\simeq M_2(\mathbb{C})$$

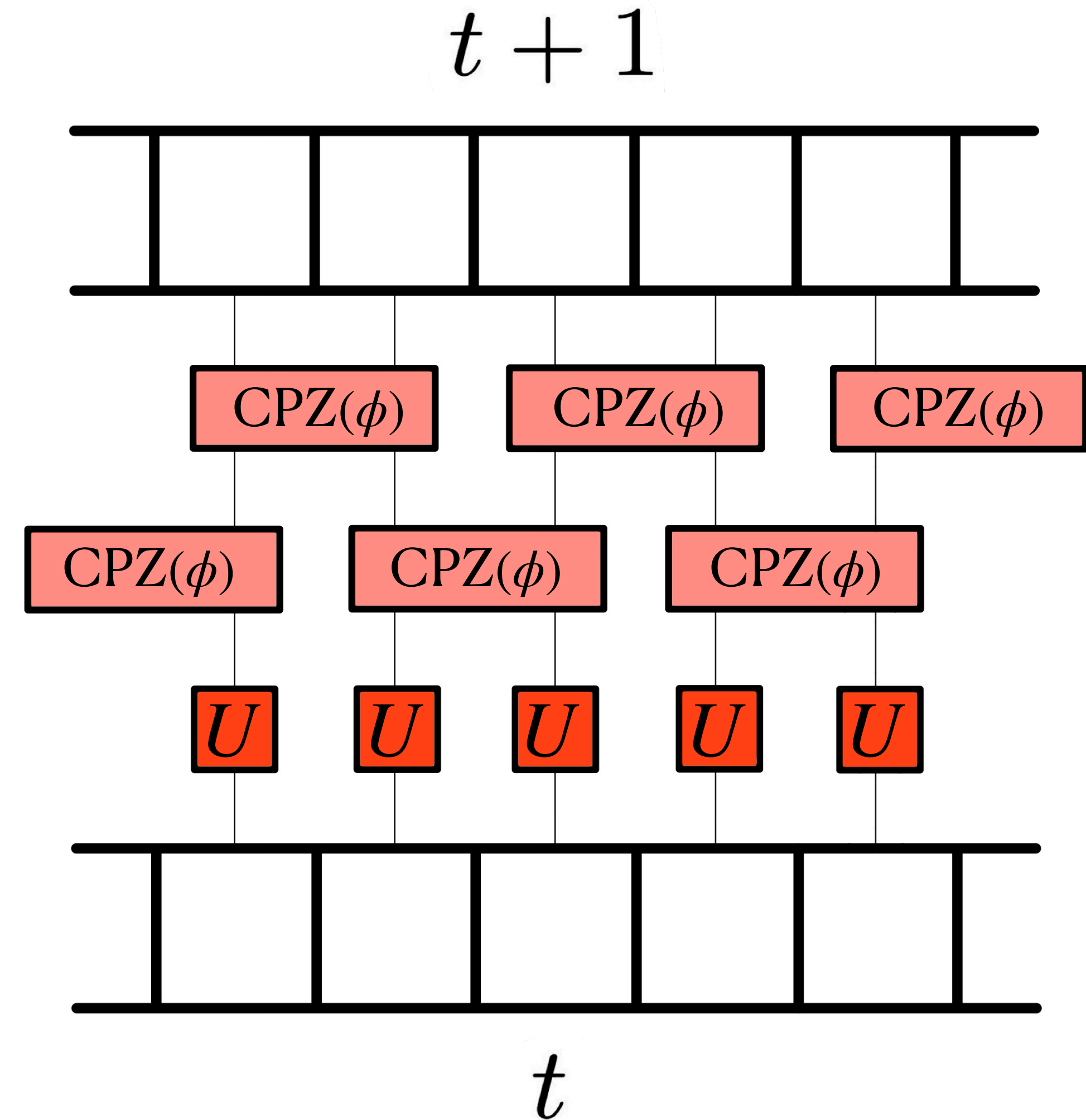


$$= \sum_{i \in \{0,1\}} |i\rangle\langle i| \otimes R_Z^i(\phi)$$

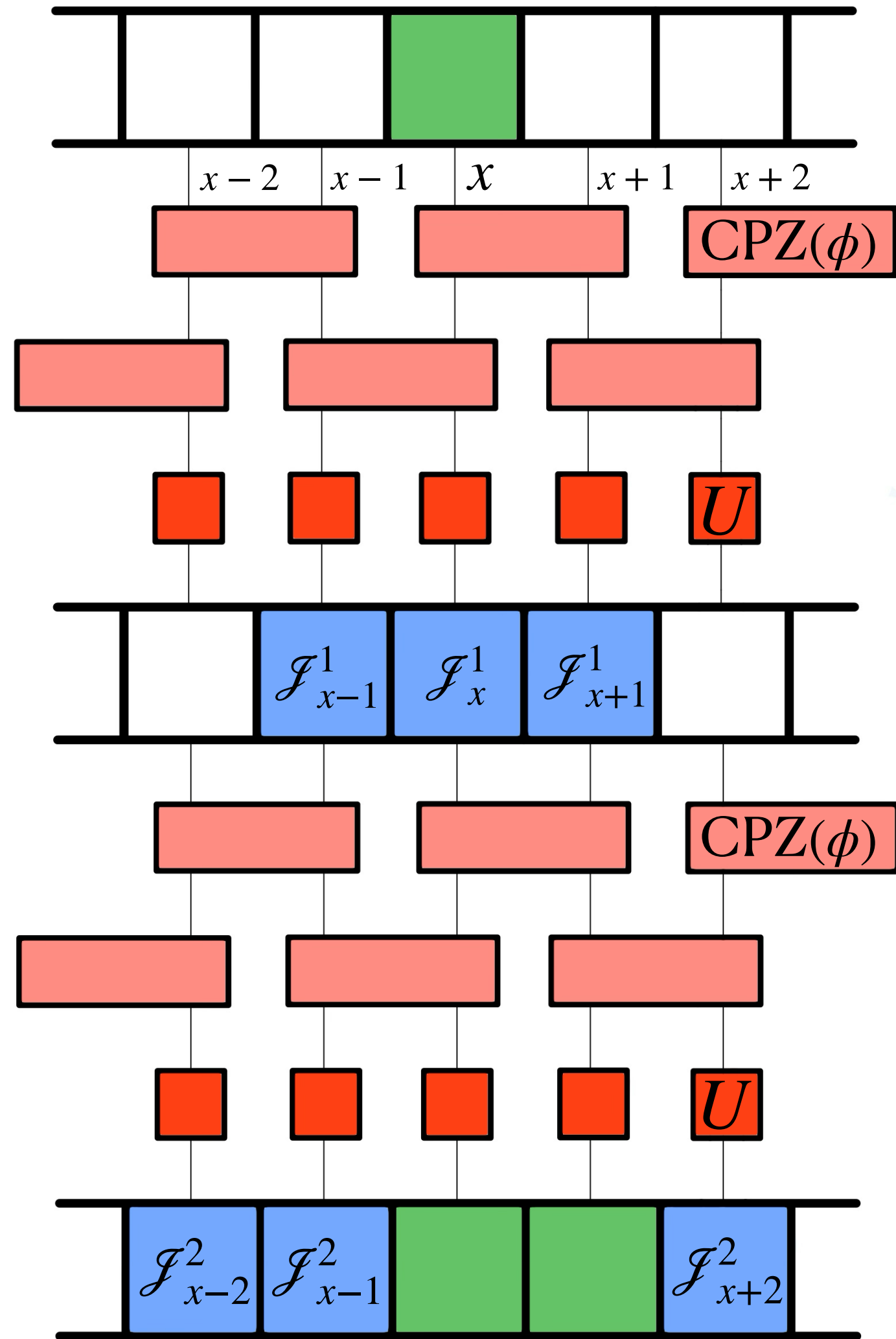


is a single qubit unitary.

T

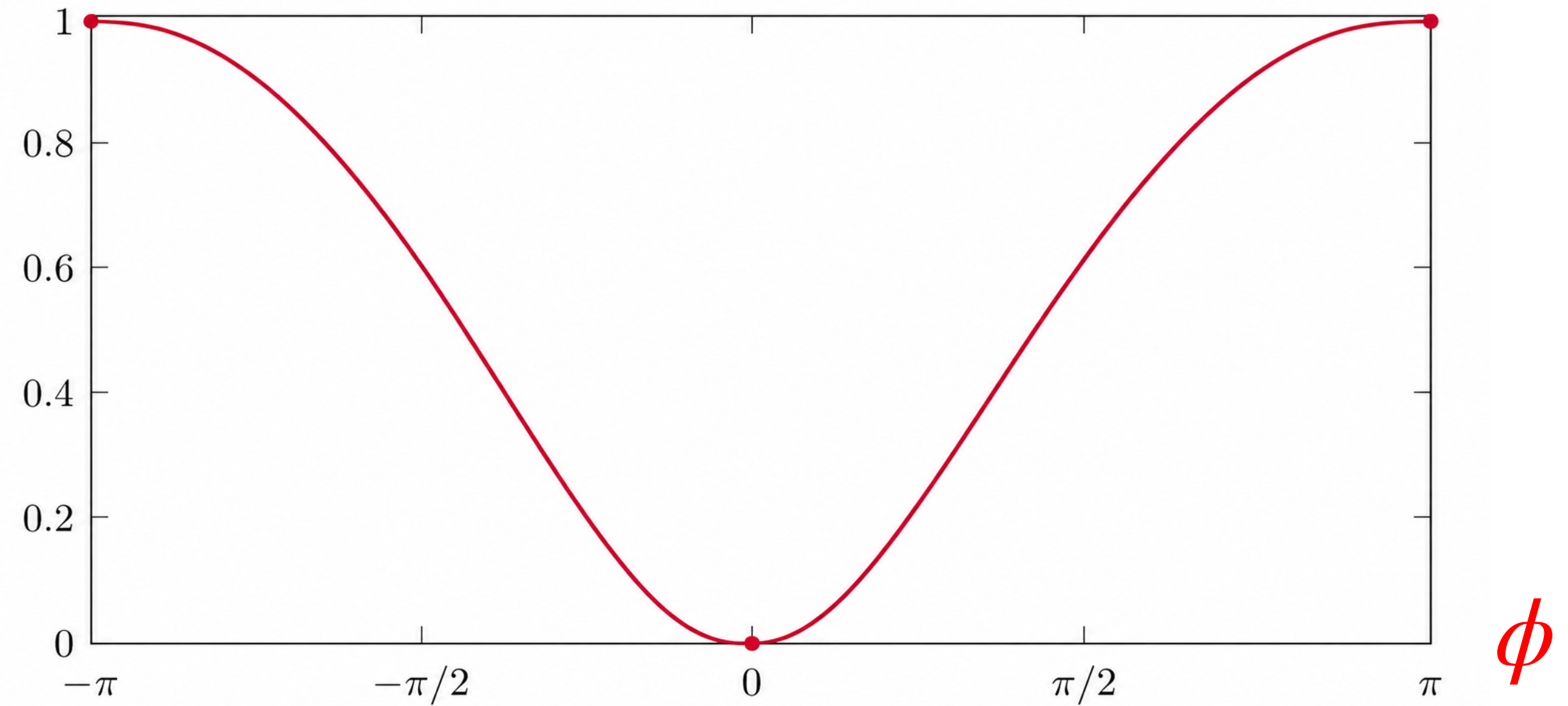


$$\{ |i\rangle\langle i|_x \}_{i=0,1}$$

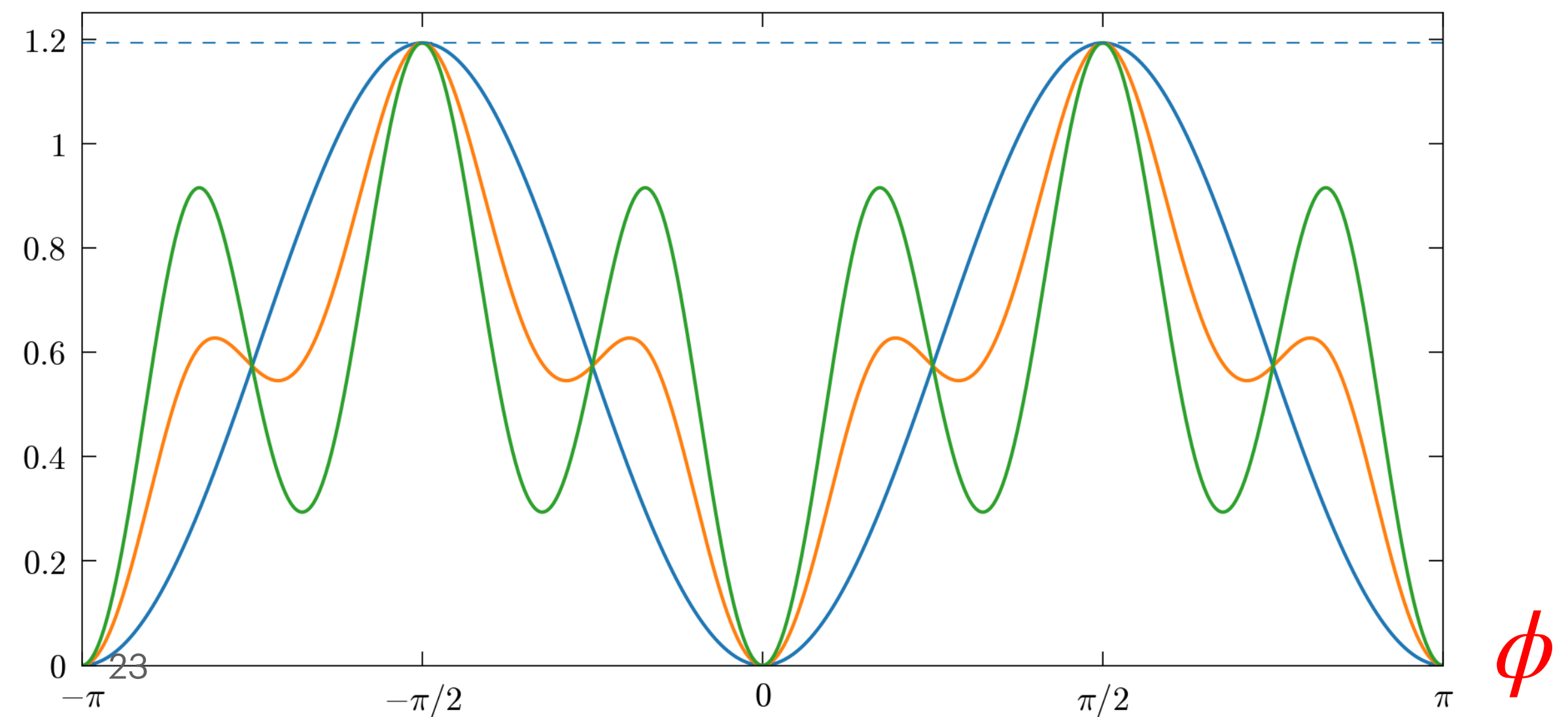


$$\{ |\tau(\phi)_j\rangle\langle\tau(\phi)_j|_{x,x+1} \}_{j=0,1,2,3}$$

Entanglement



Magic (Stabilizer Non-Classicality)



Future Perspectives

- **Fermionic Architectures** — Extending the framework to fermionic QCAs.
- **Time as a Resource** — Quantifying the time required to implement resourceful measurements as a function of the QCA parameters.
- **Beyond Measurements** — Extending the protocols to more general quantum operations.