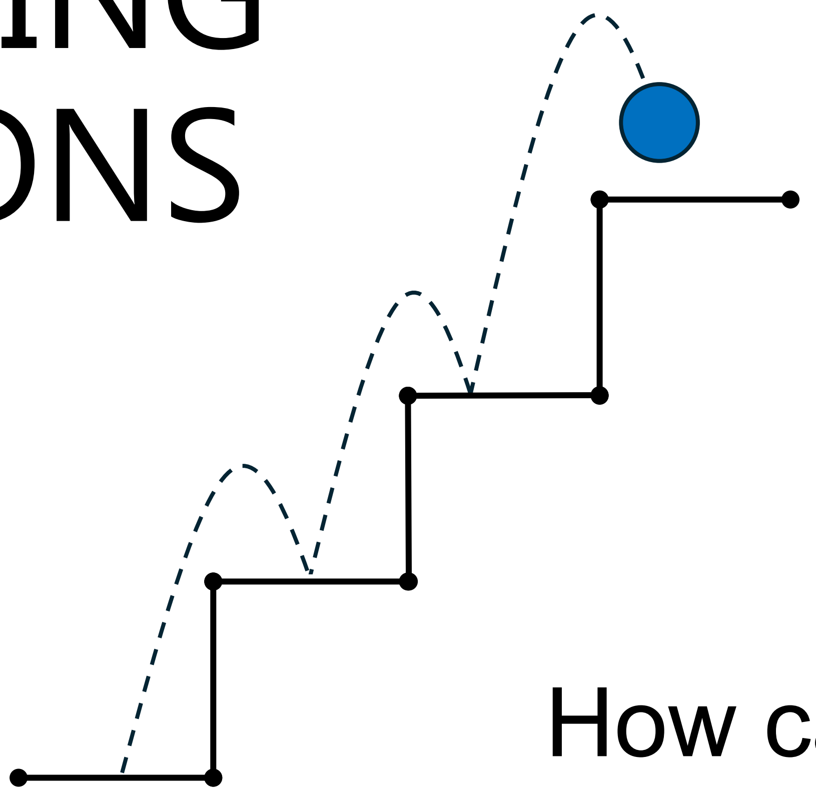
BOUNCING FERMIONS



Andrea Parpinel
XLI ciclo
EOY Seminars 2026

How can we model confined relativistic spin-½ particles?

What is a relativistic spin-½ particle?

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$$(\gamma^\mu \partial_\mu - m)\psi = 0$$

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What is a relativistic spin-½ particle?

$$(\gamma^\mu \partial_\mu - m)\psi = 0$$

$$i\partial_t \psi = i\gamma^0(\gamma^j \partial_j - m)\psi \equiv \hat{H}\psi$$

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2D confined massless Dirac field

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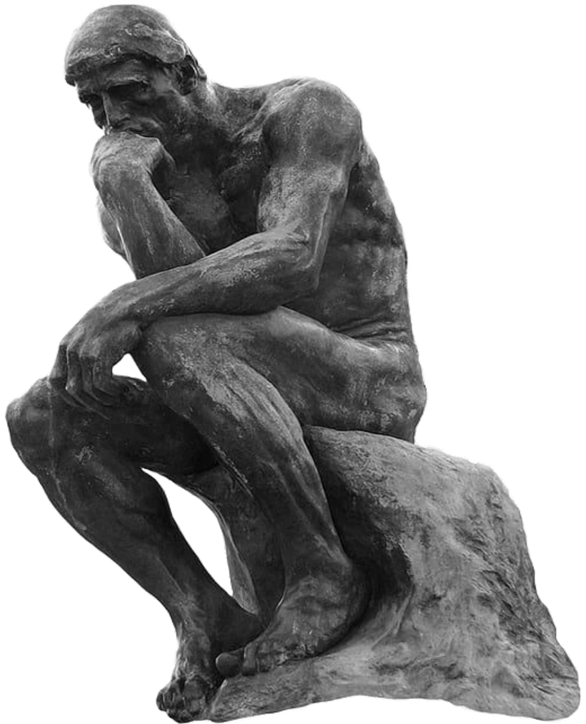
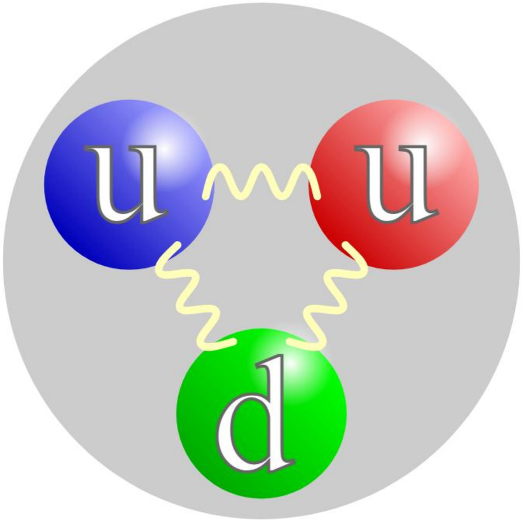
Do confined relativistic spin-½ particles even exist in nature??

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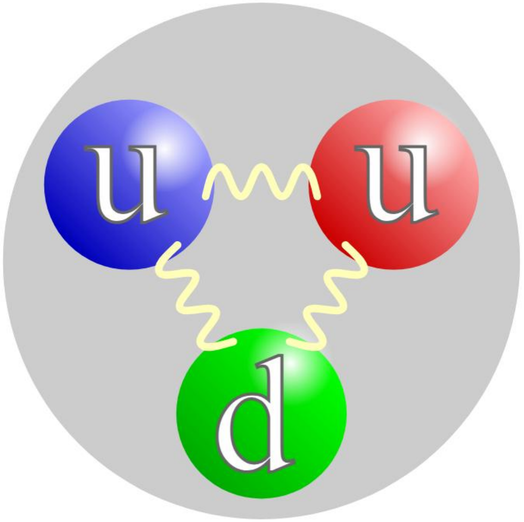
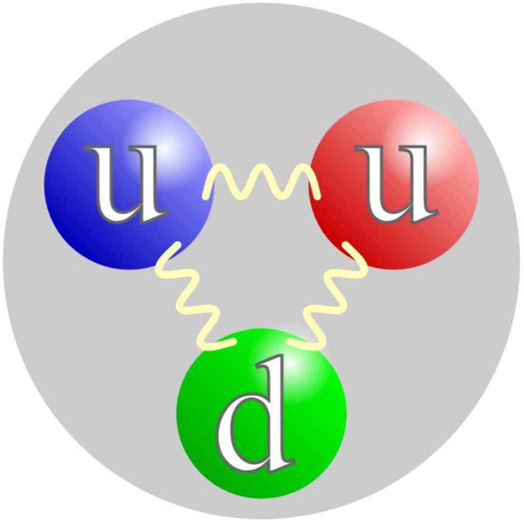


Proton $\sim 1\, GeV$
Quark $\sim 0.01\, GeV$

☑ Spin ½
☑ Confined
☑ Relativistic



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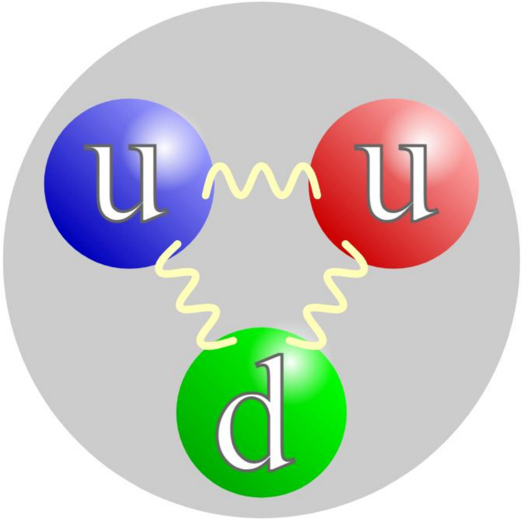
The MIT-bag model (1975)

- ☑ Spin ½
- ☑ Confined
- ☑ Relativistic



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[2] K. Johnson, *"The M.I.T. Bag Model"*, *Acta Phys. Polon. B* **6** (1975)

Fermionic boundary conditions

$$J^{\mu} = \psi^{\dagger}\gamma^{0}\gamma^{\mu}\psi$$

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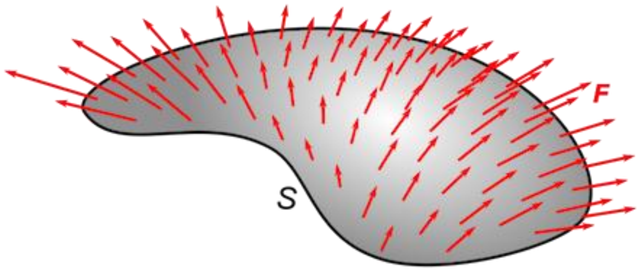

Fermionic boundary conditions

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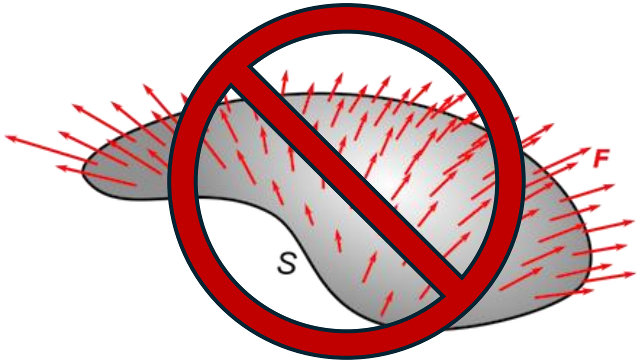


No flux of J^μ across the boundary

2D MIT-bag boundary conditions

$$i\sigma_y \partial_t \psi + \sigma_x \partial_x \psi = 0$$

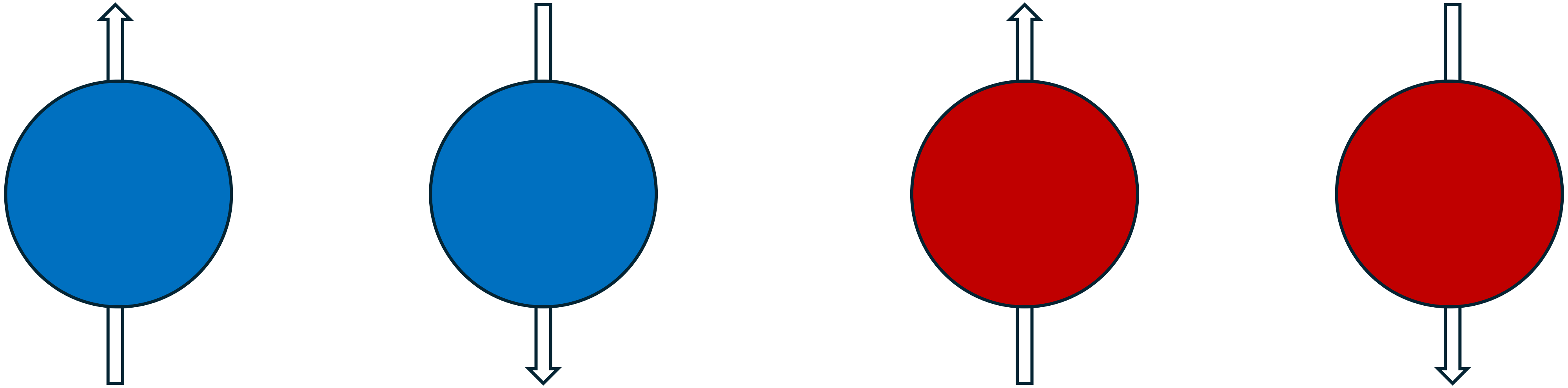
$$\sigma_x \psi_{|x=0} = +\,\psi_{|x=0}$$

particles

$$\sigma_x \psi_{|x=0} = -\,\psi_{|x=0}$$

anti-particles

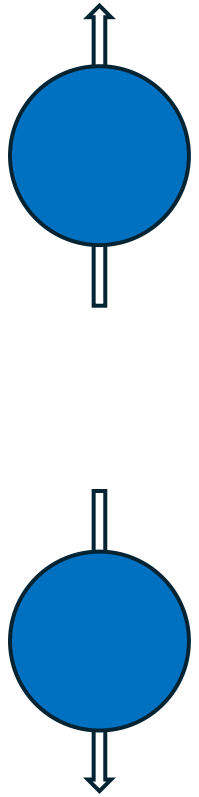
What about the spin itself?

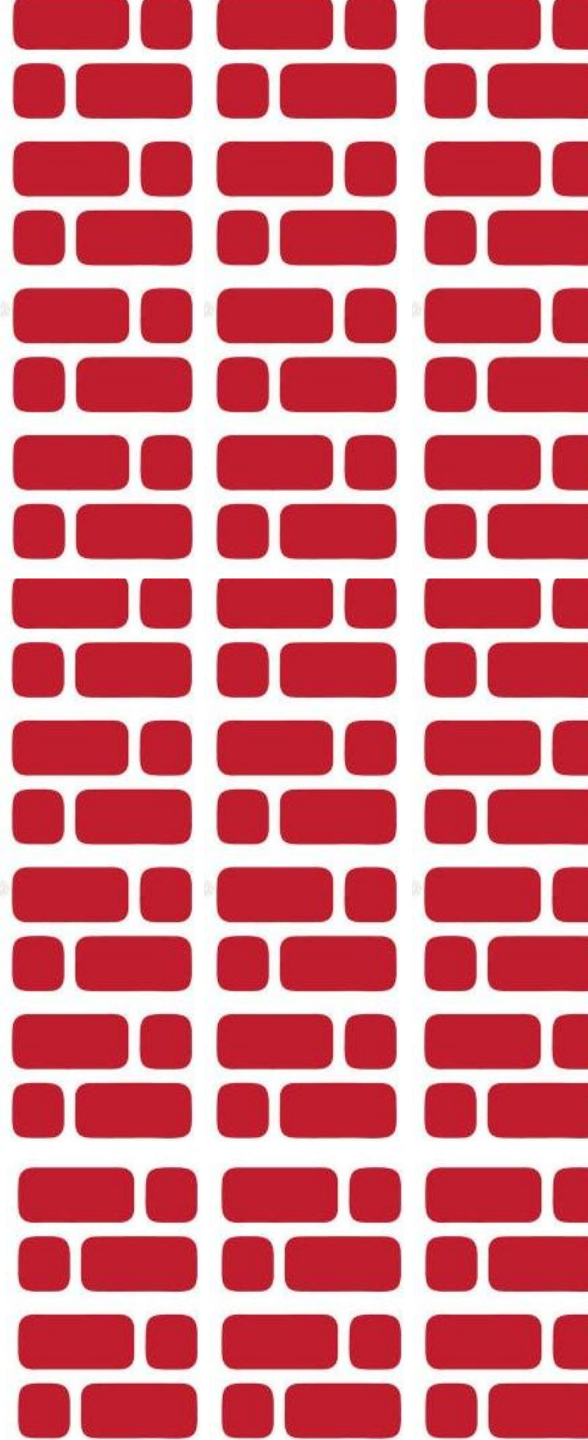


particles

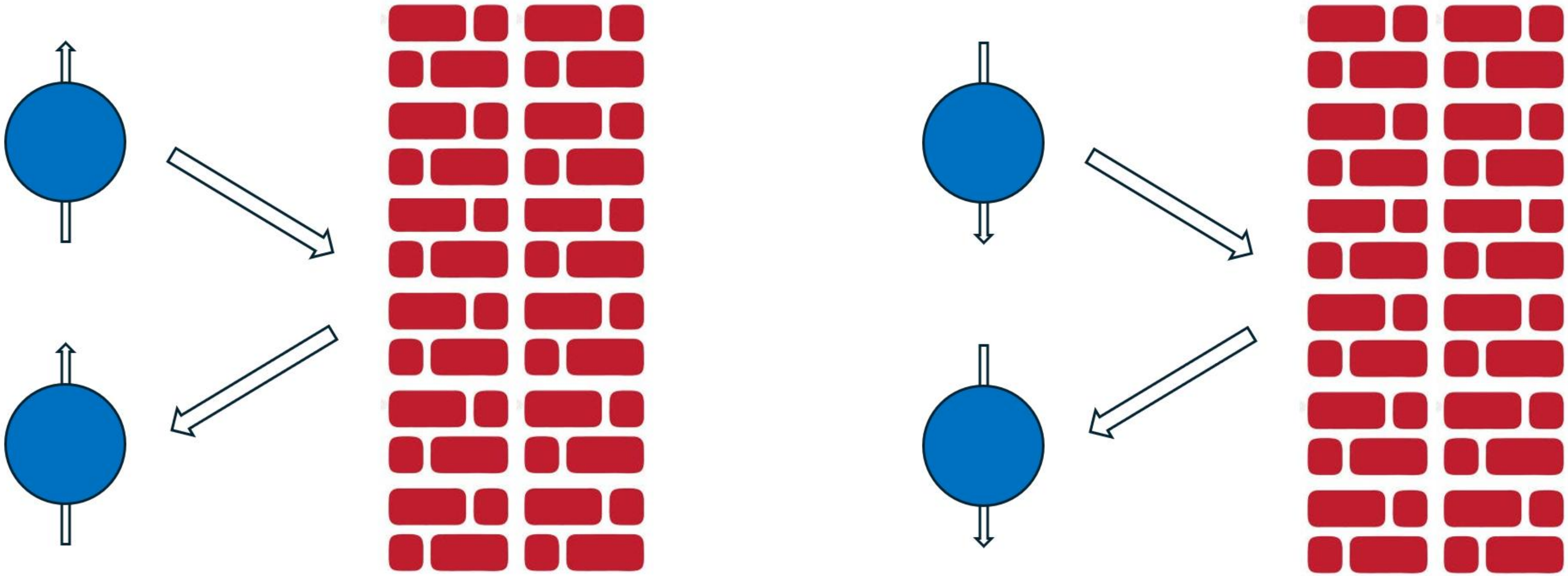
anti-particles

“+” MIT-bag boundary conditions

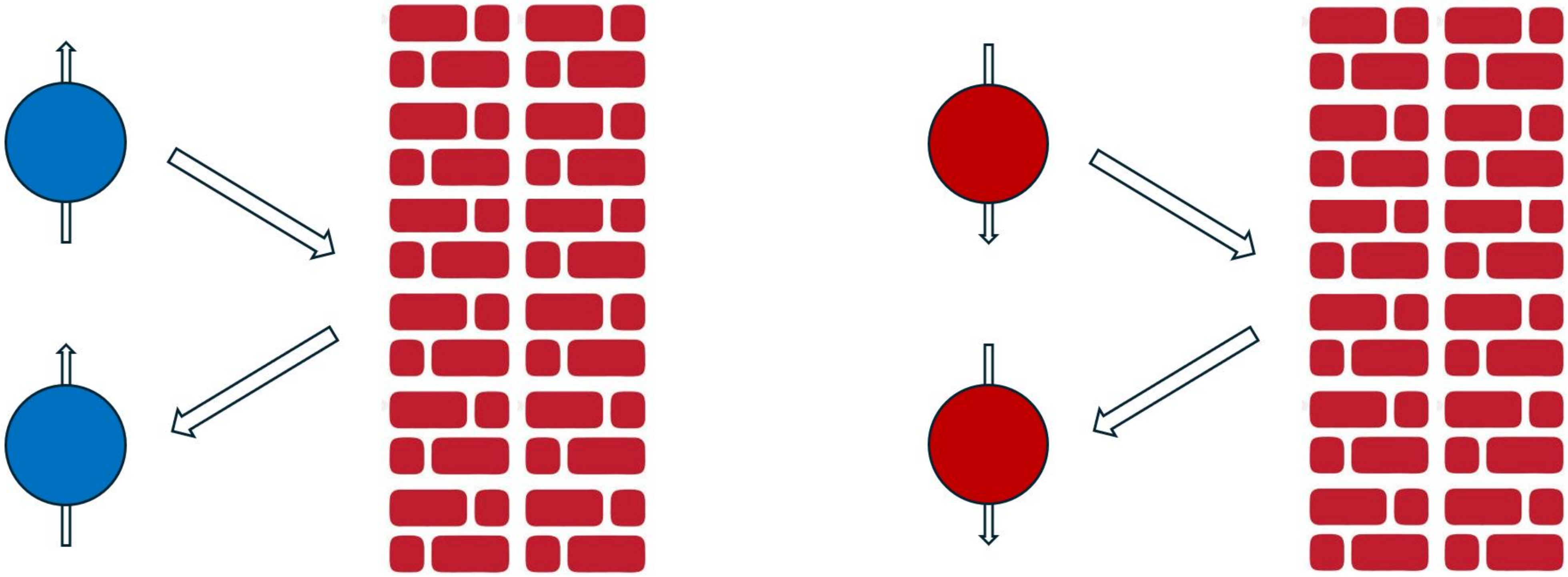




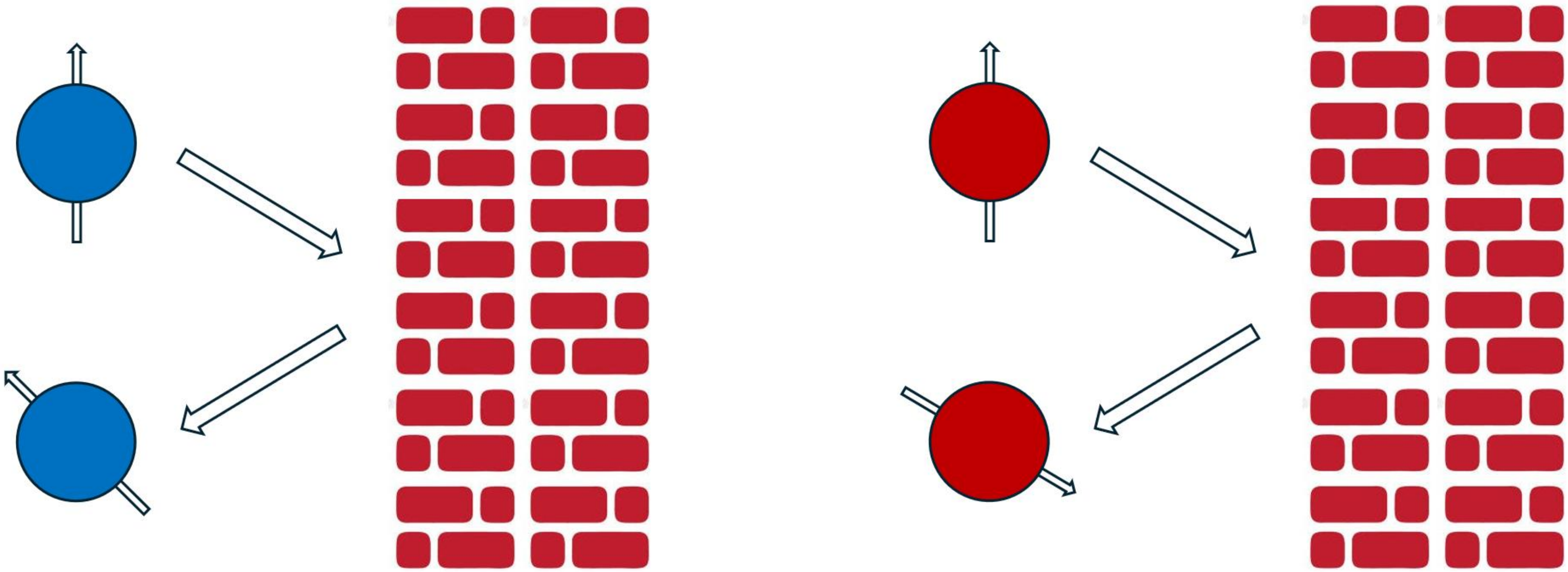
“+” MIT-bag boundary conditions



Generalised MIT-bag boundary conditions?



Generalised MIT-bag boundary conditions?



2D Generalised MIT-bag boundary conditions

Standard MIT-bag

$$\sigma_x \psi_{|x=0} = \pm\, \psi_{|x=0}$$

2D Generalised MIT-bag boundary conditions

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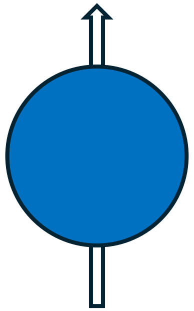
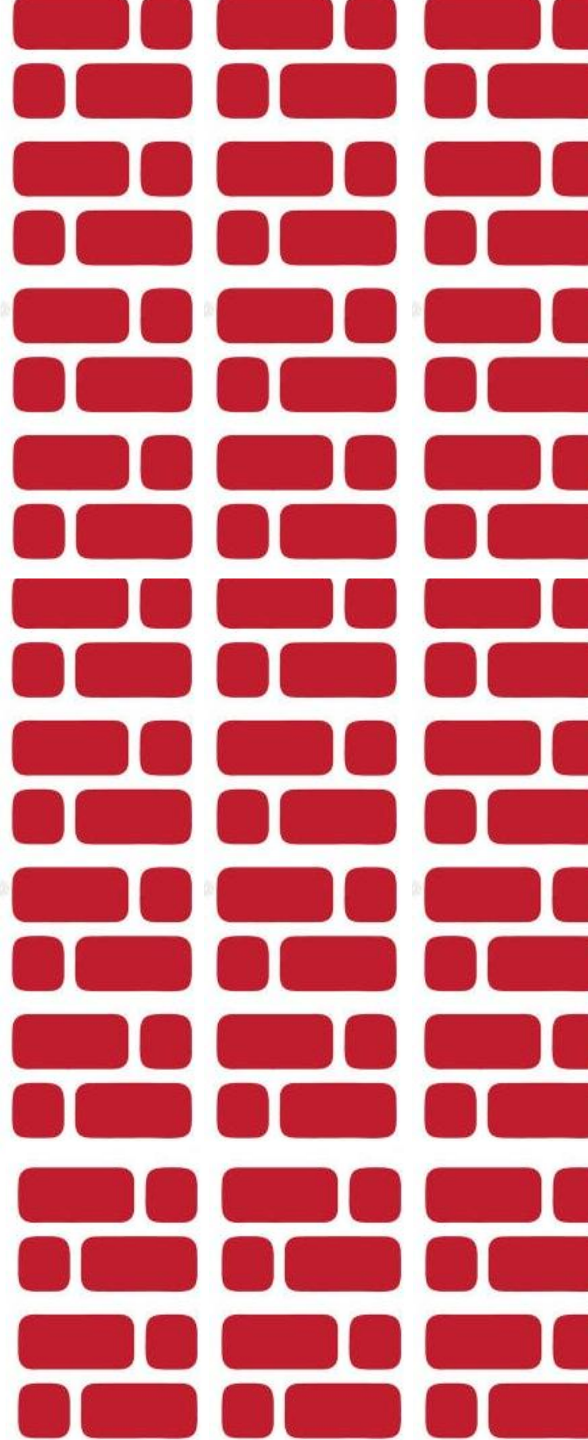
Generalised MIT-bag

$$(1+U)\,\frac{1-\sigma_x}{2}\,\psi_{|x=0} = (1-U)\,\sigma_y\,\frac{1+\sigma_x}{2}\,\psi_{|x=0}$$

[3] D. S. Blanco, "*Dirac field in AdS$_2$ and representations of SL(2, R)*", J. Math. Phys. **64** (2023)
[4] C. Dappiaggi, A. Parpinel, *"Boundary conditions and Dirac fields on AdS$_n$", Phys. Rev. D* **113** (2026)

Edge modes





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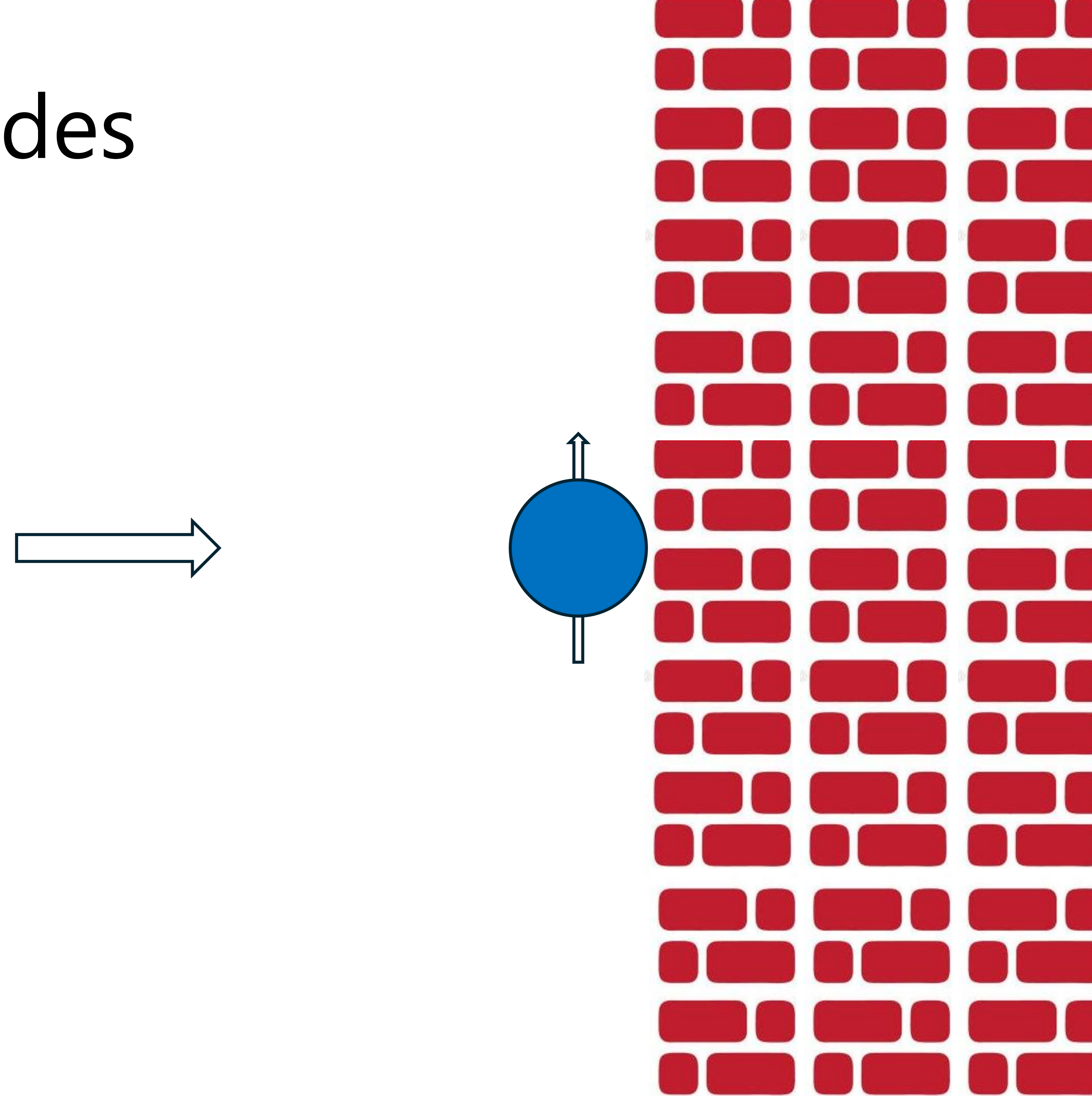
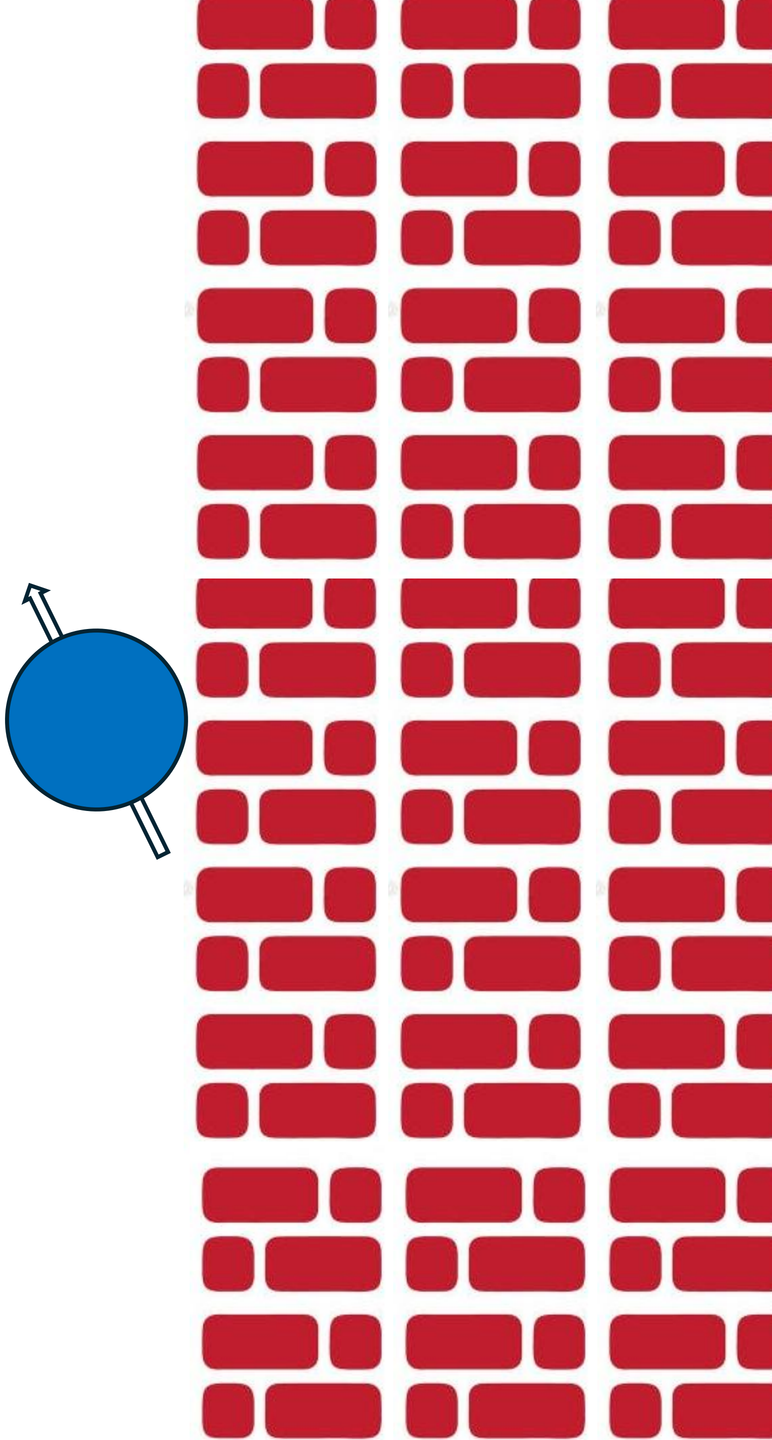


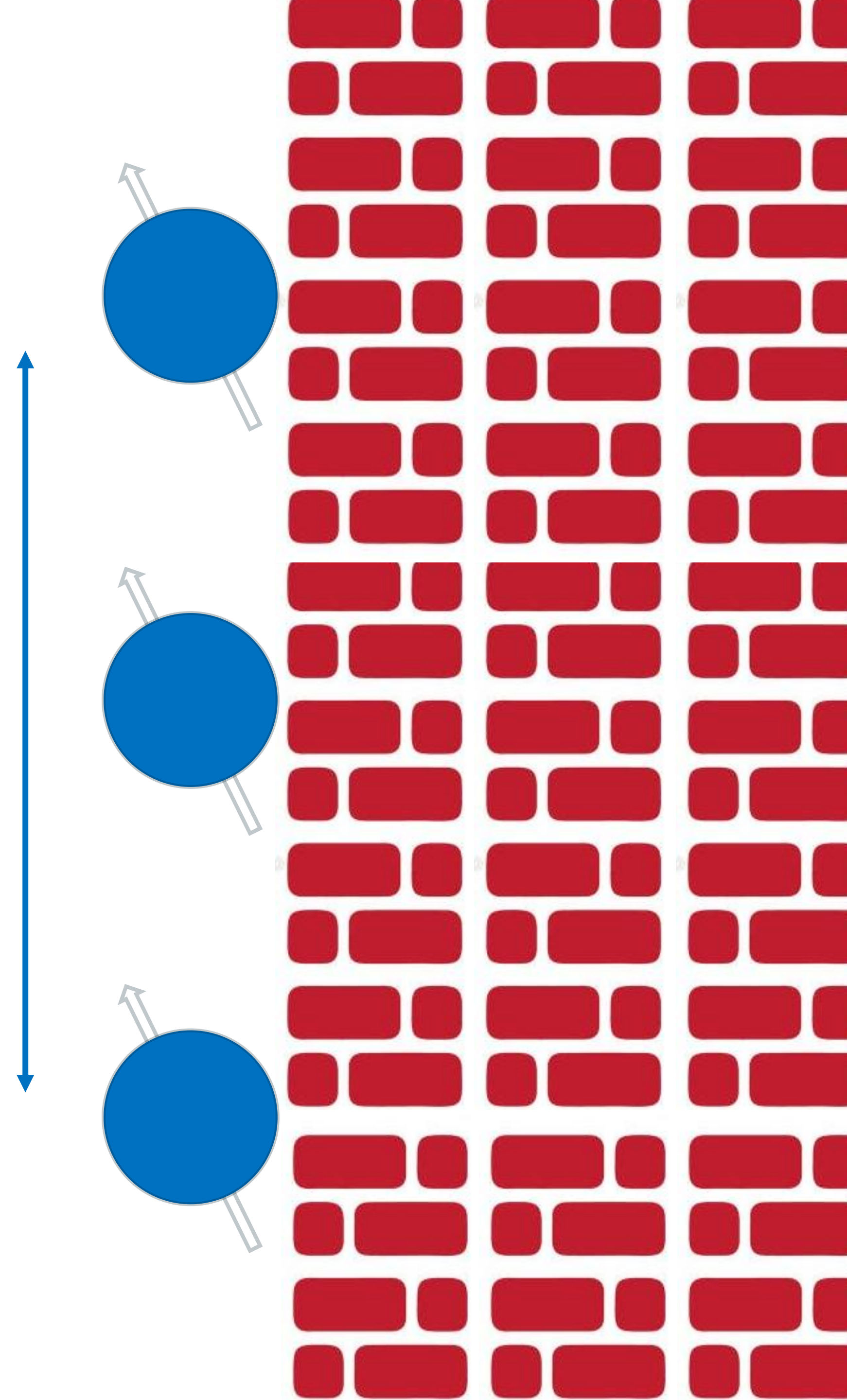
Edge modes

The particle gets a specific twist…

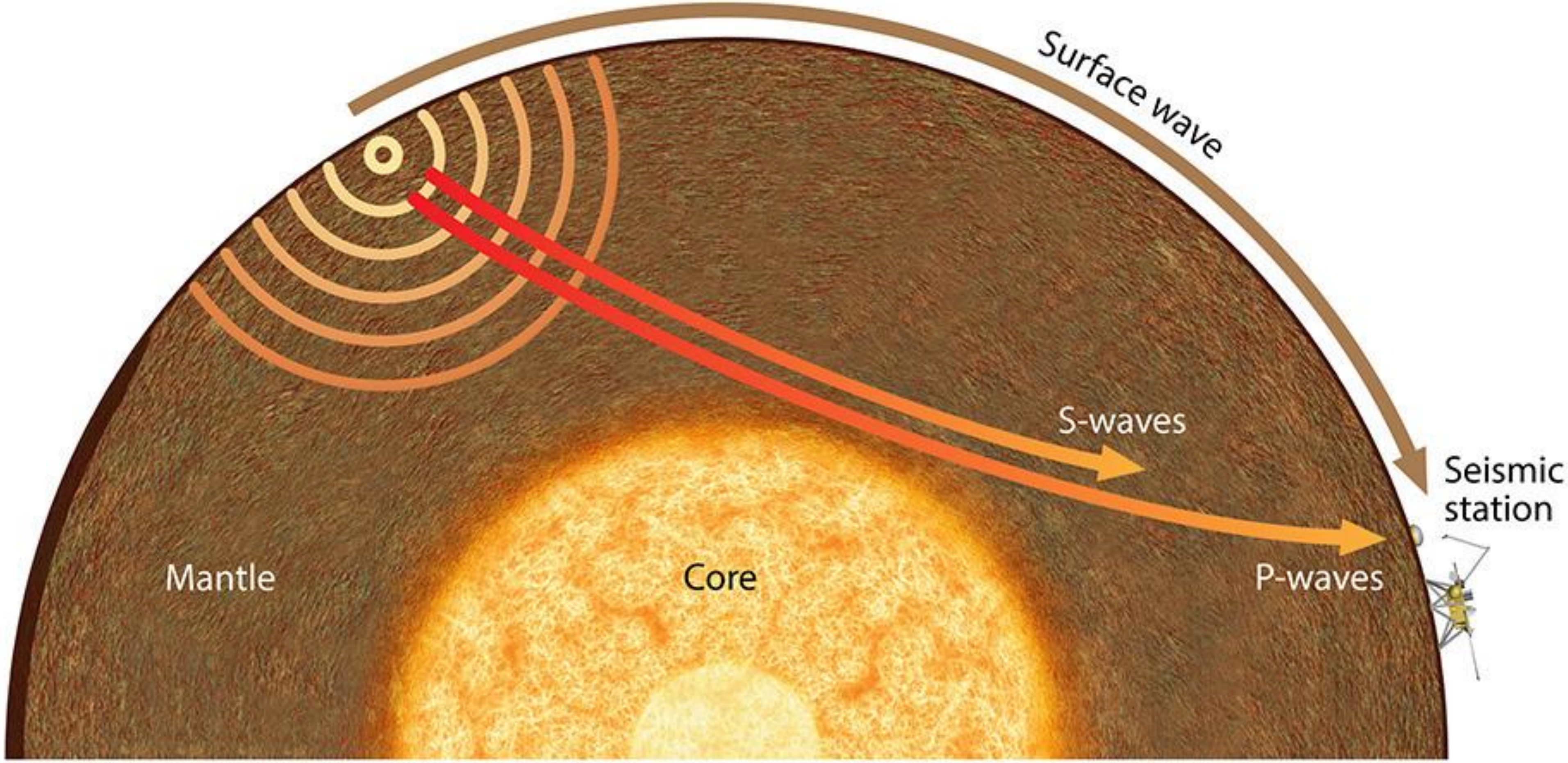


Edge modes

… which traps it on the boundary!



Edge modes



Summary

- Relativistic spin-½ particles are not easy confinable

- A confinement model could be useful for hadron physics

- It is possible to trap particles on the boundary

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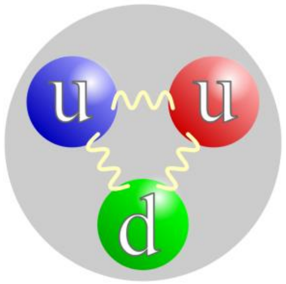

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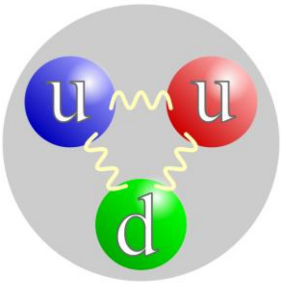

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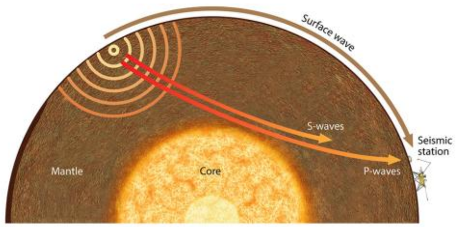

Future outlooks

Dynamical boundary conditions

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Bulk-Boundary correspondence