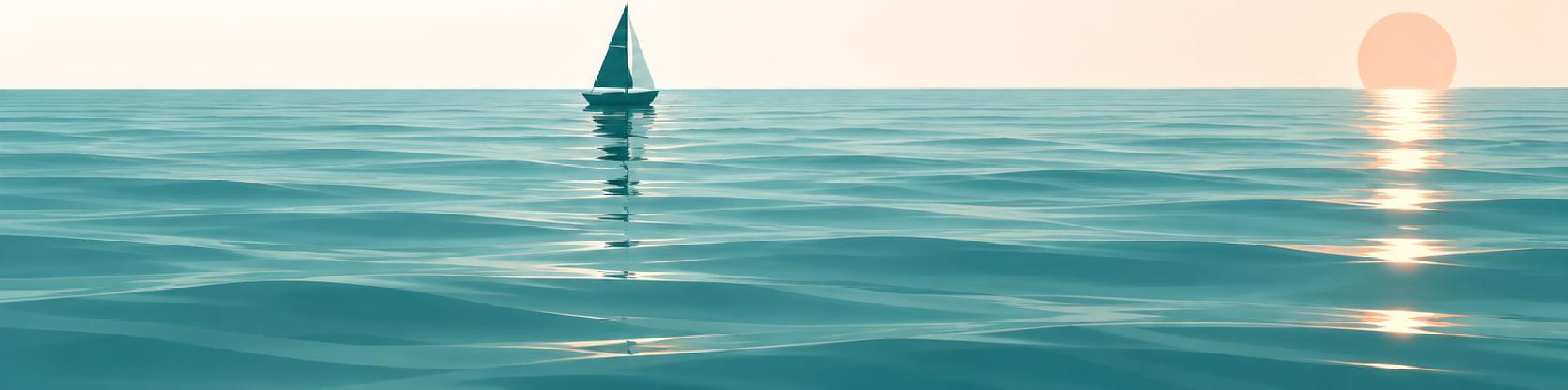
Truncate Smart, Not Hard:
Hamiltonian Truncation Effective Theory

UNIVERSITÀ
DI PAVIA

Andrea Maestri - End-of-Year seminars

Navigating the sea of quantum field theories



Knowing the theory is not the same as being able to solve it

A quantum field theory can be well defined, but finding its physical predictions can be challenging

Perturbative regime

$g \ll 1$

Systematic expansion:

$$\mathcal{O} = \mathcal{O}^{(0)} + g\mathcal{O}^{(1)} + g^2\mathcal{O}^{(2)} + \cdots$$

- ✓ systematic improvement
- ✓ reliable predictions
- ✓ controlled expansion

increasing interaction strength $\longrightarrow$

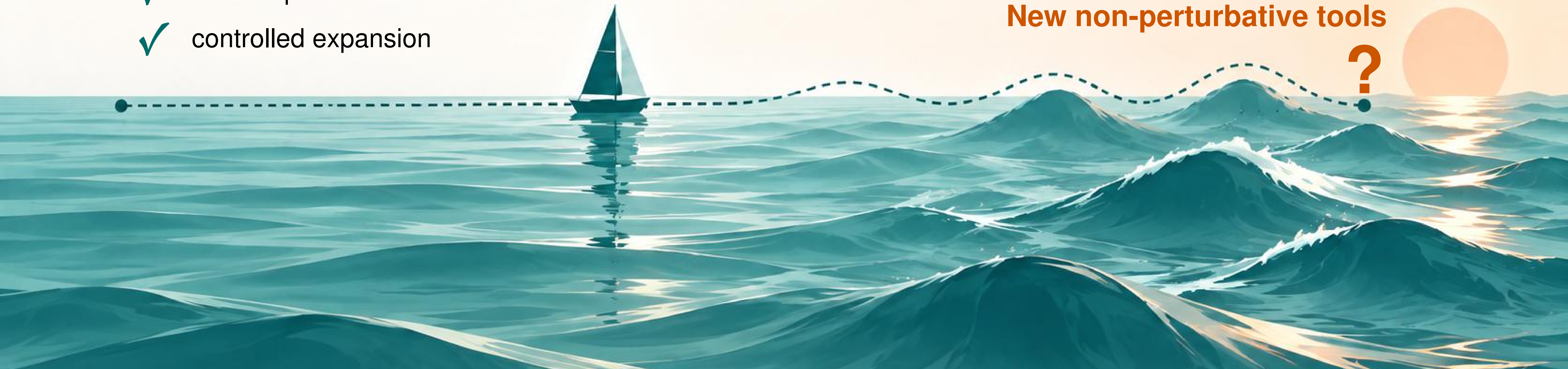
Non-perturbative regime

$g \sim \mathcal{O}(1)$

The usual expansion is not controlled

- ✗ expansion does not converge
- ✗ strongly coupled dynamics

New non-perturbative tools



Navigating non-perturbative QFT

Different tools reveal different aspects of the theory

Lattice QFT
Euclidean path integral
First principles • gauge theories

Bootstrap
Symmetry + consistency
Unitarity • crossing • bounds

Hamiltonian methods
States and spectrum
Real-time dynamics • eigenstates

Tensor networks
Entanglement structure
Low-dimensional systems

Functional methods
Correlation functions
Schwinger–Dyson • functional RG

No single method can explore the whole ocean

The Hamiltonian Route

From quantum field theory to a matrix eigenvalue problem

V. P. Yurov and A. B. Zamolodchikov
Int. J. Mod. Phys. A 5, 3221 (1990).

$$\boxed{H = H_0 + V}$$

Step 1 → **Step 2** → **Step 3**

$$H_0 \left|\mathcal{E}_i\right\rangle = \mathcal{E}_i \left|\mathcal{E}_i\right\rangle$$

Choose a solvable basis

Free Hamiltonian exactly solvable
(e.g. Fock basis)

$$V_{ij} = \left\langle\mathcal{E}_i\right| V \left|\mathcal{E}_j\right\rangle$$

Compute Interactions

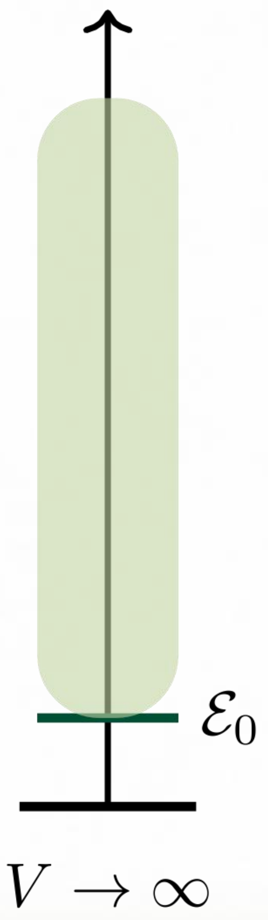
Represent the interaction as a matrix in that basis

$$H_{ij} = \mathcal{E}_i \delta_{ij} + V_{ij}$$

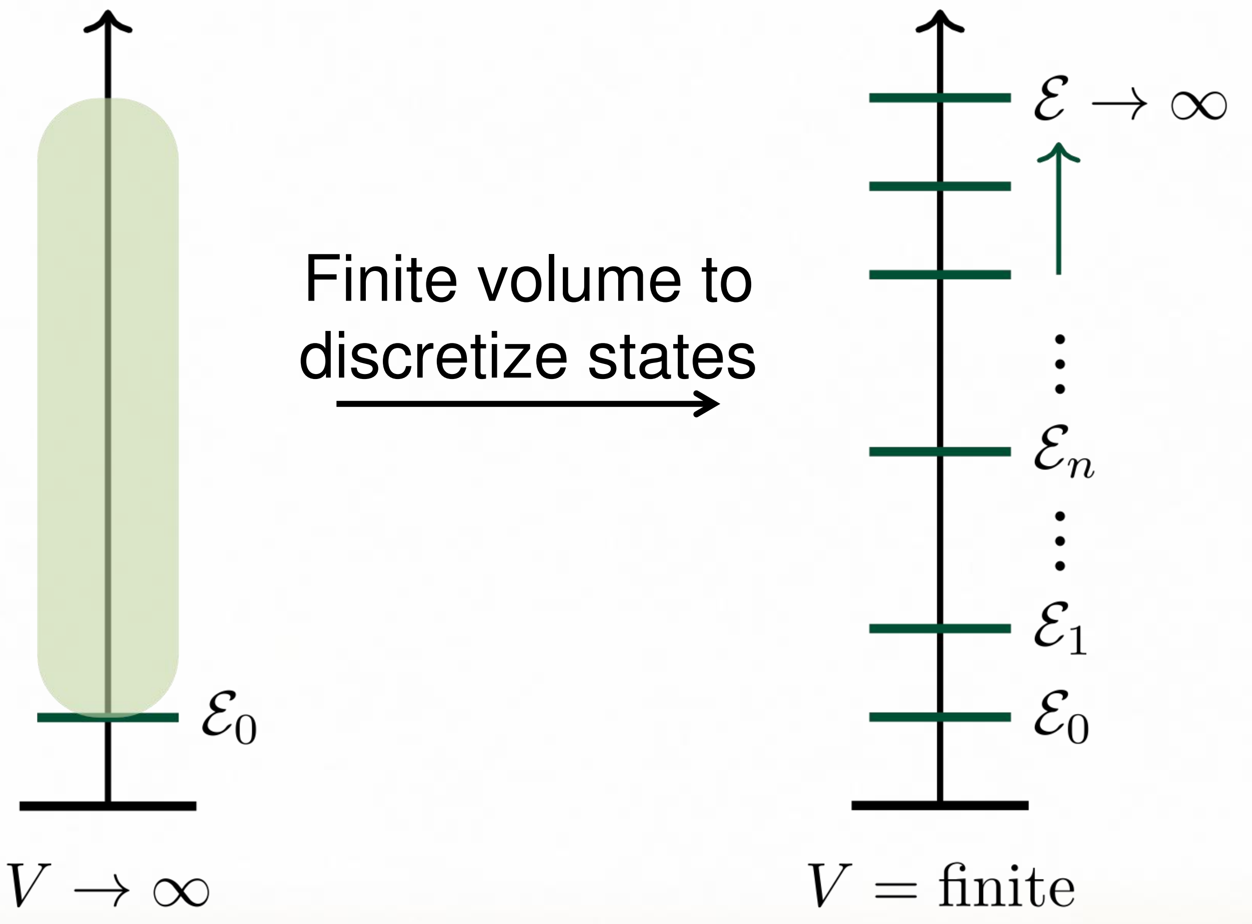
Diagonalize H

Numerically diagonalize the full Hamiltonian

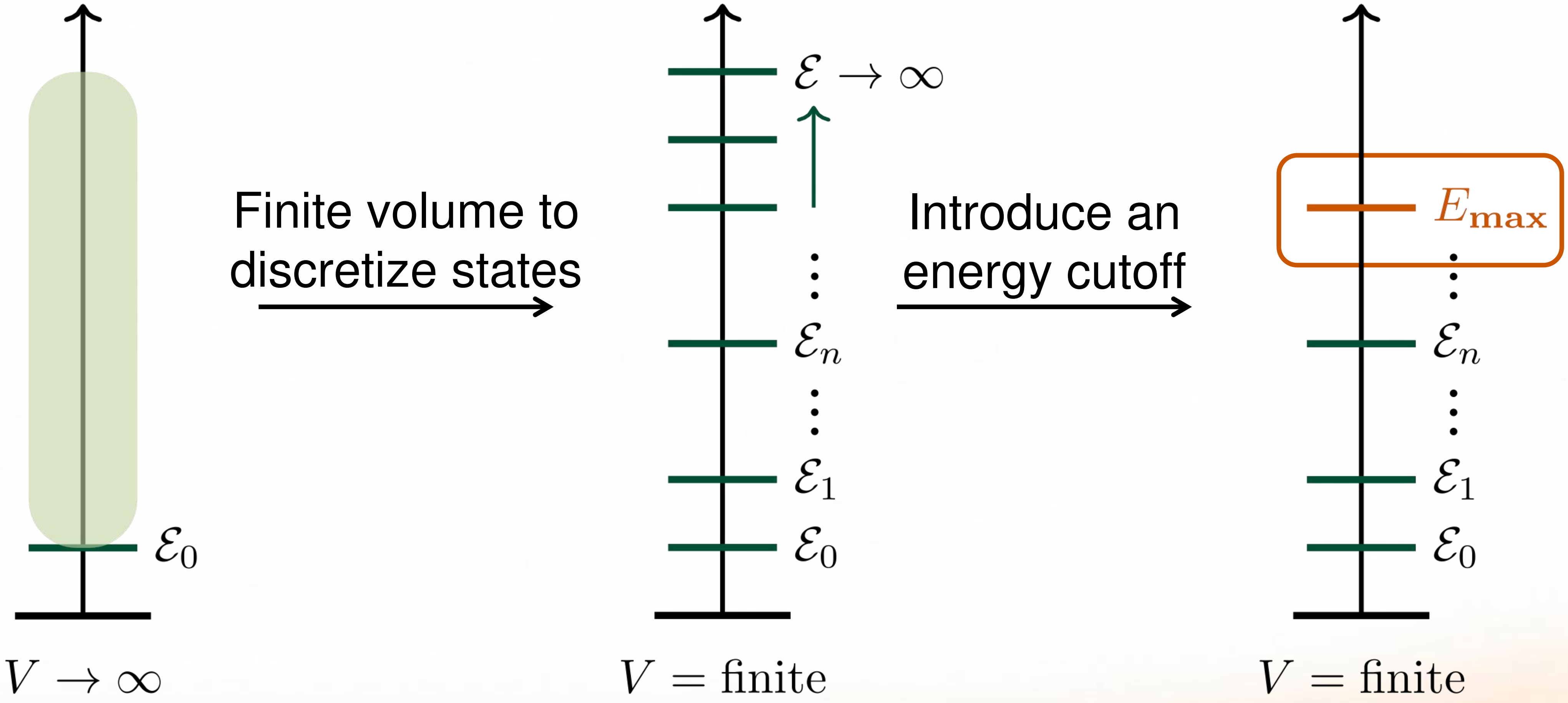
How to obtain a finite free basis



How to obtain a finite free basis



How to obtain a finite free basis



The $\lambda\phi^4$ theory in (1+1)d

A minimal interacting quantum field theory

What does it describe?

- A real scalar field with local quartic self-interaction
- Landau–Ginzburg description of systems in the Ising universality class:

$$\mathbb{Z}_2 : \phi \to -\phi$$

Hamiltonian split for HT

$$H = H_0 + V$$

$$H_0 = \frac{1}{2}\int dx \left[(\partial_t\phi)^2 + (\partial_x\phi)^2 + m^2\phi^2\right]$$

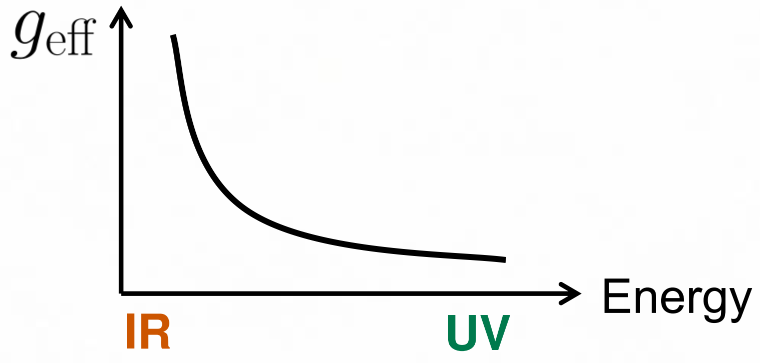
$$V = \frac{\lambda}{4!}\int dx\, \phi^4$$

Why $\lambda\phi^4$ theory in (1+1)d?

Qualitative analogy to QCD

Simple UV, strongly coupled IR, non-perturbative spectrum

$$[\lambda] = 2 \longrightarrow \boxed{g_{\text{eff}} \sim \frac{\lambda}{E^2}}$$



UV / short distances

Perturbation is small and we can use PT

IR / long distances

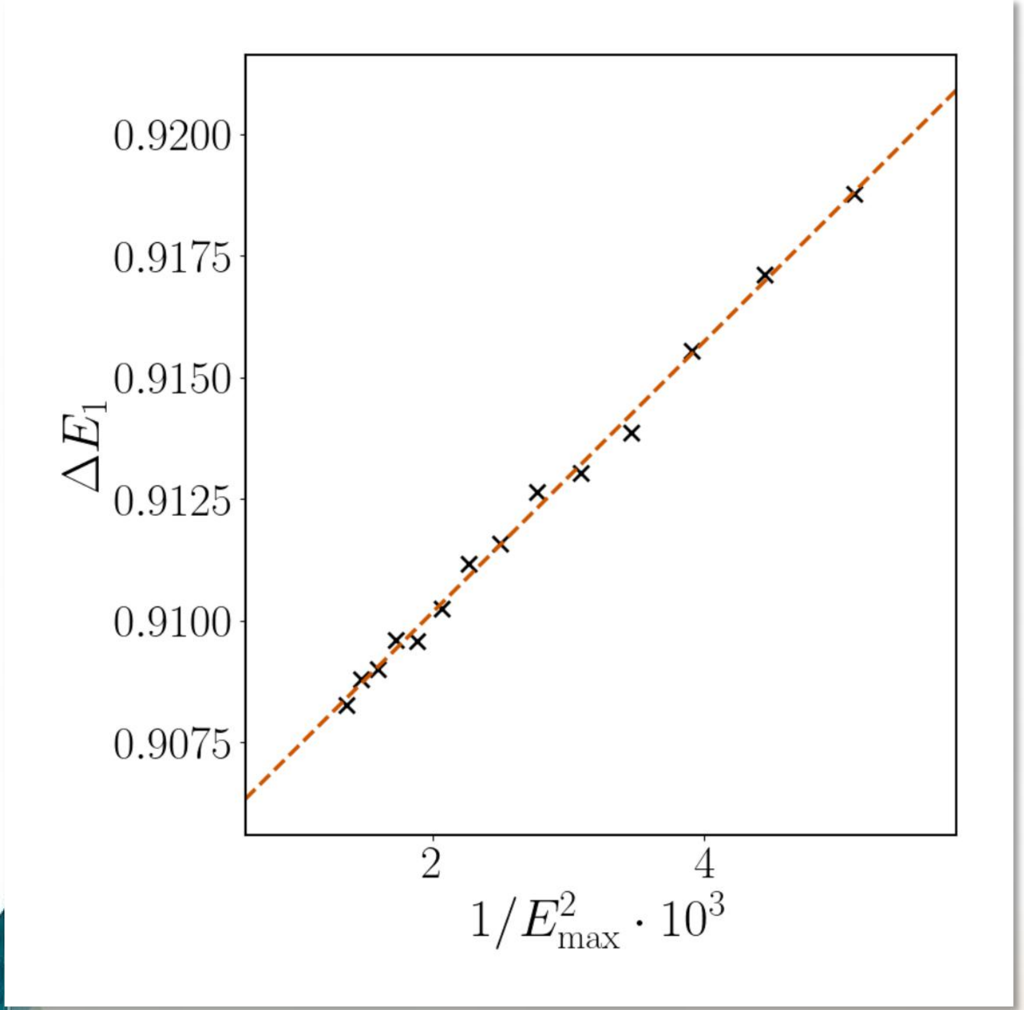
PT breaks down, so we need *nonperturbative* tools

Hamiltonian Truncation targets the low-energy spectrum

Numerical Results

Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$

Accuracy $\sim E_{\mathrm{max}}^{-2}$

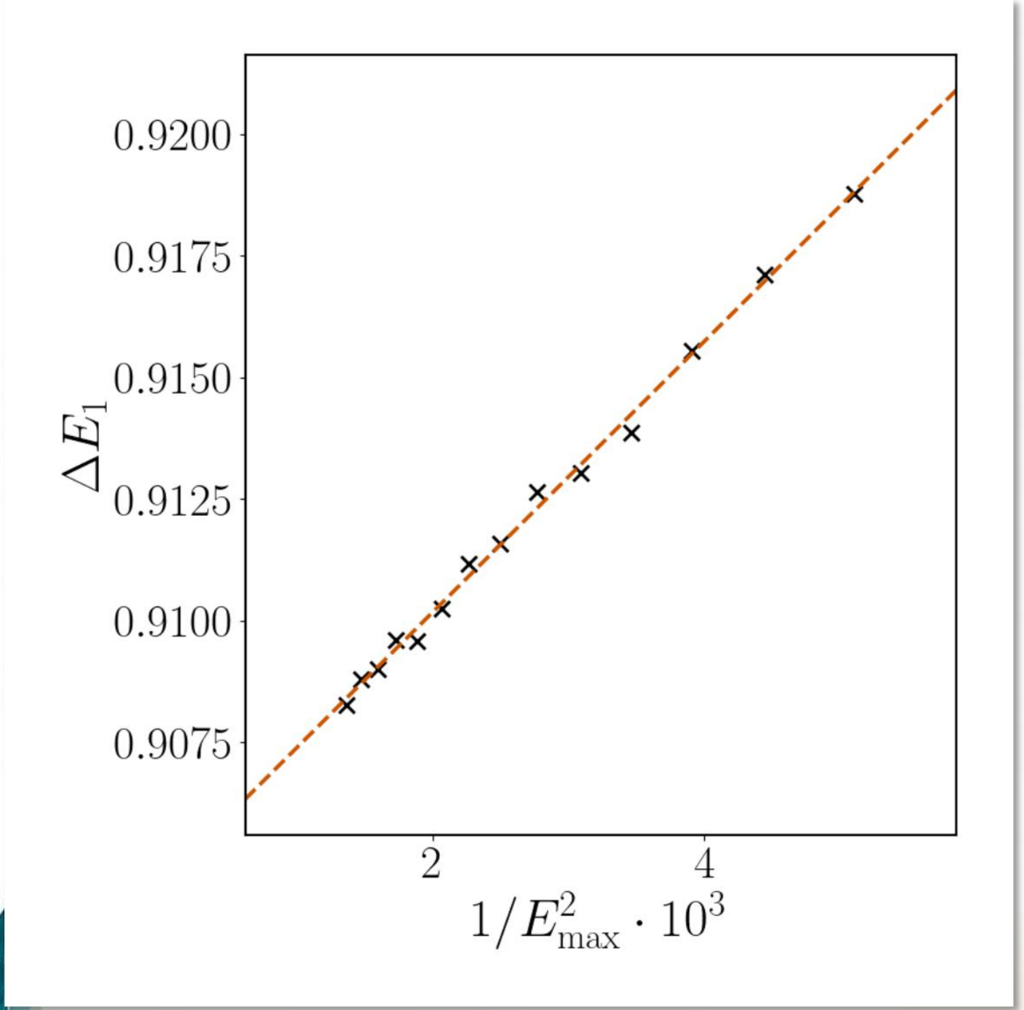


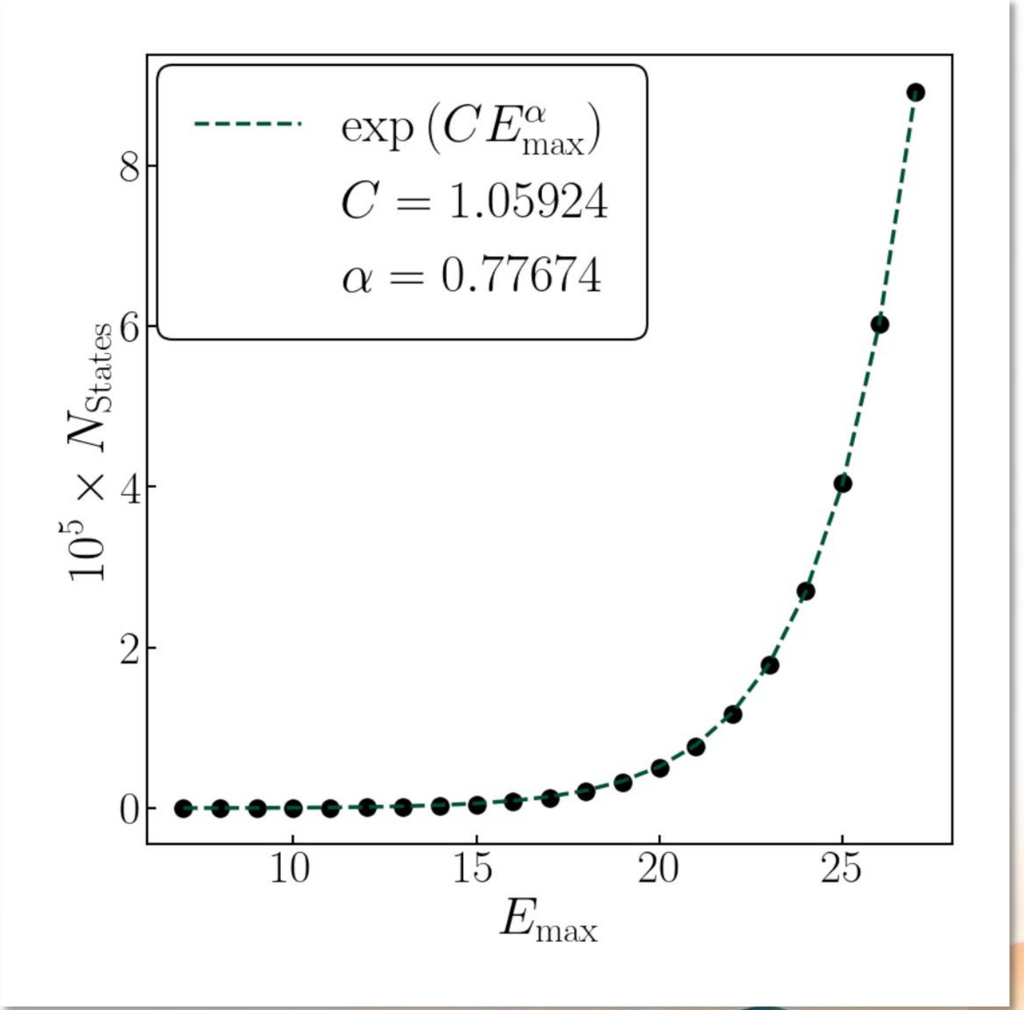
Numerical Results

Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$

Accuracy $\sim E_{\max}^{-2}$

Computational cost $\sim \exp(\sqrt{E_{\max}})$

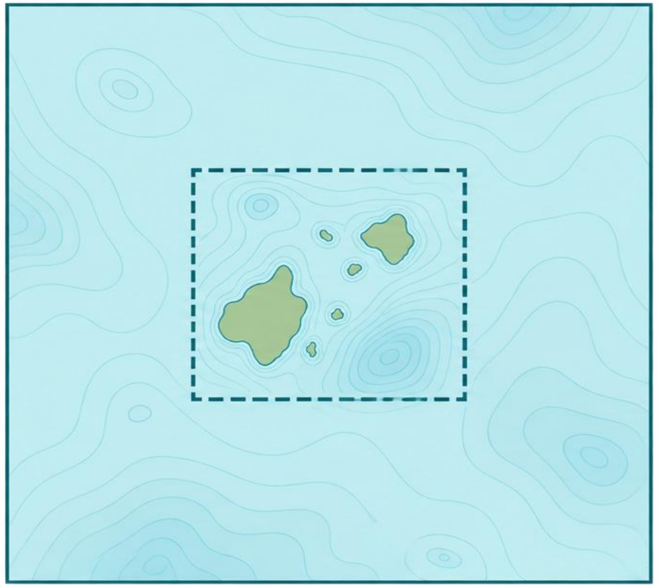




Truncate Smart, Not Hard

The HTET idea in three steps

A finite map



We cannot map the whole sea.

Finite map → truncated Hilbert space

Truncate Smart, Not Hard

The HTET idea in three steps

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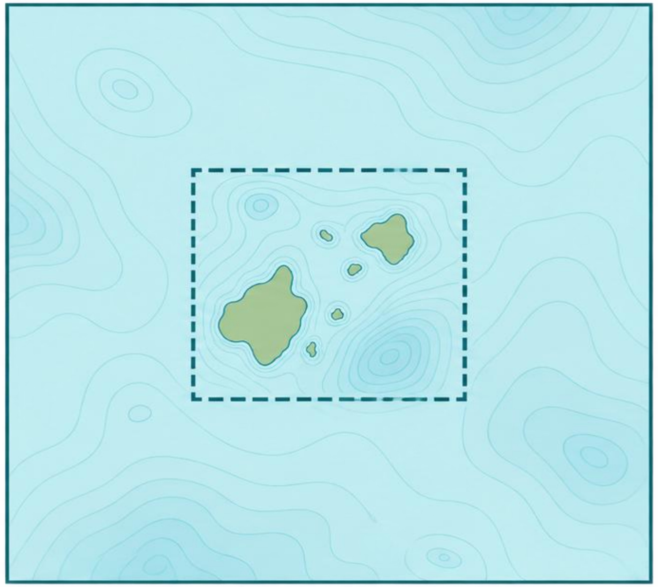
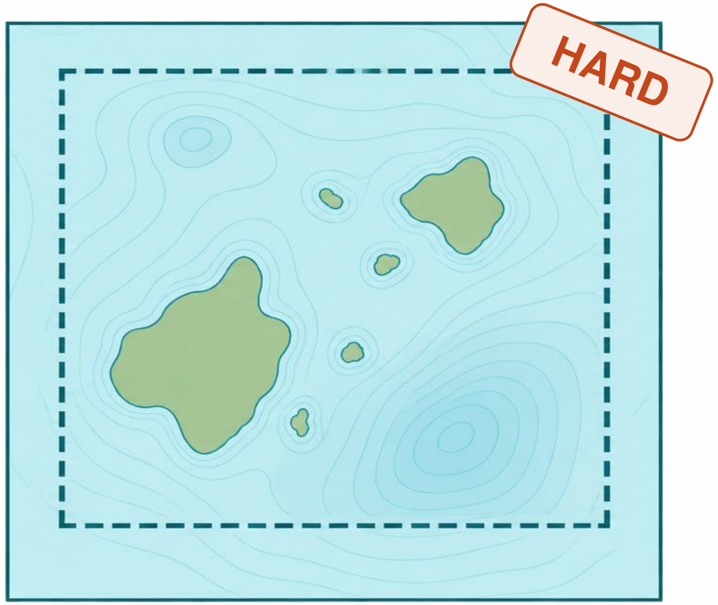


Make the map bigger



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Need more accuracy?

Bigger map → raise the cutoff

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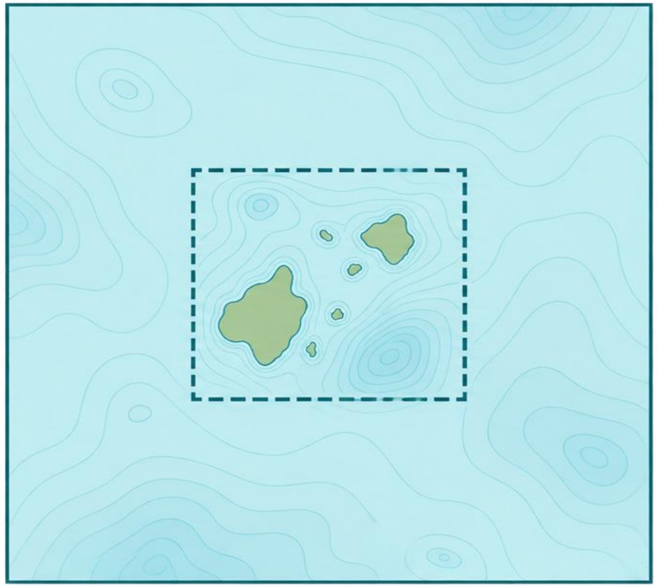

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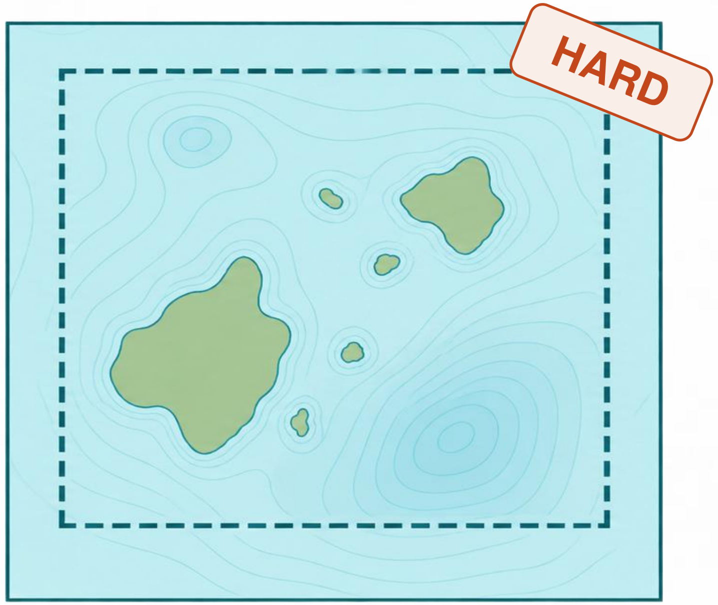
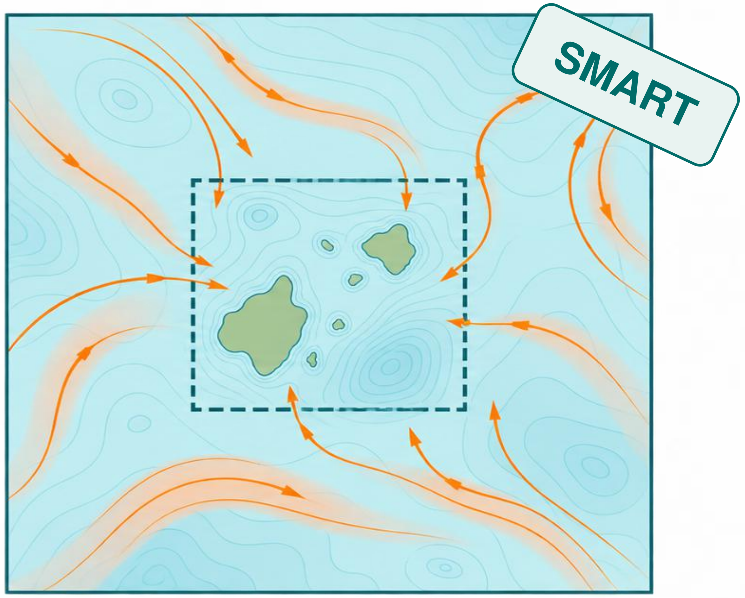


Need more accuracy?

Bigger map → raise the cutoff

Same size, better description



Keep the same map.

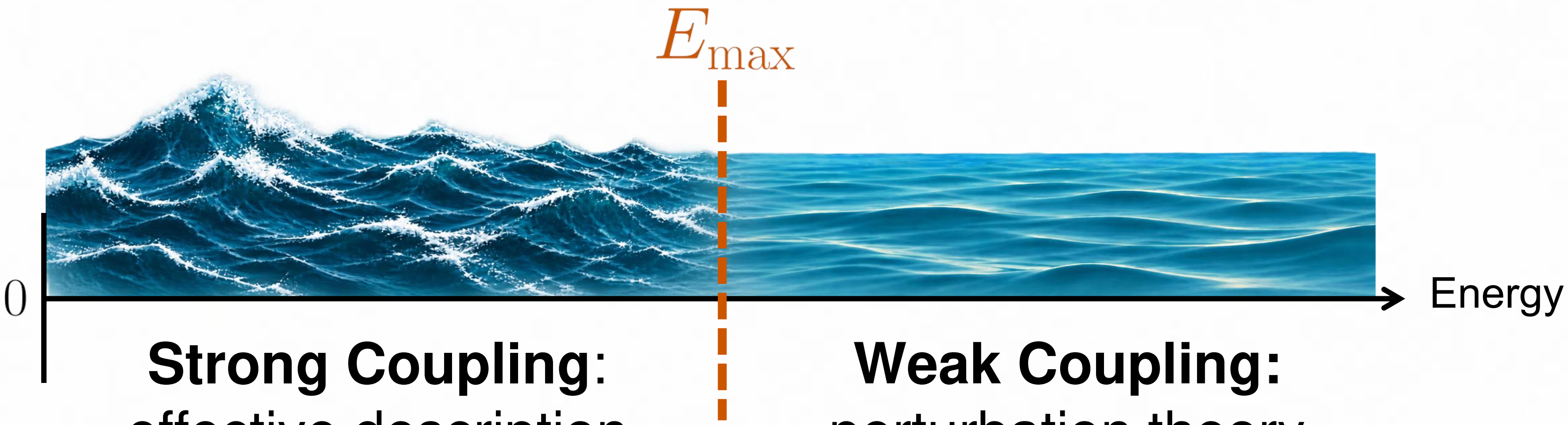
Currents → effects of excluded states

HTET: keep the map small — encode what lies beyond it!

Effective field theory to improve HT

T. Cohen, K. Farnsworth, R. Houtz, and M. A. Luty, SciPost Phys. 13, 011 (2022)

- EFT applies with a clear scale hierarchy: $E_{\mathrm{IR}} \ll E_{\max}$

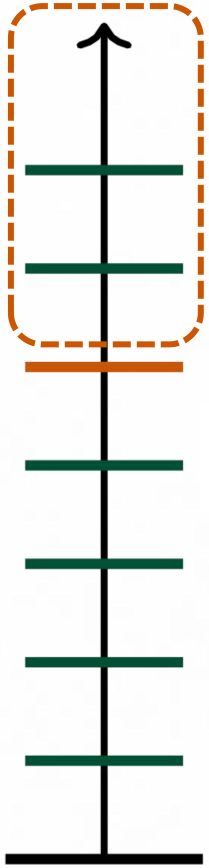


Strong Coupling: effective description

IR

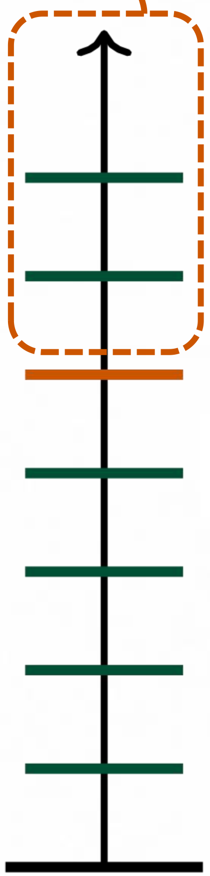
Weak Coupling: perturbation theory

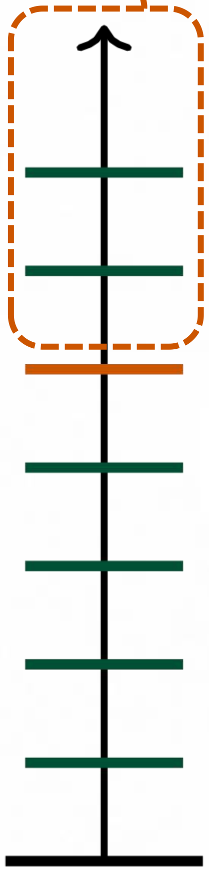
UV



effects of discarded high-energy states

Effective Hamiltonian: $$H_{\text{eff}} = H_0 + H_1 + H_2 + \cdots \ , \quad H_n \sim \mathcal{O}(V^n)$$





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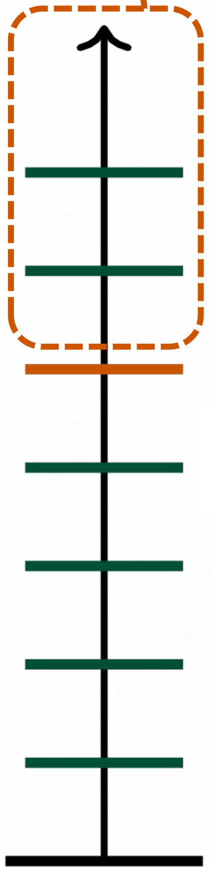
Key idea: Compare an observable in the full and effective theories

effects of discarded high-energy states



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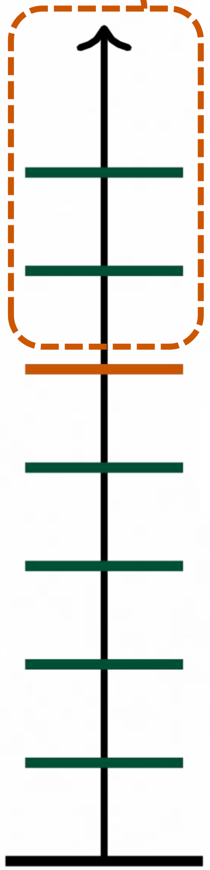
Key idea: Compare an observable in the full and effective theories

Requirements:

- Determine H_{eff} uniquely
- Allow systematic expansion in $E_{\text{IR}}/E_{\text{max}}$
- Ensure separation of scales at each order

effects of discarded high-energy states



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Key idea: Compare an observable in the full and effective theories

Requirements:

- Determine H_{eff} uniquely
- Allow systematic expansion in $E_{\text{IR}}/E_{\text{max}}$
- Ensure separation of scales at each order

Approach: Match the transition matrix

$$\langle f|\,T\,|i\rangle_{\text{full}} = \langle f|\,T\,|i\rangle_{\text{eff}}$$

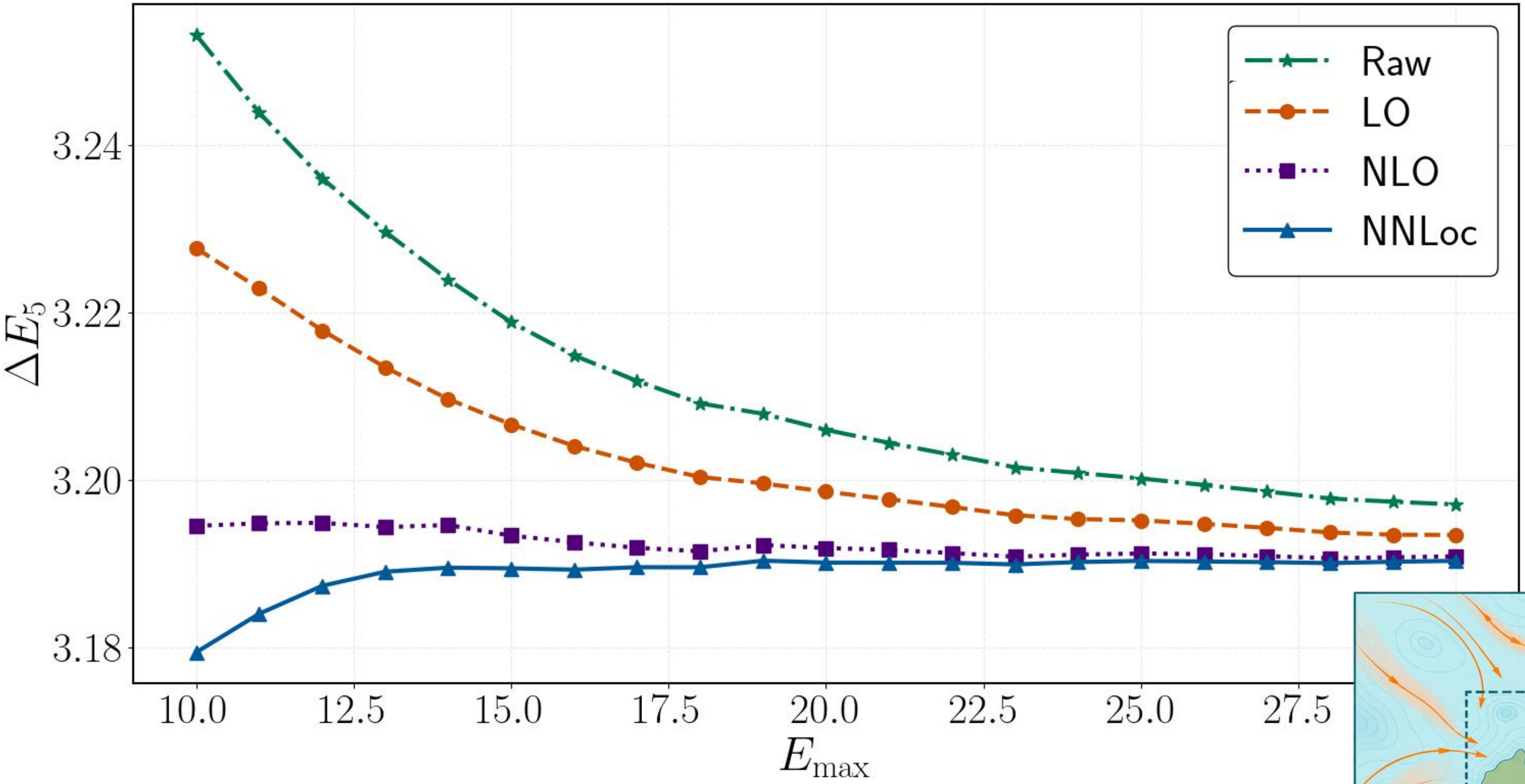
HTET expansion: mixed power counting

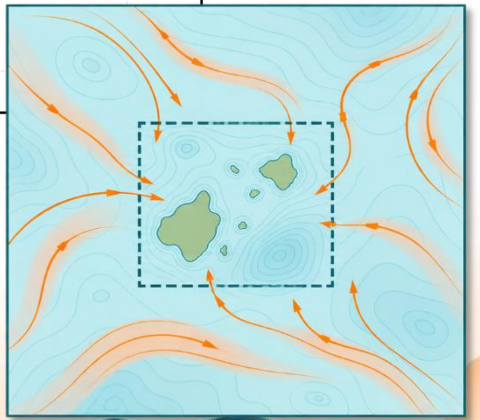
A. Maestri, S. Rodini, B. Pasquini, *Phys. Rev. D 114, 016008 (2026)*

$1/E_{\max}$	$\mathcal{O}(V^2)$	$\mathcal{O}(V^3)$	$\mathcal{O}(V^4)$
LO $\mathcal{O}(E_{\max}^{-2})$	local		
NLO $\mathcal{O}(E_{\max}^{-3})$	1st nonlocal		
$\mathcal{O}(E_{\max}^{-4})$	NNLoc: 2nd non local	local	
$\mathcal{O}(E_{\max}^{-5})$	3rd nonlocal	1st nonlocal	
$\mathcal{O}(E_{\max}^{-6})$	4th nonlocal	2nd nonlocal	local
	$\vdots$	$\vdots$	$\vdots$

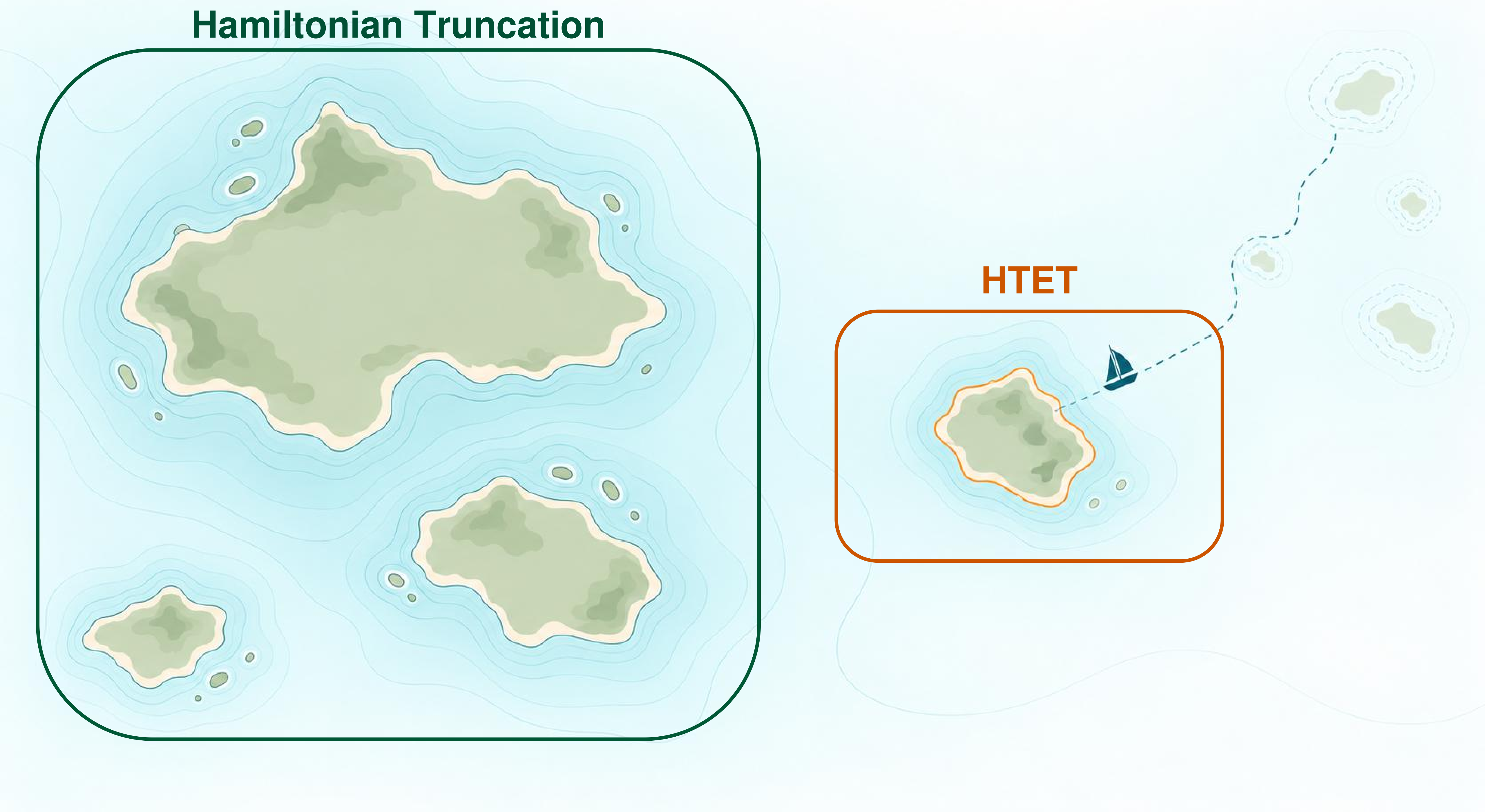
Numerical Results

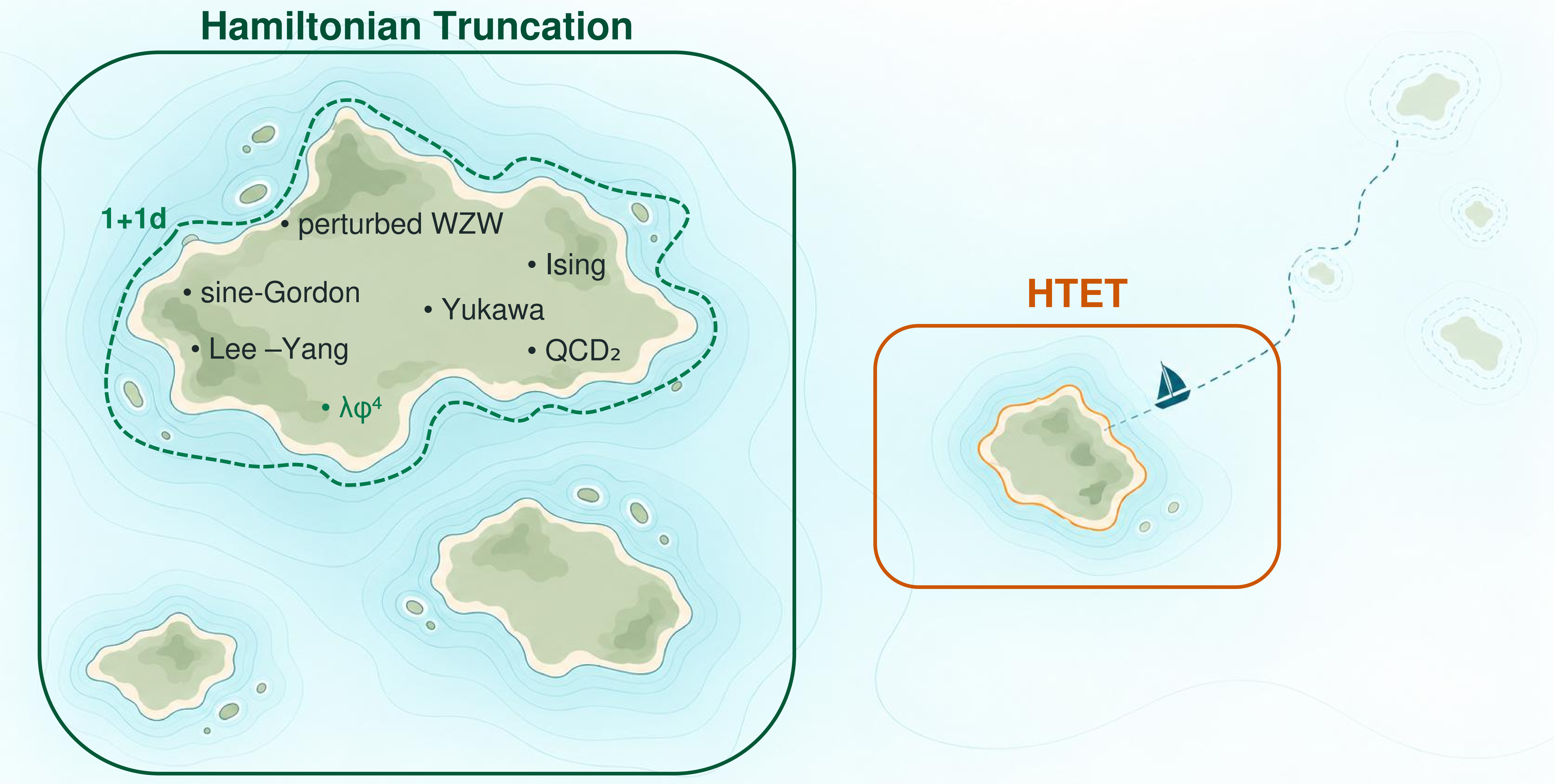
Parameters: $m^2 = 1,\ 2\pi R = 10,\ \lambda = 4\pi$



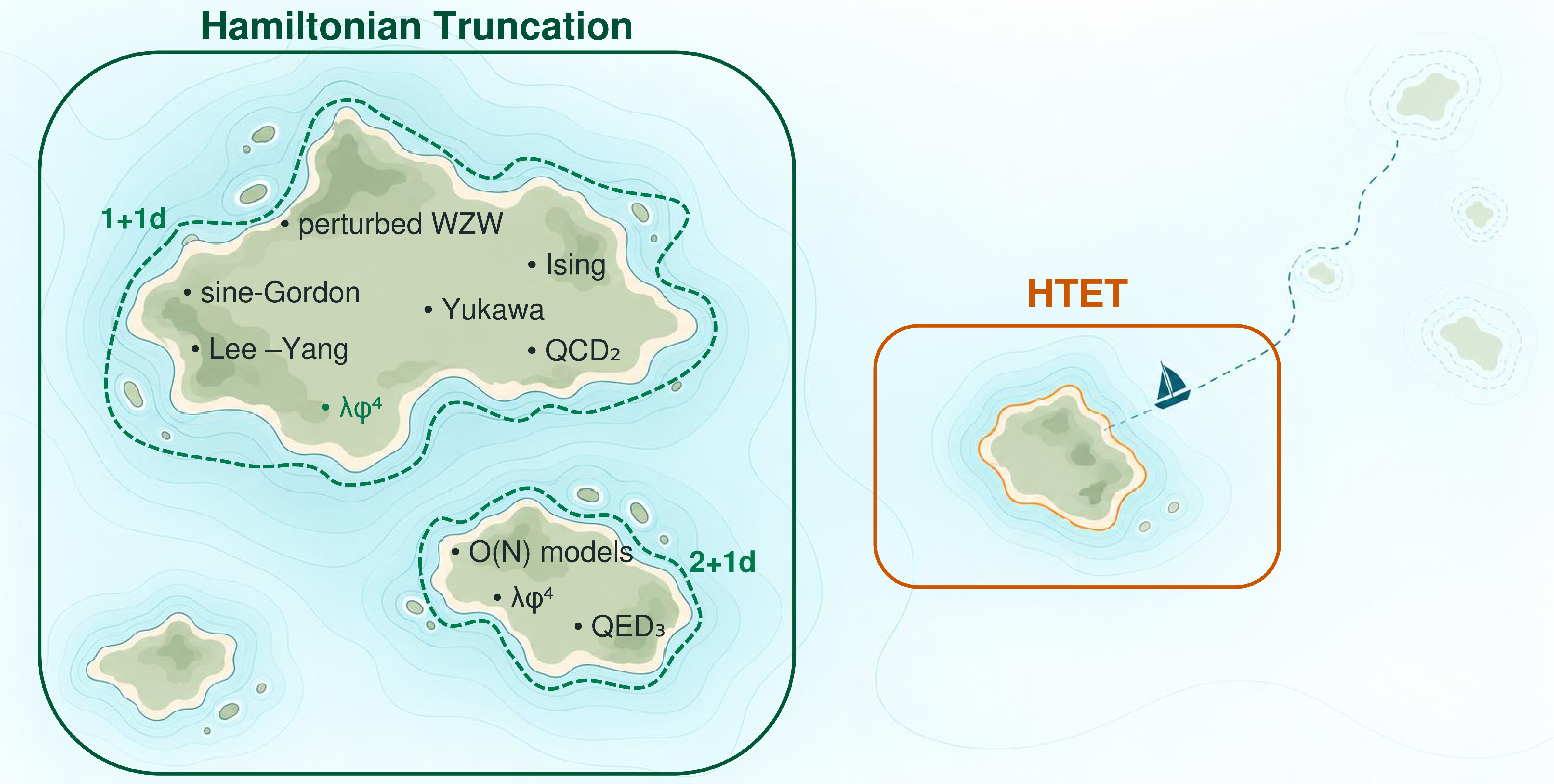


- Effective corrections flatten the E_{max} dependence


Hamiltonian Truncation
HTET



Hamiltonian Truncation
1+1d
• perturbed WZW
• Ising
• sine-Gordon
• Yukawa
• Lee –Yang
• QCD$_2$
• $\lambda\phi^4$
HTET



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• O(N) models
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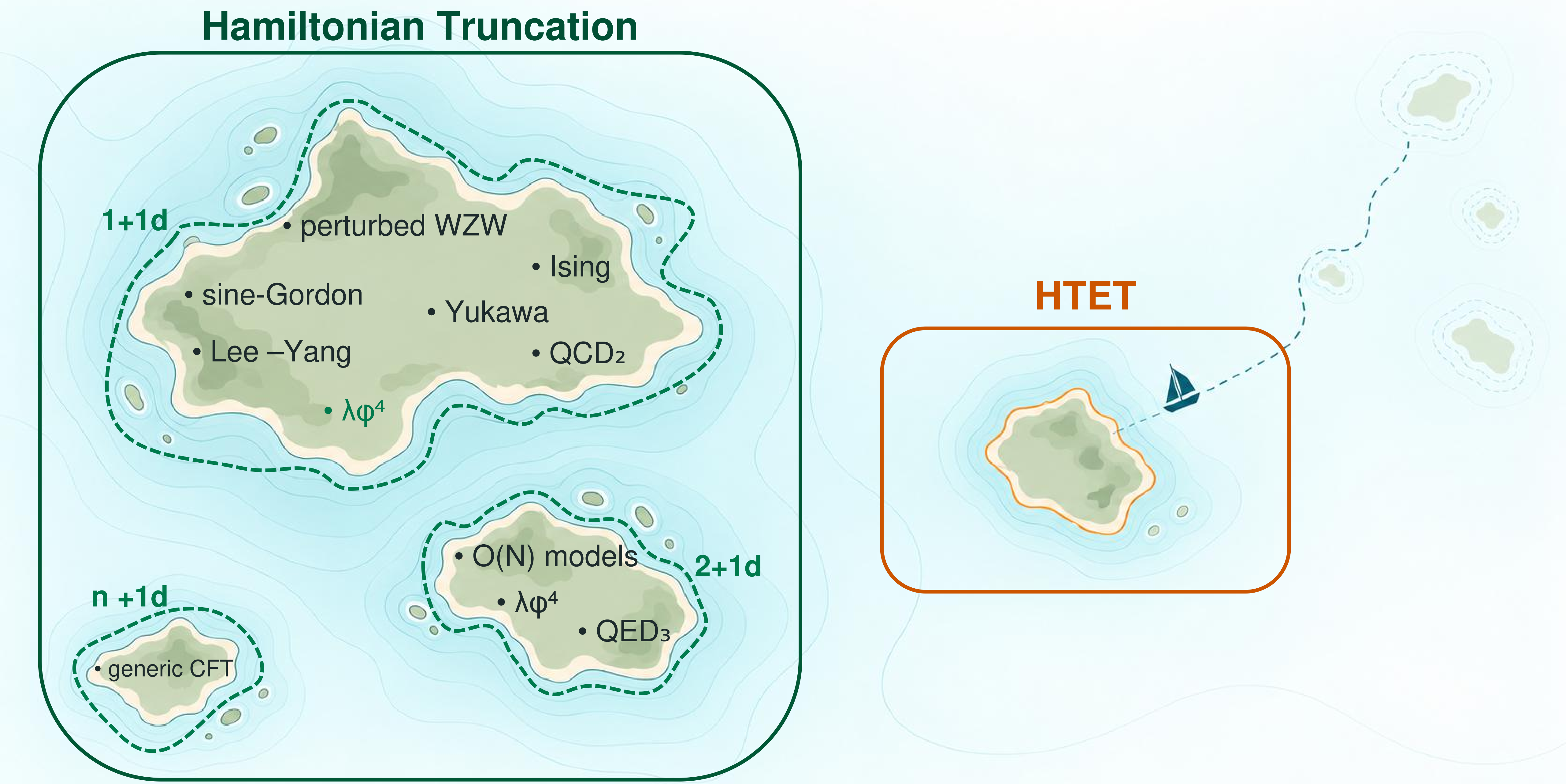
• QED₃
HTET



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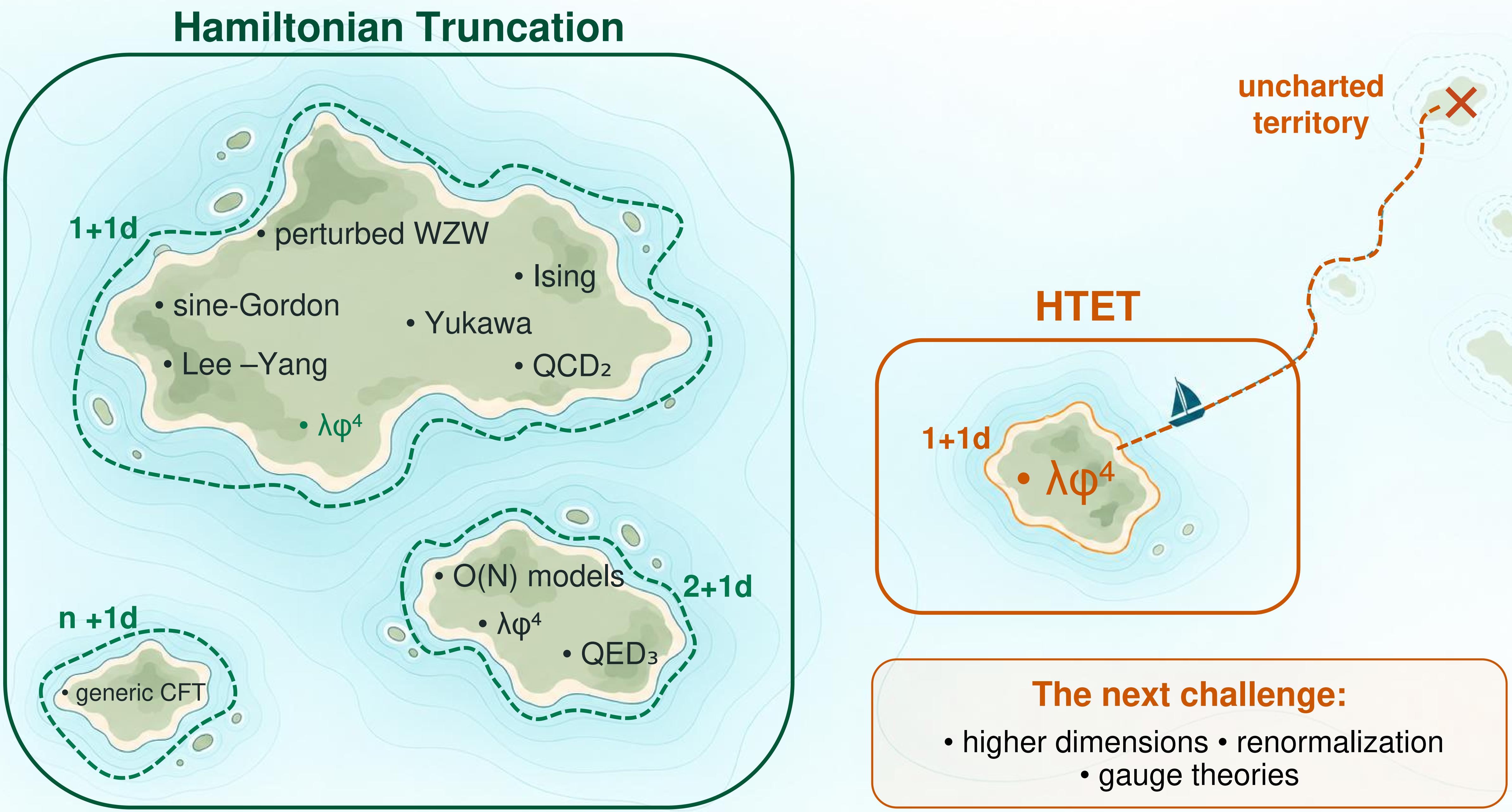
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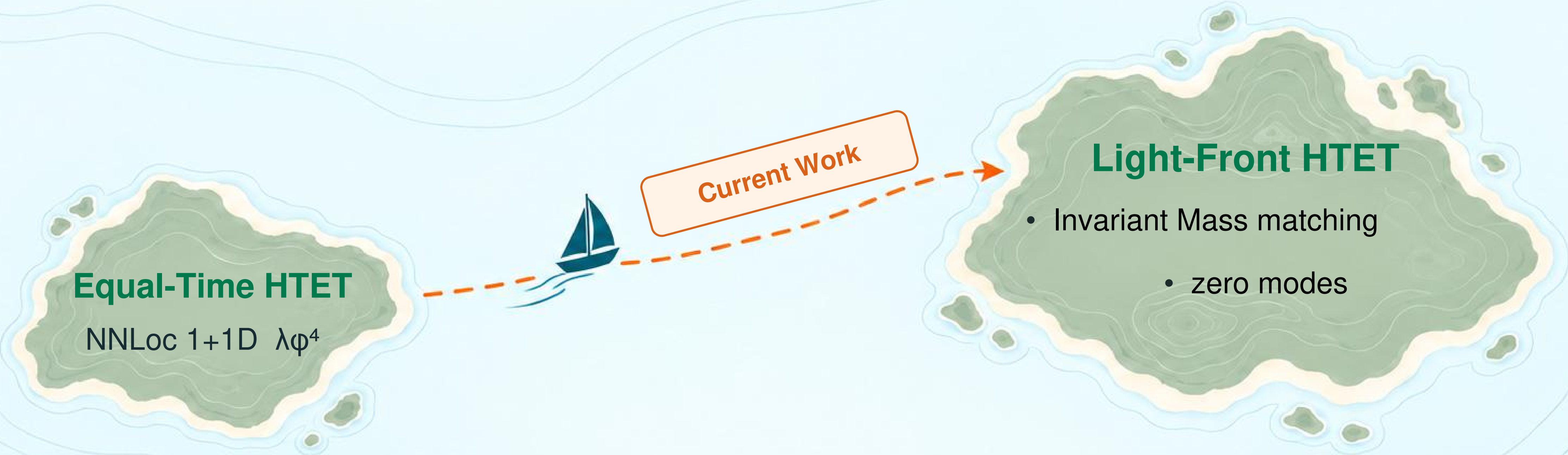
HTET
1+1d
• λφ⁴
uncharted
territory
The next challenge:
• higher dimensions • renormalization
• gauge theories

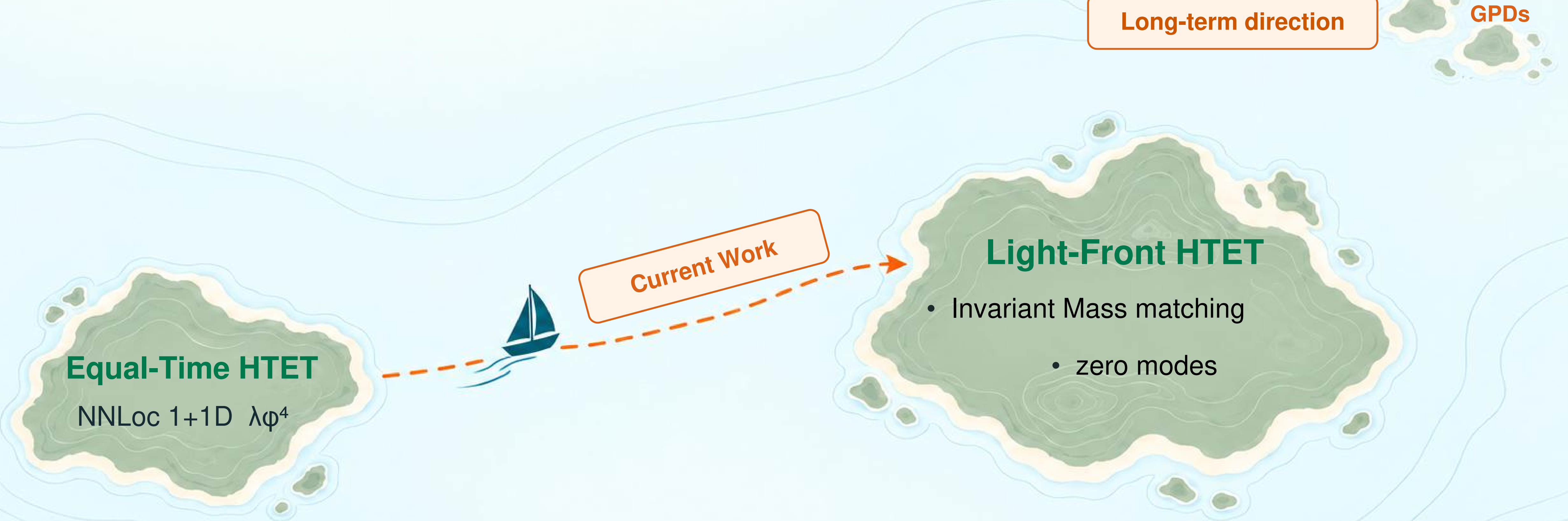
My PhD route

Extending HTET to the Light Front



My PhD route

Extending HTET to the Light Front



From effective spectra to effective partonic observables.

Thanks for your attention!

Questions?

Backup

What are these theories?

One-line glossary for the Hamiltonian Truncation landscape

1+1D — the mature territory

- **Lee–Yang**
  Non-unitary minimal model $M(2,5)$ deformed into a massive 1+1D QFT.
- **sine-Gordon**
  Real scalar field with a periodic $\cos(\beta\varphi)$ potential; supports solitons
- **Ising field theory**
  Scaling limit of the 2D Ising model, perturbed thermally or magnetically.
- **$\lambda\varphi^4$**
  Real scalar with quartic self-interaction and a Z_2 symmetry; prototype interacting QFT.
- **Perturbed WZW**
  Wess–Zumino–Witten CFT with non-Abelian current symmetry, deformed by relevant/marginal operators.
- **Yukawa / SUSY GNY**
  Fermions coupled to a scalar through a Yukawa interaction; supersymmetric variants also exist.
- **QCD_2**
  $SU(N_c)$ gauge theory in 1+1D; simpler than 4D QCD but still exhibits confinement.

2+1D

- **O(N) model**
  N scalar fields with global O(N) symmetry and quartic interactions.
- **$\lambda\varphi^4$ / 3D Ising**
  Three-dimensional scalar quartic theory in the 3D Ising universality class.
- **QED_3**
  U(1) gauge theory with fermionic matter in 2+1 dimensions.

Generic d

- **Generic CFT deformations**
  Not one specific theory: start from a known UV conformal field theory and perturb it by relevant operators.

Definitions only — no HT methodology or results shown.

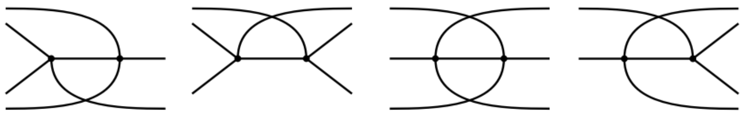
General picture of Light-front HTET

Current work: operator basis, zero-mode effects, and numerical tests in DLCQ

Clear EFT regime

$$m^2 \ll \Lambda^2 \ll m^2 K$$

New diagrams



Zero-mode effects

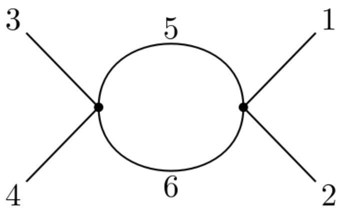
S.S. Chabysheva, J.R. Hiller, arXiv:2502.01775 (2025).

$$M_{\text{eff}}^2 = M_0^2 + M_{\text{int}}^2 + \Delta M_{\text{HTET}}^2 + \Delta M_{\text{ZM}}^2 + \cdots$$

Goal

Reach the low-energy spectrum with fewer DLCQ states by integrating out the high-invariant-mass sector

Example of a diagrammatic computation



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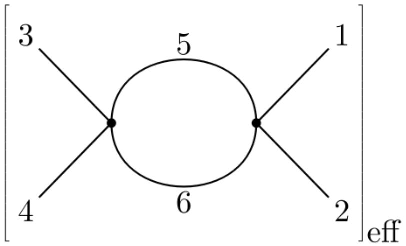

$$= \frac{1}{8}\left(\frac{\lambda}{2\pi R}\right)^2 \sum_{1,\ldots,6} \frac{\delta_{12,56}\delta_{34,56}}{2\omega_5\, 2\omega_6} \frac{\langle f|\, \phi_4^{(-)}\phi_3^{(-)}\phi_2^{(+)}\phi_1^{(+)}\, |i\rangle}{\omega_3+\omega_4-\omega_5-\omega_6}$$
$$\times\, \Theta(\mathcal{E}_f - E_{\rm max} - \omega_3 - \omega_4 + \omega_5 + \omega_6)$$

Allows systematic expansion in $E_{\rm IR}/E_{\rm max}$

First Order: **Local approximation**

$$\simeq -\frac{\lambda^2}{128\pi R}\int dx\, \langle f|[\phi^{(-)}]^2[\phi^{(+)}]^2|i\rangle \sum_k \frac{\Theta(-E_{\rm max}+2\omega_k)}{\omega_k^3} + \mathcal{O}\left(\frac{\mathcal{E}_{i,f}}{E_{\rm max}}\right)$$

$$\Theta(X+\mathcal{E}_f) = \Theta(X) + \mathcal{E}_f\,\delta(X) + \boxed{\frac{\mathcal{E}_f^2}{2}\delta'(X)} + \cdots$$

Third-order expansion: NNLoc Corrections

Structure and contributions of NNLoc corrections in the four-external-leg sector

1. Energy Monomials

$$E_f^m\ E_i^l \longrightarrow H_0^m : \phi^4 : H_0^l$$

$$m + l \leq 2$$

Nonlocality: dependence on spectator energies through H_0

$$H_0 : \phi^4 : H_0,\ \ H_0^2 : \phi^4 :,\ \ : \phi^4 : H_0^2$$

2. External Frequencies

New!

$$\omega_p^2\ \phi_p^{\pm} \longrightarrow -\partial_t^2 \phi_p^{\pm} \longrightarrow [H_0, [H_0, \phi_p^{\pm}]]$$

$$\Omega_4 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2$$

NNLoc corrections feature irreducible dependence on individual external legs energy

$$\Omega_4 : \phi^4 :$$

Take-home message:

A systematic HTET expansion is possible, but each higher order in the cutoff expansion requires an increasingly rich operator basis